Starter 1.1

1. Find the value of the expression $x^2 + 4x + 4$ if $x = -2$.
2. Solve $5n + 6 = -3n - 10$.
3. Evaluate $|x - 2y| - |2x - y| - xy$ if $x = -2$ and $y = 7$.
4. Factor $8xy^2 - 4xy$.
5. Simplify $\frac{12x^5y^3}{21xy^4}$.

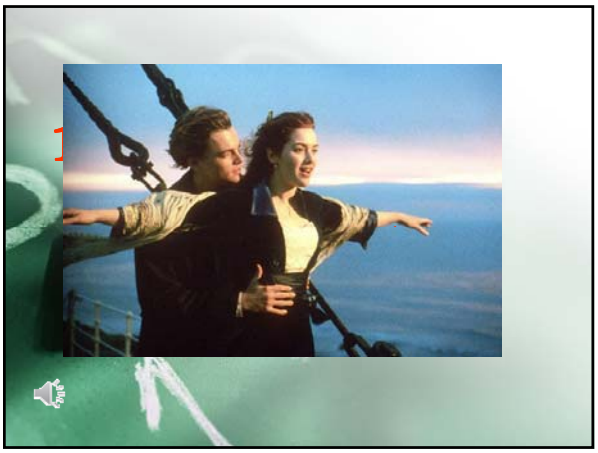
1) 0

2) -2

3) 19

4) $4xy(2y-1)$

5) $\frac{4x^4}{7y}$



1.1: Relations & Functions

You Never Escape Your...

Relations

You Never Escape Your...

Relations

You Never Escape Your...

Relations

Relation

- **A pairing of elements of one set with elements of a second set.**
- A set of ordered pairs. The set of first elements in the ordered pairs is the **domain**, while the set of second elements is the **range**.

$$(x, y)$$

Domain **– the set of first elements**
Range **– the set of second elements**

Relation

Example 1:

State the domain and range of the following relation.

{(5, 2), (30, 8), (15, 3), (17, 6), (14, 9)}

Relation

Example 2:

The domain of a relation is all odd positive integers less than 9. The range *y* of the relation is 3 more than *x*, where *x* is a member of the domain.

1. **Write the relation as a table of values and as an equation.**
2. **Then graph the relation.**

Table:

x	*y*
1	4
3	6
5	8
7	10

Equation:
$y = x + 3$

Graph:

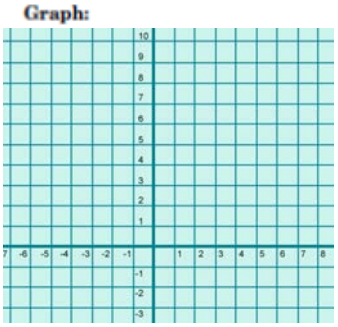
Relation

Table:

x	*y*
1	4
3	6
5	8
7	10

Equation:
$y = x + 3$

Graph:

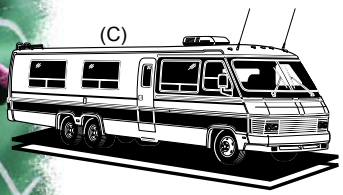


Relation

(A)

(B)

(C)



(1) 32 mpg
(2) 8 mpg
(3) 16 mpg

Domain and Range

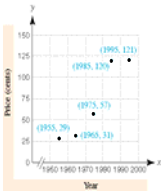
- The ***domain*** of a relation is the set of all inputs, or ***x***-coordinates, of the ordered pairs.
- The values that make up the set of ***independent*** values are the ***domain.***
- The ***range*** of a relation is the set of all outputs, or ***y***-coordinates, of the ordered pairs.
- The values that make up the set of ***dependent*** values are the ***range***.

