1-5 Properties of Exponents

Starter 1.5 **HW 1.3???, Short Quiz 1.3 & 1.4**

Simplify.

1. $4 \bullet 4 \bullet 4$ 64

2. $\frac{1}{2 \cdot 2 \cdot 2 \cdot 2}$ $\frac{1}{16}$

3. $\frac{10 \cdot 10 \cdot 2}{10}$ 20

4. $\left(\frac{2}{3}\right)^2$ $\frac{4}{9}$

5. $\frac{1}{4^2}$ $\frac{1}{16}$

6. 10^5 100,000

7. 3×10^4 30,000

1-5 Properties of Exponents

Objectives

Simplify expressions involving exponents.

Use scientific notation.

1-5 Properties of Exponents

Vocabulary

scientific notation

1-5 Properties of Exponents

In an expression of the form a^n, a is the **base**, n is the **exponent**, and the quantity a^n is called a **power**. The exponent indicates the number of times that the base is used as a factor.

Base Exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a \cdot a \cdot a}_{a \text{ is a factor } n \text{ times}}$$

1-5 Properties of Exponents

When the base includes more than one symbol, it is written in parentheses.

Exponential Form	Base	Expanded Form
$-2x^3$	x	$-2(x \cdot x \cdot x)$
$-(2x)^3$	$2x$	$-(2x)(2x)(2x)$
$(-2x)^3$	$-2x$	$(-2x)(-2x)(-2x)$

Reading Math

A ***power*** includes a base and an exponent. The expression 2^3 is a power of 2. It is read "2 to the third power" or "2 cubed."

1-5 Properties of Exponents

Example 1A: Writing Exponential Expressions in Expanded Form

Write the expression in expanded form.

$(5z)^2$

$(5z)^2$ *The base is 5z, and the exponent is 2.*

$(5z)(5z)$ *5z is a factor 2 times.*

1-5 Properties of Exponents

Example 1B: Writing Exponential Expressions in Expanded Form

Write the expression in expanded form.

$-s^4$

$-s^4$ *The base is s, and the exponent is 4.*

$-(s \bullet s \bullet s \bullet s) = -s \bullet s \bullet s \bullet s$ *s is a factor 4 times.*

1-5 Properties of Exponents

Example 1C: Writing Exponential Expressions in Expanded Form

Write the expression in expanded form.

$3h^3(k + 3)^2$

$3h^3(k + 3)^2$ *There are two bases: h and k + 3.*

$3(h)(h)(h)(k + 3)(k + 3)$ *h is a factor 3 times, and k + 3 is a factor 2 times.*

1-5 Properties of Exponents

Check It Out! Example 1a

Write the expression in expanded form.

$(2a)^5$

$(2a)^5$ *The base is 2a, and the exponent is 5.*

$(2a)(2a)(2a)(2a)(2a)$ *2a is a factor 5 times.*

1-5 Properties of Exponents

Check It Out! Example 1b

Write the expression in expanded form.

$3b^4$

$3b^4$ *The base is b, and the exponent is 4.*

$3 \bullet b \bullet b \bullet b \bullet b$ *b is a factor 4 times.*

1-5 Properties of Exponents

Check It Out! Example 1c

Write the expression in expanded form.

$-(2x - 1)^3y^2$

$-(2x - 1)^3y^2$ *There are two bases: 2x–1, and y.*

$-(2x - 1)(2x - 1)(2x - 1) \bullet y \bullet y$ *2x–1 is a factor 3 times, and y is a factor 2 times.*

1-5 Properties of Exponents

Zero and Negative Exponents

For all nonzero real numbers a and integers n,

WORDS	NUMBERS	ALGEBRA
Zero Exponent Property A nonzero quantity raised to the zero power is equal to 1.	$100^0 = 1$	$a^0 = 1$
Negative Exponent Property A nonzero base raised to a negative exponent is equal to the reciprocal of the base raised to the opposite, positive exponent.	$7^{-2} = \left(\frac{1}{7}\right)^2 = \frac{1}{7^2}$ $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4$	$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

1-5 Properties of Exponents

Caution!

Do not confuse a negative exponent with a negative expression.

$$a^{-n} \neq -a^{n} \neq \frac{1}{-a^{n}}$$

1-5 Properties of Exponents

Example 2A: Simplifying Expressions with Negative Exponents

Simplify the expression.

3^{-2}

$\frac{1}{3^2}$ *The reciprocal of 3 is* $\frac{1}{3}$.

$\frac{1}{3 \cdot 3} = \frac{1}{9}$

1-5 Properties of Exponents

Example 2B: Simplifying Expressions with Negative Exponents

Simplify the expression.

$\left(\frac{2}{3}\right)^{-2}$

$\left(\frac{3}{2}\right)^{2}$ *The reciprocal of* $\left(\frac{2}{3}\right)$ *is* $\left(\frac{3}{2}\right)$.

$\frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}$

1-5 Properties of Exponents

Check It Out! Example 2a

Simplify the expression.

$\left(\frac{1}{3}\right)^{-2}$

3^2 *The reciprocal of* $\frac{1}{3}$ *is 3.*

$3 \cdot 3 = 9$

1-5 Properties of Exponents

Check It Out! Example 2b

Write the expression in expanded form.

