Turn in:

- **HW 1.1 ???**
- **Signed Contract**

1.2: Composition of Functions

Starter 1.2

1. State the domain and range of the relation $\{(-2, 2), (1, 2), (2, 3)\}$.
2. Is the relation in Exercise 1 a function? Explain.

Evaluate each function for the given value $f(x)$.

3. $f(-2)$ if $f(x) = 6 - x^2$
4. $f(m)$ if $f(x) = \frac{5}{2-x}$
5. $f(3a)$ if $f(x) = \sqrt{x^2 - 4}$

1. D5 $\{-2, 1, 2\}$; R5 $\{2, 3\}$
2. Yes; each point in the domain is paired with exactly one point in the range.
3. 2
4. $\frac{5}{2-m}$
5. $\sqrt{9a^2 - 4}$

Operations with Functions

Sum of f and g: $(f+g)(x) = f(x) + g(x)$

Difference of f and g: $(f-g)(x) = f(x) - g(x)$

Product of f and g: $(f \bullet g)(x) = f(x) \bullet g(x)$

Quotient of f and g: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

Operations with Functions

Given , $f(x) = 3x^2 - 4$ and $g(x) = 4x + 5$, find each function.

1) $(f+g)(x) = f(x) + g(x)$

2) $(f-g)(x) = f(x) - g(x)$

3) $(f \bullet g)(x) = f(x) \bullet g(x)$

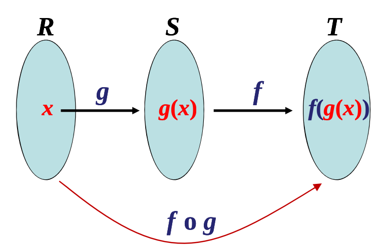
4) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

DEFINITION

Given two functions f and g, the **composite function**, denoted by $f \circ g$ (read as "f composed with g"), is defined by

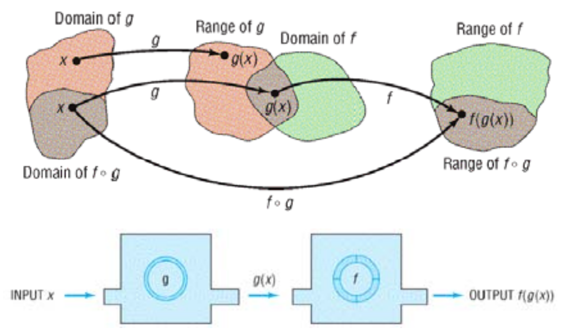
$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.



$(f \text{ o } g)(x) = f(g(x))$

Composite Function



Evaluating a Composite Function

EXAMPLE

Suppose that $f(x) = 2x^2 + 3$ $g(x) = 4x^3 + 1$. Find:

(a) $(f \circ g)(1)$ (b) $(g \circ f)(1)$ (c) $(f \circ f)(-2)$ (d) $(g \circ g)(-1)$

Finding a Composite Function

EXAMPLE

Given $f(x) = x^3$ $g(x) = x + 2$ the composite function

$$f \circ g(x) = f(g(x)) = f(x+2) = (x+2)^3 = x^3 + 6x^2 + 8x + 8$$

Replace $g(x)$ with $x+2$

Replace the variable x in the f function with $x+2$

Expand

Finding a Composite Function

EXAMPLE

Given $h(x) = \sqrt{2x-4}$ $k(x) = x^2 + 2$ the composite function

$$k \circ h(x) = k(h(x)) = k\left(\sqrt{2x-4}\right) = \left(\sqrt{2x-4}\right)^2 + 2 = (2x-4) + 2 = 2x - 2$$

The result of the function h becomes the input to k

Replace the variable x in $k(x)$ with $\sqrt{2x-4}$

Simplify

Finding a Composite Function

EXAMPLE

Now see what happens when we take the same two functions and reverse the order of the composition.

$h(x) = \sqrt{2x-4}$ $k(x) = x^2 + 2$ The composite function

$$h \circ k(x) = h(k(x)) = h(x^2+2) = \sqrt{2(x^2+2)-4} = \sqrt{2x^2+4-4} = \sqrt{2x^2} = \sqrt{2}|x|$$

Notice, the result here is **not the same** as the previous result. This is usually the case with composite functions. Changing the order of the composition (changing which function is the “inner” function and which is the “outer” function) usually changes the result.

Finding a Composite Function

EXAMPLE

For the functions $f(x) = \frac{1}{x}$ $g(x) = 3x - 5$ find

$f \circ g(x)$ (click mouse to see answer)

$$f(g(x)) = f(3x-5) = \frac{1}{3x-5}$$

$g \circ f(x)$ (click mouse to see answer)

$$g(f(x)) = g\left(\frac{1}{x}\right) = 3\left(\frac{1}{x}\right) - 5 = \frac{3}{x} - 5$$

Finding a Composite Function and Its Domain

EXAMPLE

Suppose that $f(x)=2x^2+3$ $g(x)=4x^3+1$. Find:

Find: (a) $f \circ g$ (b) $g \circ f$

Then find the domain of each composite function.

1. $g(x)$ must be defined so any x not in the domain of g must be excluded.
2. $f(g(x))$ must be defined so any x for which $g(x)$ is not in the domain of f must be excluded.

Finding a Composite Function and Its Domain

EXAMPLE

Find the domain of $(f \circ g)(x)$ if $f(x)=\frac{1}{x+4}$ and $g(x)=\frac{4}{x-2}$

Finding a Composite Function and Its Domain

EXAMPLE

Suppose that $f(x)=\frac{1}{x}$ and $g(x)=\sqrt{x-1}$

Find: (a) $f \circ g$ (b) $f \circ f$

Then find the domain of each composite function.

Showing that Two Composite Functions are Equal

EXAMPLE

If $f(x)=\frac{1}{2}(x-1)$ and $g(x)=2x+1$, show that $(f \circ g)(x)=(g \circ f)(x)=x$ for every x in the domain of $f \circ g$ and $g \circ f$.