1-1 Sets of Numbers

Starter 1.1

Write in decimal form.

1. $-\frac{9}{2}$ **−4.5**   **2.** $\frac{2}{3}$ $0.\overline{6}$

3. Write $\sqrt{2}$ as a decimal approximation.

≈1.414

Order from least to greatest.

4. 10, −5, −10, 0, 5 **−10, −5, 0, 5, 10**

5. 0.1, 1, 1.1, 0.01, 0.11, 0.009

0.009, 0.01, 0.1, 0.11, 1, 1.1

1-1 Sets of Numbers

Objective

Classify and order real numbers.

1-1 Sets of Numbers

Vocabulary

set	finite set
element	infinite set
subset	interval notation
empty set	set-builder notation
roster notation	

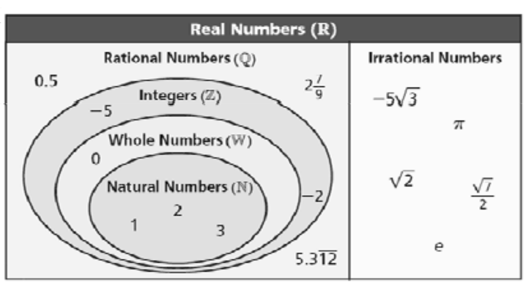
1-1 Sets of Numbers

A **set** is a collection of items called **elements**. The rules of 8-ball divide the set of billiard balls into three *subsets:* solids (1 through 7), stripes (9 through 15), and the 8 ball.

A **subset** is a set whose elements belong to another set. The **empty set**, denoted ∅, is a set containing no elements.

1-1 Sets of Numbers

The diagram shows some important subsets of the real numbers.



1-1 Sets of Numbers

Reading Math

Note the symbols for the sets of numbers.

ℝ: real numbers
ℚ: rational numbers
ℤ: integers
𝕎: whole numbers
ℕ: natural numbers

1-1 Sets of Numbers

Rational numbers can be expressed as a quotient (or *ratio*) of two integers, $\frac{a}{b}$ where ***b***, the denominator is not zero. The decimal form of a rational number either terminates or repeats.

Irrational numbers, such as $\sqrt{2}$ and π, *cannot* be expressed as a quotient of two integers, and their decimal forms do not terminate or repeat. However, you can approximate these numbers using terminating decimals.

1-1 Sets of Numbers

Example 1A: Ordering and Classifying Real Numbers

Consider the numbers $2.\overline{3}$, π, $\sqrt{5}$, $-\frac{11}{2}$, and 2.7652.

Order the numbers from least to greatest.

Write each number as a decimal to make it easier to compare them.

$\sqrt{5} \approx 2.23$ — *Use a decimal approximation for $\sqrt{5}$.*

$\pi \approx 3.14$ — *Use a decimal approximation for π.*

$-\frac{11}{2} \approx -5.5$ — *Rewrite $-\frac{11}{2}$ in decimal form.*

$-5.5 < 2.23 < 2.\overline{3} < 2.7652 < 3.14$ — *Use < to compare the numbers.*

The numbers in order from least to great are $-\frac{11}{2}$, $\sqrt{5}$, $2.\overline{3}$, 2.7652, and π

1-1 Sets of Numbers

Example 1B: Ordering and Classifying Real Numbers

Consider the numbers $2.\overline{3}$, π, $\sqrt{5}$, $-\frac{11}{2}$, and 2.7652.

Classify each number by the subsets of the real numbers to which it belongs.

Numbers	Real	Rational	Integer	Whole	Natural	Irrational
$2.\overline{3}$	✓	✓				
π	✓					✓
$\sqrt{5}$	✓					✓
$-\frac{11}{2}$	✓	✓				
2.7652	✓	✓				

1-1 Sets of Numbers

Check It Out! Example 1A

Consider the numbers –2, π, –0.321, $\frac{3}{2}$ and $-\sqrt{3}$.

Order the numbers from least to greatest.

Write each number as a decimal to make it easier to compare them.

$-\sqrt{3} \approx -1.313$ — *Use a decimal approximation for $-\sqrt{3}$.*

$\frac{3}{2} = 1.5$ — *Rewrite $\frac{3}{2}$ in decimal form.*

$\pi \approx 3.14$ — *Use a decimal approximation for π.*

$-2 < -1.313 < -0.321 < 1.50 < 3.14$ *Use < to compare the numbers.*

The numbers in order from least to great are –2, $-\sqrt{3}$, –0.321, $\frac{3}{2}$, and π.

1-1 Sets of Numbers

Check It Out! Example 1B

Consider the numbers –2, π, –0.321, $\frac{3}{2}$ and $-\sqrt{3}$.

Classify each number by the subsets of the real numbers to which it belongs.

Numbers	Real	Rational	Integer	Whole	Natural	Irrational
–2	✓	✓	✓			
π	✓					✓
–0.321	✓	✓				
$\frac{3}{2}$	✓	✓				
$-\sqrt{3}$	✓					✓

1-1 Sets of Numbers

There are many ways to represent sets. For instance, you can use words to describe a set. You can also use **roster notation**, in which the elements in a set are listed between braces, **{ }**.

Words	Roster Notation
The set of billiard balls is numbered 1 through 15.	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}

1-1 Sets of Numbers

A set can be *finite* like the set of billiard ball numbers or *infinite* like the natural numbers {1, 2, 3, 4 …}.

A **finite set** has a definite, or finite, number of elements.

An **infinite set** has an unlimited, or infinite number of elements.