$(-5)^{-5}$

$\left(-\frac{1}{5}\right)^{5}$ *The reciprocal of* -5 *is* $-\frac{1}{5}$.

$\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right)\left(-\frac{1}{5}\right) = -\frac{1}{3125}$

1-5 Properties of Exponents

Properties of Exponents

For all nonzero real numbers a and b and integers m and n,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$4^3 \cdot 4^2 = 4^{3+2} = 4^5$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{3^7}{3^2} = 3^{7-2} = 3^5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$(4^3)^2 = 4^{3 \cdot 2} = 4^6$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, apply the exponent to each factor.	$(3 \cdot 4)^2 = 3^2 \cdot 4^2$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, apply the exponent to the numerator and denominator.	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

1-5 Properties of Exponents

Example 3A: Using Properties of Exponents to Simplify Expressions

Simplify the expression. Assume all variables are nonzero.

$3z^7(-4z^2)$

$3 \bullet (-4) \bullet z^7 \bullet z^2$

$-12z^{7+2}$ *Product of Powers*

$-12z^9$ *Simplify.*

1-5 Properties of Exponents

Example 3B: Using Properties of Exponents to Simplify Expressions

Simplify the expression. Assume all variables are nonzero.

$$\left(\frac{yz^3}{z^5}\right)^3$$

$(yz^{3-5})^3 = (yz^{-2})^3$ *Quotient of Powers*

$y^3(z^{-2})^3$ *Power of a Product*

$y^3z^{(-2)(3)}$ *Power of a Product*

$y^3z^{-6} = \frac{y^3}{z^6}$ *Negative of Exponent Property*

1-5 Properties of Exponents

Check It Out! Example 3a

Simplify the expression. Assume all variables are nonzero.

$(5x^6)^3$

$5^3(x^6)^3$ *Power of a Product*

$125x^{(6)(3)}$ *Power of a Power*

$125x^{18}$

1-5 Properties of Exponents

Check It Out! Example 3b

Simplify the expression. Assume all variables are nonzero.

$(-2a^3b)^{-3}$

$\frac{1}{(-2a^3b)^3}$ *Negative Exponent Property*

$\frac{1}{(-2)^{(3)}a^{(3)(3)}b^{(3)}}$ *Power of a Power*

$-\frac{1}{8a^9b^3}$

1-5 Properties of Exponents

Scientific notation is a method of writing numbers by using powers of 10. In scientific notation, a number takes a form $m \times 10^n$, where $1 \le m < 10$ and n is an integer.

Scientific Notation	Move the decimal	Standard Notation
1.275×10^7	Right 7 places	12,750,000
3.5×10^{-7}	Left 7 places	0.00000035

You can use the properties of exponents to calculate with numbers expressed in scientific notation.

1-5 Properties of Exponents

Example 4A: Simplifying Expressions Involving Scientific Notation

Simplify the expression. Write the answer in scientific notation.

$$\frac{4.5\times10^{-5}}{1.5\times10^{6}}$$

$\left(\frac{4.5}{1.5}\right)\times\left(\frac{10^{-5}}{10^{6}}\right)$ $\frac{a \bullet b}{c \bullet b} = \frac{a}{c} \bullet \frac{b}{d}$

3.0×10^{-11} *Divide 4.5 by 1.5 and subtract exponents: –5 – 6 = –11.*

1-5 Properties of Exponents

Example 4B: Simplifying Expressions Involving Scientific Notation

Simplify the expression. Write the answer in scientific notation.

$(2.6 \times 10^4)(8.5 \times 10^7)$

$(2.6)(8.5) \times (10^4)(10^7)$

22.1×10^{11} *Multiply 2.6 and 8.5 and add exponents: 4 + 7 = 11.*

2.21×10^{12} *Because 22.1 > 10, move the decimal point left 1 place and add 1 to the exponent.*

1-5 Properties of Exponents

Check It Out! Example 4a

Simplify the expression. Write the answer in scientific notation.

$$\frac{2.325 \times 10^6}{9.3 \times 10^9}$$

$\left(\frac{2.325}{9.3}\right) \times \left(\frac{10^6}{10^9}\right)$ $\frac{a \cdot b}{c \cdot b} = \frac{a}{c} \cdot \frac{b}{d}$

0.25×10^{-3} *Divide 2.325 by 9.3 and subtract exponents: 6 – 9 = –3.*

2.5×10^{-4} *Because 0.25 < 10, move the decimal point right 1 place and subtract 1 from the exponent.*

1-5 Properties of Exponents

Check It Out! Example 4b

Simplify the expression. Write the answer in scientific notation.

$(4 \times 10^{-6})(3.1 \times 10^{-4})$

$(4)(3.1) \times (10^{-6})(10^{-4})$

12.4×10^{-10} *Multiply 4 by 3.1 and add exponents: –6 – 4 = –10.*

1.24×10^{-9} *Because 12.4 >10, move the decimal point left 1 place and add 1 to the exponent.*

1-5 Properties of Exponents

Example 5: Problem-Solving Application

Light travels through space at a speed of about 3×10^5 kilometers per second. Pluto is approximately 5.9×10^{12} m from the Sun. How many minutes, on average, does it take light to travel from the Sun to Pluto?