Representations

- **Set of ordered pairs**
  (3, 5), (4, 7), (8, 15)
- **Table of values**

x	*y*
3	5
4	7
8	15

- **Pictorial representation /graph**



- **Rule or equation** $y = 3x + 4$

Domain and Range

- **Find the Domain and Range**
  (-2, 4), (-1, 1), (0,0), (1,1), (2,4)
- **For the domain, grab the *x*'s**
  Domain: {-2, -1, 0, 1, 2}
- **For the range, grab the *y*'s**
  Range: {0, 1, 4}

Function

- **A relation in which each element of the domain is paired with exactly ONE element in the range.**
  - A function is a relation in which the members of the domain (*x*-values) **DO NOT** repeat.
  - Each element of the domain is mapped to **one and only one element of the range**.
  - So, for every *x*-value there is only one *y*-value that corresponds to it.
  - *y*-values can be repeated.

Function

- If we think of the ***domain*** as the set of boys and the ***range*** the set of girls, then a **function** is a **monogamous** relationship from the domain to the range. Each boy gets to go out with one and only one girl.

  But… It does not say anything about the girls. Maybe they get to live in Utah.

Do the ordered pairs represent a function?

{(3, 4), (7, 2), (0, -1), (-2, 2), (-5, 0), (3, 3)}

No, 3 is repeated in the domain.

{(4, 1), (5, 2), (8, 2), (9, 8)}

Yes, no *x*-coordinate is repeated.

Function Machine Graphic

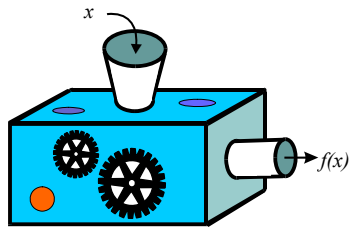


Diagram of a function

Demonstrating the Process

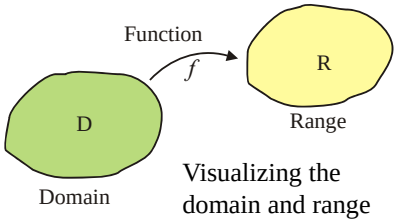
How to represent a function?

- By words: A function is a relation that for each element of a given set (called the ***domain***) associates exactly one value in another set. The set of values obtained is called the **range**.

Too abstract?

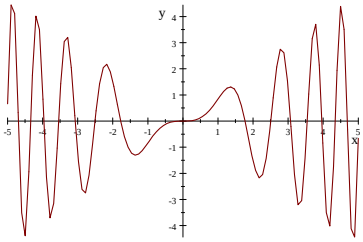
Visualizing a function

Mapping form



Visualizing the domain and range

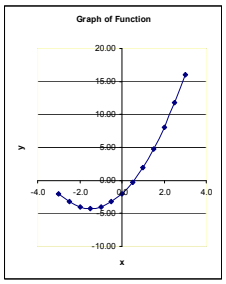
Graphical (if there is a graph)



Tabular → Graphical

x	y
-3.0	-2.00
-2.5	-3.25
-2.0	-4.00
-1.5	-4.25
-1.0	-4.00
-0.5	-3.25
0.0	-2.00
0.5	-0.25
1.0	2.00
1.5	4.75
2.0	8.00
2.5	11.75
3.0	16.00

