

1-8 Exploring Transformations

Starter 1.8 **HW 1.7 ???**

Plot each point.

1. A(0,0)
2. B(5,0)
3. C(-5,0)
4. D(0,5)
5. E(0,-5)
6. F(-5,-5)

1-8 Exploring Transformations

Transformation – a change in the position, size, or shape of a figure.

Translation (or slide) – moves right, left, up, or down.

Horizontal Translation (right or left) The x-coordinate changes. $(x, y) \rightarrow (x + h, y)$	Vertical Translation (up or down) The y-coordinate changes. $(x, y) \rightarrow (x, y + k)$
---	--

Translate the function $y = f(x)$ left 2 units.

Move each point 2 units left. Connect the points.
 $(x, y) \rightarrow (x - 2, y)$

1-8 Exploring Transformations

translate **left or right – x-coordinate** changes.
translate **up or down – y-coordinate** changes.

Translations

Horizontal Translation Each point shifts <i>right</i> or <i>left</i> by a number of units. The x-coordinate changes. $(1, 2) \rightarrow (1 + 3, 2)$ $(x, y) \rightarrow (x + h, y)$ left if $h < 0$ right if $h > 0$	Vertical Translation Each point shifts <i>up</i> or <i>down</i> by a number of units. The y-coordinate changes. $(1, 2) \rightarrow (1, 2 + 3)$ $(x, y) \rightarrow (x, y + k)$ down if $k < 0$ up if $k > 0$
---	---

1-8 Exploring Transformations

Example 1: Translating Points
Perform the given translation on the point $(-3, 4)$. Give the coordinates of the translated point.

Translation	Translated Point
1) 5 units right	
2) 2 units left	
3) 4 units down	
4) 2 units down, 2 units left	
5) 2 units down, 4 units right	
6) 4 units up	
7) 6 units up, 2 units right	
8) 8 up, 5 down, 4 left, then 6 right	
9) 4 down, 3 left, 5 up, then 2 right	

1-8 Exploring Transformations

Example 1A: Translating Points
Perform the given translation on the point $(-3, 4)$. Give the coordinates of the translated point.
5 units right

Translating $(-3, 4)$ 5 units right results in the point $(2, 4)$.

1-8 Exploring Transformations

Example 1B: Translating Points
Perform the given translation on the point $(-3, 4)$. Give the coordinates of the translated point.
2 units left and 2 units down

Translating $(-3, 4)$ 2 units left and 2 units down results in the point $(-5, 2)$.

1-8 Exploring Transformations

Check It Out! Example 1a

Perform the given translation on the point $(-1, 3)$. Give the coordinates of the translated point.

4 units right

Translating $(-1, 3)$ 4 units right results in the point $(3, 3)$.

1-8 Exploring Transformations

Check It Out! Example 1b

Perform the given translation on the point $(-1, 3)$. Give the coordinates of the translated point.

1 unit left and 2 units down

Translating $(-1, 3)$ 1 unit left and 2 units down results in the point $(-2, 1)$.

1-8 Exploring Transformations

Reflection – a transformation that **flips** a figure across a line called the **line of reflection**.

Reflection Across y-axis	Reflection Across x-axis
The x-coordinate changes. $(x, y) \rightarrow (-x, y)$	The y-coordinate changes. $(x, y) \rightarrow (x, -y)$

Symmetry with respect to the y-axis

Symmetry with respect to the x-axis

1-8 Exploring Transformations

Reflections

Reflection Across y-axis	Reflection Across x-axis
Each point flips across the y-axis. The x-coordinate changes. $(1, 2) \rightarrow (-1, 2)$ $(x, y) \rightarrow (-x, y)$	Each point flips across the x-axis. The y-coordinate changes. $(1, 2) \rightarrow (1, -2)$ $(x, y) \rightarrow (x, -y)$

1-8 Exploring Transformations

Example 1: Translating Points

Perform the given translation on the point $(-3, 4)$. Give the coordinates of the translated point.

Translation	Translated Point
1) 5 units right	
2) 2 units left	
3) 4 units down	
4) 2 units down, 2 units left	
5) 2 units down, 4 units right	
6) 4 units up	
7) 6 units up, 2 units right	
8) 8 up, 5 down, 4 left, then 6 right	
9) 4 down, 3 left, 5 up, then 2 right	

1-8 Exploring Transformations

Example 2: Translating and Reflecting Functions

Use a table to perform each transformation of $y=f(x)$. Use the same coordinate plane as the original function.

translation 2 units up

Identify important points from the graph and make a table.

(x, y)	$y + 2$	Trans. Point
$(-5, -3)$	$-3 + 2 = -1$	$(-5, -1)$
$(-2, 0)$	$0 + 2 = 2$	$(-2, 2)$
$(0, -2)$	$-2 + 2 = 0$	$(0, 0)$
$(2, 0)$	$0 + 2 = 2$	$(2, 2)$
$(5, -3)$	$-3 + 2 = -1$	$(5, -1)$

Add 2 to each y-coordinate.
The entire graph shifts 2 units up.

1-8 Exploring Transformations

Example 2: Translating and Reflecting Functions
Use a table to perform each transformation of $y=f(x)$. Use the same coordinate plane as the original function.

translation 2 units up

(x, y)	$y + 2$	Trans. Point
(-5, -3)	$-3 + 2 = -1$	(-5, -1)
(-2, 0)	$0 + 2 = 2$	(-2, 2)
(0, -2)	$-2 + 2 = 0$	(0, 0)
(2, 0)	$0 + 2 = 2$	(2, 2)
(5, -3)	$-3 + 2 = -1$	(5, -1)

1-8 Exploring Transformations

Example 2: Translating and Reflecting Functions

translation 2 units up

Identify important points from the graph and make a table.

x	y	$y + 2$
-5	-3	$-3 + 2 = -1$
-2	0	$0 + 2 = 2$
0	-2	$-2 + 2 = 0$
2	0	$0 + 2 = 2$
5	-3	$-3 + 2 = -1$

Add 2 to each y-coordinate.
The entire graph shifts 2 units up.