1-5 Properties of Exponents

Example 5 Continued

1 Understand the Problem

The **answer** will be the time it takes for light to travel from the Sun to Pluto.

List the important information:

- The speed of light in space is about 3×10^5 kilometers per second.
- The distance from the Sun to Pluto is 5.9×10^{12} meters.

1-5 Properties of Exponents

Example 5 Continued

2 Make a Plan

Use the relationship: rate, or speed, equals distance divided by time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \text{ so time} = \frac{\text{distance}}{\text{speed}}$$

1-5 Properties of Exponents

Example 5 Continued

3 Solve

First, convert the speed of light from $\frac{\text{kilometers}}{\text{seconds}}$ to $\frac{\text{meters}}{\text{minute}}$.

$$3\times10^5\frac{\text{km}}{\text{s}}\left(\frac{10^3\text{m}}{1\text{km}}\right)\left(\frac{60\text{s}}{1\text{min}}\right)$$

There are 1000, or 10^3 meters in every kilometers and 60 seconds in every minute

$$(3\cdot60)\times\left(10^5\cdot10^3\right)\frac{\text{m}}{\text{min}}$$

$$180\times10^8\frac{\text{m}}{\text{min}}=1.8\times10^{10}\frac{\text{m}}{\text{min}}$$

1-5 Properties of Exponents

Example 5 Continued

3 Solve

Use the relationship between time, distance, and speed to find the number of minutes it takes light to travel from the Sun to Pluto.

$$\text{time}=\frac{\text{distance}}{\text{speed}}=\frac{5.9\times10^{12}\,\text{m}}{1.8\times10^{10}\,\frac{\text{m}}{\text{min}}}$$

$$\frac{m}{\left(\frac{m}{min}\right)}=\not{m}\left(\frac{min}{\not{m}}\right)=min$$

$$=3.2\overline{7}\times10^2\,\text{min}\approx328\,\text{min}$$

It takes light approximately 328 minutes to travel from the Sun to Pluto.

1-5 Properties of Exponents

Example 5 Continued

4 Look Back

Light travels at 3×10^5 km/s for $328(60)\approx19{,}666$ seconds travels a distance of $5{,}899{,}560{,}000 = 5.89\times10^9$ km or 5.89×10^{12} m. The answer is reasonable.

1-5 Properties of Exponents

Check It Out! Example 5

Light travels through space at a speed of about 3×10^5 kilometers per second. Earth is approximately 1.5×10^{11} m from the Sun. How many minutes, on average, does it take light to travel from the Sun to Earth?

1-5 Properties of Exponents

Check It Out! Example 5 Continued

1 Understand the Problem

The **answer** will be the time it takes for light to travel from the Sun to Earth.

List the important information:

- The speed of light is about 3×10^5 kilometers per second.
- The distance from the Sun to Earth is 1.5×10^{11} meters.

1-5 Properties of Exponents

Check It Out! Example 5 Continued

2 Make a Plan

Use the relationship: rate, or speed, equals distance divided by time.

$$\text{speed}=\frac{\text{distance}}{\text{time}},\ \text{so time}=\frac{\text{distance}}{\text{speed}}$$

1-5 Properties of Exponents

Check It Out! Example 5 Continued

3 Solve

First, convert the speed of light from $\frac{\text{kilometers}}{\text{seconds}}$ to $\frac{\text{meters}}{\text{minute}}$.

$$3\times10^5\frac{\text{km}}{\text{s}}\left(\frac{10^3\text{m}}{1\text{km}}\right)\left(\frac{60\text{s}}{1\text{min}}\right)$$

There are 1000, or 10^3 meters in every kilometers and 60 seconds in every minute

$$(3\cdot60)\times\left(10^5\cdot10^3\right)\frac{\text{m}}{\text{min}}$$

$$180\times10^8\frac{\text{m}}{\text{min}}=1.8\times10^{10}\frac{\text{m}}{\text{min}}$$

1-5 Properties of Exponents

Check It Out! Example 5 Continued

3 Solve

Use the relationship between time, distance, and speed to find the number of minutes it takes light to travel from the Sun to Earth.

$$\text{time}=\frac{\text{distance}}{\text{speed}}=\frac{1.5\times10^{11}\text{m}}{1.8\times10^{10}\frac{\text{m}}{\text{min}}}\qquad \frac{m}{\left(\frac{m}{min}\right)}=m\left(\frac{min}{m}\right)=min$$

$$=0.8\bar{3}\times10^1\,\text{min}\approx8.\bar{3}\,\text{min}$$

It takes light approximately 8.333 minutes to travel from the Sun to Earth.

1-5 Properties of Exponents

Check It Out! Example 5 Continued

4 Look Back

Light travels at 1.49×10^8 km or 1.49×10^{11} m for $8.33(60)\approx499.99$ seconds travels a distance of $149{,}999{,}400=3\times10^5$ kilometers per second.

The answer is reasonable.