→



Connecting: Data and Graphic

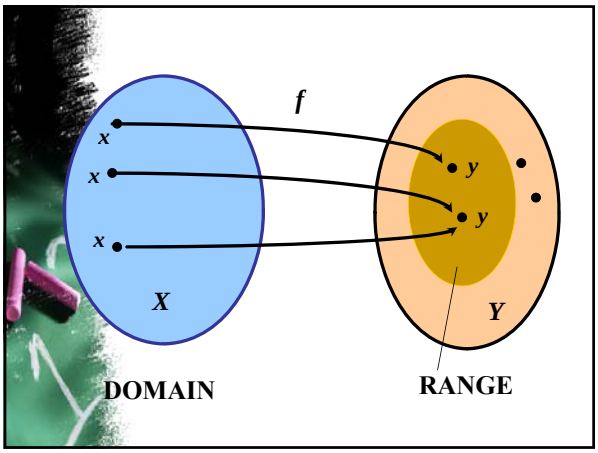
Formula-type forms

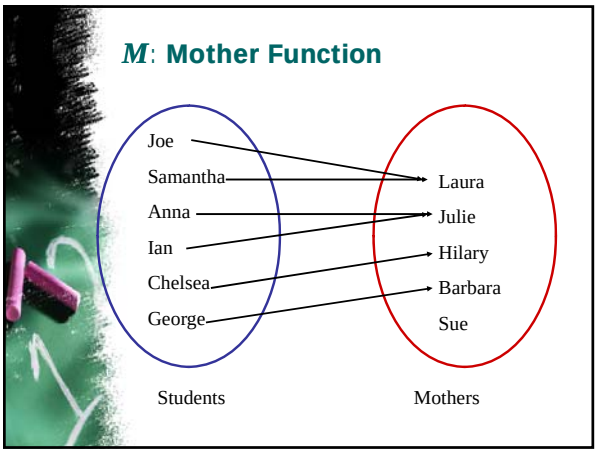
- Function Form

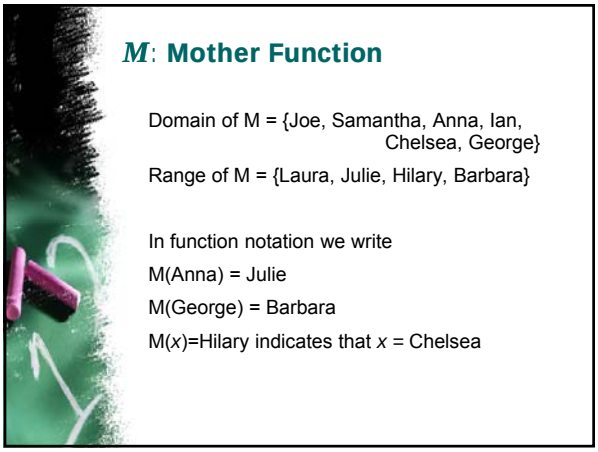
$$f(x) = x\sin(x^2)$$

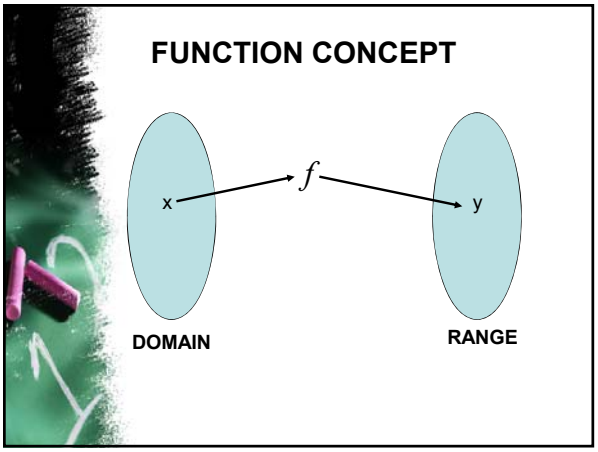
- Domain-Range form (*x*-*y* form)