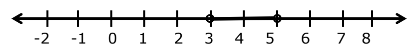
Helpful Hint

The Density Property states that between any two numbers there is another real number. So any interval that includes more than one point contains infinitely many points.

1-1 Sets of Numbers

Many infinite sets, such as the real numbers, cannot be represented in roster notation. There are other methods of representing these sets. For example, the number line represents the sets of all real numbers.

The set of real numbers between 3 and 5, which is also an infinite set, can be represented on a number line or by an inequality.

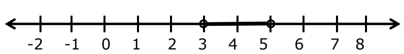


3 < x < 5

1-1 Sets of Numbers

An interval is the set of all numbers between two endpoints, such as 3 and 5. In **interval notation** the symbols [and] are used to include an endpoint in an interval, and the symbols (and) are used to exclude an endpoint from an interval.

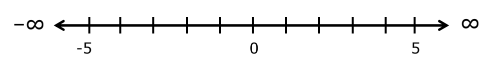
(3, 5) *The set of real numbers between but not including 3 and 5.*



3 < x < 5

1-1 Sets of Numbers

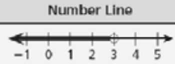
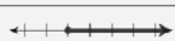
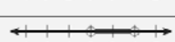
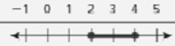
An interval that extends forever in the positive direction goes to infinity (∞), and an interval that extends forever in the negative direction goes to negative infinity (−∞).



1-1 Sets of Numbers

Because ∞ and −∞ are not numbers, they cannot be included in a set of numbers, so parentheses are used to enclose them in an interval. The table shows the relationship among some methods of representing intervals.

Methods of Representing Intervals

Words	Number Line	Inequality	Interval Notation
Numbers less than 3		$x < 3$	$(-\infty, 3)$
Numbers greater than or equal to −2		$x \geq -2$	$[-2, \infty)$
Numbers between 2 and 4		$2 < x < 4$	$(2, 4)$
Numbers 1 through 3		$1 \leq x \leq 3$	$[1, 3]$

1-1 Sets of Numbers

Example 2A: Interval Notation

Use interval notation to represent the set of numbers.

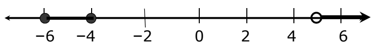
$7 < x \leq 12$

(7, 12] *7 is not included, but 12 is.*

1-1 Sets of Numbers

Example 2B: Interval Notation

Use interval notation to represent the set of numbers.



There are two intervals graphed on the number line.

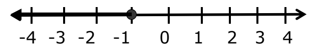
[−6, −4]	*–6 and –4 are included.*
(5, ∞)	*5 is not included, and the interval continues forever in the positive direction.*
[−6, −4] or (5, ∞)	*The word "or" is used to indicate that a set includes more than one interval.*

1-1 Sets of Numbers

Check It Out! Example 2

Use interval notation to represent each set of numbers.

a.



(−∞, −1] *–1 is included, and the interval continues forever in the negative direction.*

b. $x \le 2$ **or** $3 < x \le 11$

(−∞, 2] *2 is included, and the interval continues forever in the negative direction.*

(3, 11] *3 is not included, but 11 is.*

(−∞, 2] or (3, 11]

1-1 Sets of Numbers

Another way to represent sets is *set-builder notation.* **Set-builder notation** uses the properties of the elements in the set to define the set. Inequalities and the element symbol ∈ are often used in the set-builder notation. The set of striped-billiard-ball numbers, or {9, 10, 11, 12, 13, 14, 15}, is represented in set-builder notation on the following slide.

1-1 Sets of Numbers

The set of all numbers *x* such that *x* has the given properties

$\{x \mid 8 < x \le 15 \text{ and } x \in \mathbb{N}\}$

Read the above as "**the set of all numbers *x* such that *x* is greater than 8 and less than or equal to 15 and *x* is a natural number**."

Helpful Hint

The symbol ∈ means "is an element of." So $x \in \mathbb{N}$ is read "*x* is an element of the set of natural numbers," or "*x* is a natural number."

1-1 Sets of Numbers

Some representations of the same sets of real numbers are shown.

Methods of Set Notation

Words	Roster Notation	Interval Notation	Set-Builder Notation
All real numbers except 1	Cannot be written in roster notation	(−∞, 1) or (1, ∞)	$\{x \mid x \ne 1\}$
Positive odd numbers	{1, 3, 5, 7,...}	Cannot be notated using interval notation	$\{x \mid x = 2n - 1 \text{ and } n \in \mathbb{N}\}$
Numbers within 3 units of 2	Cannot be written in roster notation	[−1, 5]	$\{x \mid -1 \le x \le 5\}$

1-1 Sets of Numbers

Example 3: Translating Between Methods of Set Notation

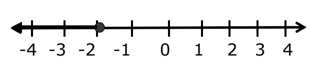
Rewrite each set in the indicated notation.

A. $\{x \mid x > -5.5, x \in \mathbb{Z}\}$**; words**

integers greater than -5.5

B. positive multiples of 10; roster notation

{10, 20, 30, ...} *The order of elements is not important.*

C.  **; set-builder notation**

$\{x \mid x \le -2\}$

1-1 Sets of Numbers

Check It Out! Example 3

Rewrite each set in the indicated notation.

a. **{2, 4, 6, 8}; words**

even numbers between 1 and 9; even numbers from 2 through 8

b. **$\{x \mid 2 < x < 8 \text{ and } x \in N\}$; roster notation**

{3, 4, 5, 6, 7} *The order of the elements is not important.*

c. **[99, ∞}; set-builder notation**

$\{x \mid x \geq 99\}$