1-8 Exploring Transformations

Example 2: Translating and Reflecting Functions
Use a table to perform each transformation of $y=f(x)$. Use the same coordinate plane as the original function.

Reflection across the x-axis

Identify important points from the graph and make a table.

(x, y)	$-y$	Ref. Point
(-5, -3)	3	(-5, 3)
(-2, 0)	0	(-2, 0)
(0, -2)	2	(0, 2)
(2, 0)	0	(2, 0)
(5, -3)	3	(5, 3)

Multiply each y-coordinate by -1 .
The entire graph flips across the x-axis.

1-8 Exploring Transformations

Example 2B: Translating and Reflecting Functions

reflection across x-axis

Identify important points from the graph and make a table.

x	y	$-y$
-5	-3	$-1(-3) = 3$
-2	0	$-1(0) = 0$
0	-2	$-1(-2) = 2$
2	0	$-1(0) = 0$
5	-3	$-1(-3) = 3$

Multiply each y-coordinate by -1 .
The entire graph flips across the x-axis.

1-8 Exploring Transformations

Check It Out! Example 2a
Use a table to perform the transformation of $y = f(x)$. Use the same coordinate plane as the original function.

translation 3 units right

x	y	$x + 3$
-2	4	$-2 + 3 = 1$
-1	0	$-1 + 3 = 2$
0	2	$0 + 3 = 3$
2	2	$2 + 3 = 5$

Add 3 to each x-coordinate.
The entire graph shifts 3 units right.

1-8 Exploring Transformations

Check It Out! Example 2b
Use a table to perform the transformation of $y = f(x)$. Use the same coordinate plane as the original function.

reflection across x-axis

x	y	$-y$
-2	4	-4
-1	0	0
0	2	-2
2	2	-2

Multiply each y-coordinate by -1 .
The entire graph flips across the x-axis.

1-8 Exploring Transformations

Horizontal Stretch – pull the points away from the y-axis

Horizontal Compression – you push the points towards the y-axis

1-8 Exploring Transformations

Stretches and compressions are not congruent to the original graph.

Stretches and Compressions		
	Horizontal	Vertical
Stretch	Each point is <i>pulled away</i> from the y-axis. The x-coordinate changes. $(4, 0) \rightarrow (2(4), 0)$ $(x, y) \rightarrow (bx, y)$ $ b > 1$	Each point is <i>pulled away</i> from the x-axis. The y-coordinate changes. $(0, 4) \rightarrow (0, 2(4))$ $(x, y) \rightarrow (x, ay)$ $ a > 1$
Compression	Each point is <i>pushed toward</i> the y-axis. The x-coordinate changes. $(4, 0) \rightarrow (\frac{1}{2}(4), 0)$ $(x, y) \rightarrow (bx, y)$ $0 < b < 1$	Each point is <i>pushed toward</i> the x-axis. The y-coordinate changes. $(0, 4) \rightarrow (0, \frac{1}{2}(4))$ $(x, y) \rightarrow (x, ay)$ $0 < a < 1$

1-8 Exploring Transformations

Example 3: Stretching and Compressing Functions

Use a table to perform a horizontal stretch of the function $y = f(x)$ by a factor of 3. Graph the function and the transformation on the same coordinate plane.

Identify important points from the graph and make a table.

$3x$	x	y
$3(-1) = -3$	-1	3
$3(0) = 0$	0	0
$3(2) = 6$	2	2
$3(4) = 12$	4	2

Multiply each x-coordinate by 3.

1-8 Exploring Transformations

Check It Out! Example 3

Use a table to perform a vertical stretch of $y = f(x)$ by a factor of 2. Graph the transformed function on the same coordinate plane as the original figure.

Identify important points from the graph and make a table.

x	y	$2y$
-1	3	$2(3) = 6$
0	0	$2(0) = 0$
2	2	$2(2) = 4$
4	2	$2(2) = 4$

Multiply each y-coordinate by 2.

1-8 Exploring Transformations

Example 4: Business Application

The graph shows the cost of painting based on the number of cans of paint used. Sketch a graph to represent the cost of a can of paint doubling, and identify the transformation of the original graph that it represents.

If the cost of painting is based on the number of cans of paint used and the cost of a can of paint doubles, the cost of painting also doubles. This represents a vertical stretch by a factor of 2.

1-8 Exploring Transformations

Check It Out! Example 4

Recording studio fees are usually based on an hourly rate, but the rate can be modified due to various options. The graph shows a basic hourly studio rate.

1-8 Exploring Transformations

Check It Out! Example 4 Continued
What if...? Suppose that a discounted rate is $\frac{3}{4}$ of the original rate. Sketch a graph to represent the situation and identify the transformation of the original graph that it represents.

If the price is discounted by $\frac{3}{4}$ of the hourly rate, the value of each y-coordinate would be multiplied by $\frac{3}{4}$.

1-8 Exploring Transformations

Short Quiz 1.8

1. Translate the point $(4, -6)$ 6 units right and 7 units up. Give the coordinates on the translated point.

(10, 1)

1-8 Exploring Transformations

Short Quiz 1.8

Use a table to perform the transformation of $y = f(x)$. Graph the function and the transformation on the same coordinate plane.

2. Reflection across y-axis 3. vertical compression by a factor of $\frac{1}{2}$.