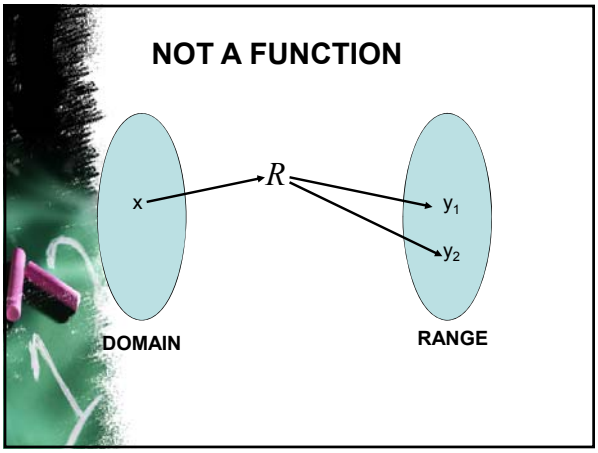
$$y = x^4 - 3x^3 + 6x^2 - 7$$

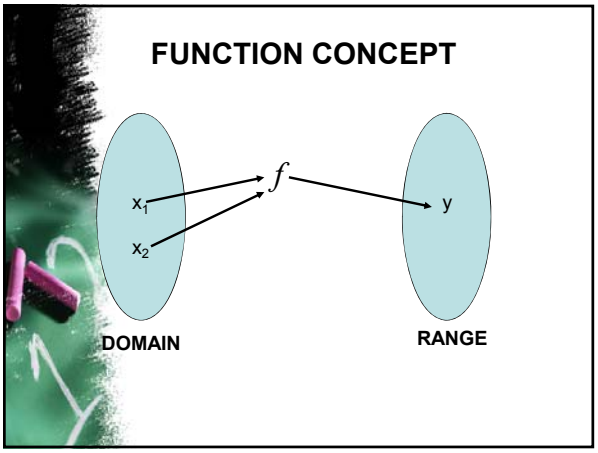

f
x
x
x
y
y
X
Y
DOMAIN
RANGE


M: Mother Function
Joe
Samantha
Anna
Ian
Chelsea
George
Laura
Julie
Hilary
Barbara
Sue
Students
Mothers


M: Mother Function
Domain of M = {Joe, Samantha, Anna, Ian, Chelsea, George}
Range of M = {Laura, Julie, Hilary, Barbara}
In function notation we write
M(Anna) = Julie
M(George) = Barbara
M(x)=Hilary indicates that x = Chelsea


FUNCTION CONCEPT
x
f
y
DOMAIN
RANGE


NOT A FUNCTION
x
R
y1
y2
DOMAIN
RANGE


FUNCTION CONCEPT
x1
x2
f
y
DOMAIN
RANGE

Examples

- Decide if the following relations are functions.

x	y
1	2
-5	7
-1	2
3	3

x	y
1	1
-5	1
-1	1
3	1

x	y
1	2
1	7
1	2
1	3

x	y
1	π
π	1
-1	5
π	3

Ways to Represent a Function

- **Symbolic**

$$\{(x,y) | y = 2x\}$$
or
$$y = 2x$$

- **Graphical**

- **Numeric**

X	Y
1	2
5	10
-1	-2
3	6

- **Verbal**
  The cost is twice the original amount.

Function Notation

- When we know that a relation is a function, the "*y*" in the equation can be replaced with ***f*(*x*)**.
- ***f*(*x*)** is simply a notation to designate a function. It is pronounced '***f* of *x***'.
- The '***f***' names the function, the '***x***' tells the variable that is being used.

Function Notation: The Symbolic Form

- A truly excellent notation. It is concise and useful.

$$y = f(x)$$

$$y = f(x)$$

Name of the function

- Output Value
- Member of the Range
- Dependent Variable

These are all equivalent names for the *y*.

- Input Value
- Member of the Domain
- Independent Variable

These are all equivalent names for the *x*.

Example of Function Notation

- The *f* notation

$$f(x) = \sqrt{x+1}$$

$$f(2) = \sqrt{(2)+1}$$

Value of a Function

Since the equation $y = x - 2$ represents a function, we can also write it as $f(x) = x - 2$.

Find $f(4)$:

$f(4) = 4 - 2$

$f(4) = 2$

Value of a Function

If $g(s) = 2s + 3$, find $g(-2)$.

$g(-2) = 2(-2) + 3$

$= -4 + 3$

$= -1$

$g(-2) = -1$

Value of a Function

If $h(x) = x^2 - x + 7$, find $h(2c)$.

$h(2c) = (2c)^2 - (2c) + 7$

$= 4c^2 - 2c + 7$

Value of a Function

If $f(k) = k^2 - 3$, find $f(a - 1)$

$f(a - 1) = (a - 1)^2 - 3$

(Remember FOIL?!)

$= (a - 1)(a - 1) - 3$

$= a^2 - a - a + 1 - 3$

$= a^2 - 2a - 2$

State the Domain of each function

1) $f(x) = 3x^2$

2) $g(x) = \dfrac{x^3 + 5x}{x^2 - 4x}$

3) $h(x) = \dfrac{x^2 + 2x}{x^2 + 5}$

4) $f(x) = \sqrt{x}$

5) $g(x) = \sqrt{x - 6}$

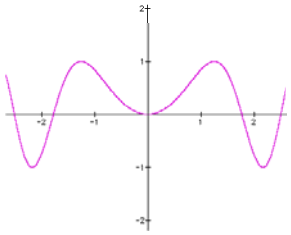
6) $f(x) = \sqrt{x^2 - 4}$

Graphical Representation

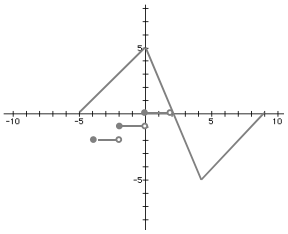
- Graphical representation of functions have the advantage of conveying lots of information in a compact form. There are many types and styles of graphs but in algebra we concentrate on graphs in the rectangular (Cartesian) coordinate system.

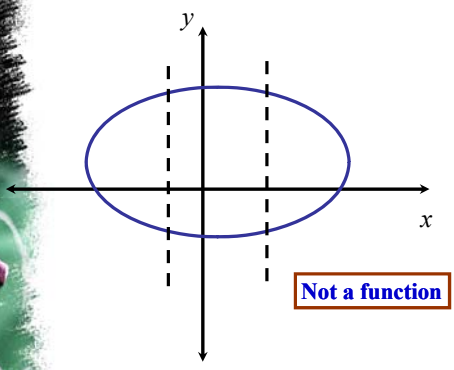
Vertical Line Test for Functions

- If a vertical line is passed over the graph and it intersects the graph in exactly one point for each element of the domain, then the graph represents a ***function***.

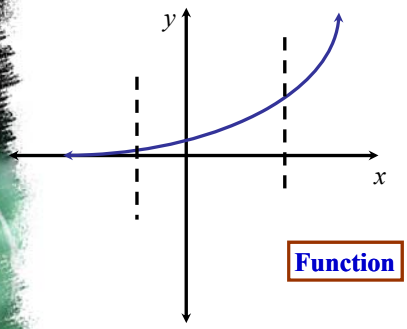


Determine the Domain and Range for Each Function From Their Graph



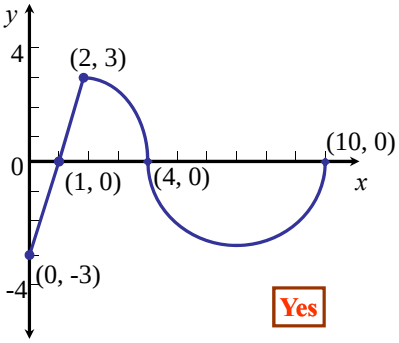


Not a function



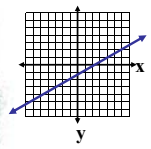
Function

Is this a graph of a function?



Yes

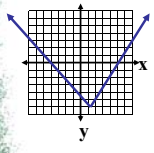
Does the graph represent a function? Name the domain and range.



Yes

D: all reals

R: all reals

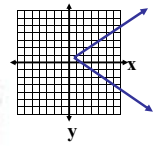


Yes

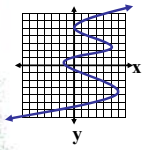
D: all reals

R: $y \geq -6$

Does the graph represent a function? Name the domain and range.

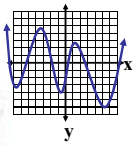


No
D: $x \geq 1/2$
R: all reals

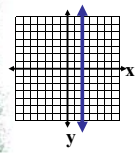


No
D: all reals
R: all reals

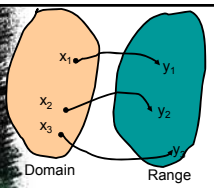
Does the graph represent a function? Name the domain and range.



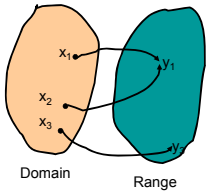
Yes
D: all reals
R: $y \geq -6$



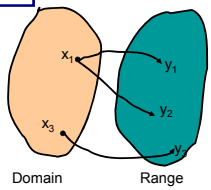
No
D: $x = 2$
R: all reals



One-to-one function

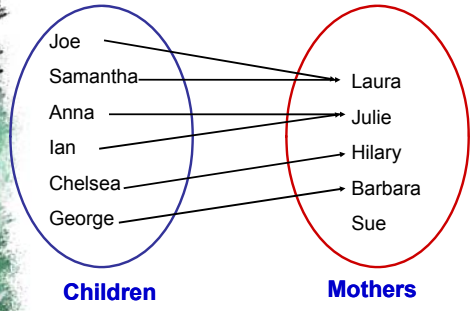


NOT One-to-one function

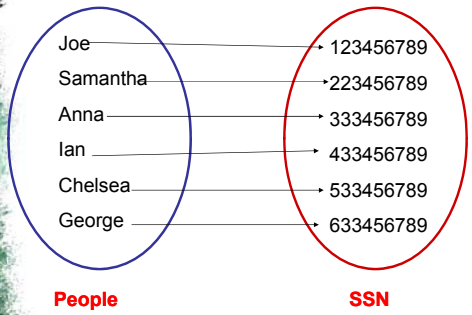


Not a function

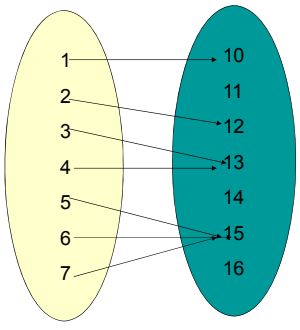
M: Mother Function is **NOT** one-one



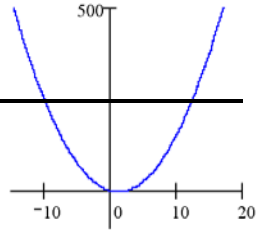
S: Social Security function **IS** one-one



Is the function f below one-to-one?



Use the graph to determine whether the function $f(x) = 2x^2 - 5x + 1$ is one-to-one.

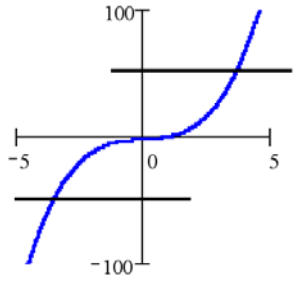


Not one-to-one.

Theorem Horizontal Line Test

If horizontal lines intersect the graph of a function ***f*** in at most one point, then ***f*** is one-to-one.

Use the graph to determine whether the function $f(x) = x^3 + x - 2$ is one-to-one.

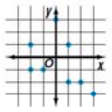


One-to-one

CW 1.1

Read and study the lesson to answer each question.

1. **Represent** the relation $\{(-4, 2), (6, 1), (0, 5), (8, -4), (2, 2), (-4, 0)\}$ in two other ways.
2. **Draw** the graph of a relation that is not a function.
3. **Describe** how to use the vertical line test to determine whether the graph at the right represents a function.



CW 1.1

4. **You Decide** Keisha says that all functions are relations but not all relations are functions. Kevin says that all relations are functions but not all functions are relations. Who is correct and why?

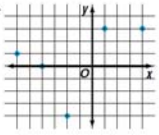
5. The domain of a relation is all positive integers less than 8. The range y of the relation is x less 4, where x is a member of the domain. Write the relation as a table of values and as an equation. Then graph the relation.

State each relation as a set of ordered pairs. Then state the domain and range.

6.

x	y
-3	4
0	0
3	-4
6	-8

7.



CW 1.1

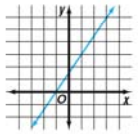
Given that x is an integer, state the relation representing each equation by making a table of values. Then graph the ordered pairs of the relation.

8. $y = 3x + 5$ and $-4 \le x \le 4$
9. $y = -5$ and $1 \le x \le 8$

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*. Explain.

10. $\{(1, 2), (2, 4), (-3, -6), (0, 0)\}$
11. $\{(6, -2), (3, 4), (6, -6), (-3, 0)\}$

12. Study the graph at the right.
    a. State the domain and range of the relation.
    b. State whether the graph represents a function. Explain.



Evaluate each function for the given value.

13. $f(-3)$ if $f(x) = 4x^3 + x^2 - 5x$
14. $g(m + 1)$ if $g(x) = 2x^2 - 4x + 2$
15. State the domain of $f(x) = \sqrt{x + 1}$.

CW 1.1

16. Sports The table shows the heights and weights of members of the Los Angeles Lakers basketball team during a certain year.

a. State the relation of the data as a set of ordered pairs. Also state the domain and range of the relation.

b. Graph the relation.

c. Determine whether the relation is a function.

Los Angeles Lakers

Height (in.)	Weight (lb)
83	240
81	220
82	245
78	200
83	255
73	200
80	215
77	210
78	190
73	180
86	300
77	220
82	260

Source: Preview Sports

CW 1.1

Write each relation as a table of values and as an equation. Graph the relation.

17. the domain is all positive integers less than 10, the range is 3 times x, where x is a member of the domain

18. the domain is all negative integers greater than -7, the range is x less 5, where x is a member of the domain

19. the domain is all integers greater than -5 and less than or equal to 4, the range is 8 more than x, where x is a member of the domain

State each relation as a set of ordered pairs. Then state the domain and range.

20.

x	y
−5	−5
−3	−3
−1	−1
1	1

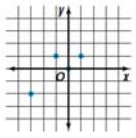
21.

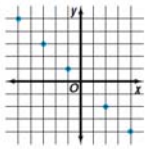
x	y
−10	0
−5	0
0	0
5	0

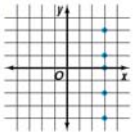
22.

x	y
4	0
5	1
8	0
13	1

CW 1.1

23. 

24. 

25. 

Given that x is an integer, state the relation representing each equation by making a table of values. Then graph the ordered pairs of the relation.

26. $y = x - 5$ and $-4 \le x \le 1$

27. $y = -x$ and $1 \le x < 7$

28. $y = |x|$ and $-5 \le x \le 1$

29. $y = 3x - 3$ and $0 < x < 6$

30. $y^2 = x - 2$ and $x = 11$

31. $|2y| = x$ and $x = 4$

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*. Explain.

32. $\{(4, 4), (5, 4), (6, 4)\}$

33. $\{(1, -2), (1, 4), (1, -6), (1, 0)\}$

34. $\{(4, -2), (4, 2), (1, -1), (1, 1), (0, 0)\}$

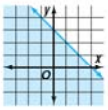
35. $\{(0, 0), (2, 2), (2, -2), (5, 8), (5, -8)\}$

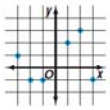
CW 1.1

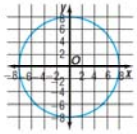
36. $\{(-1.1, -2), (-0.4, -1), (-0.1, -1)\}$

37. $\{(2, -3), (9, 0), (8, -3), (-9, 8)\}$

For each graph, state the domain and range of the relation. Then explain whether the graph represents a function.

38. 

39. 

40. 

CW 1.1

Evaluate each function for the given value.

41. $f(3)$ if $f(x) = 2x + 3$

42. $g(-2)$ if $g(x) = 5x^2 + 3x - 2$

43. $h(0.5)$ if $h(x) = \frac{1}{x}$

44. $j(2a)$ if $j(x) = 1 - 4x^3$

45. $f(n - 1)$ if $f(x) = 2x^2 - x + 9$

46. $g(b^2 + 1)$ if $g(x) = \frac{3 - x}{5 + x}$

47. Find $f(5m)$ if $f(x) = |x^2 - 13|$.

State the domain of each function.

48. $f(x) = \frac{3x}{x^2 - 5}$

49. $g(x) = \sqrt{x^2 - 9}$

50. $h(x) = \frac{x + 2}{\sqrt{x^2 - 7}}$

51. You can use the table feature of a graphing calculator to find the domain of a function. Enter the function into the Y= list. Then observe the y-values in the table. An error indicates that an x-value is excluded from the domain. Determine the domain of each function.

a. $f(x) = \frac{3}{x - 1}$

b. $g(x) = \frac{3 - x}{5 + x}$

c. $h(x) = \frac{x^2 - 12}{x^2 - 4}$

52. Education The table shows the number of students who applied and the number of students attending selected universities.

a. State the relation of the data as a set of ordered pairs. Also state the domain and range of the relation.