# 2.1: Solving Systems of Equations in Two Variables

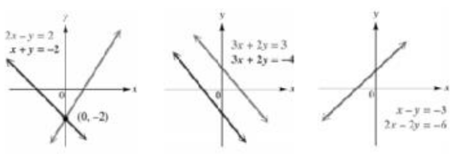


## 2.1: Solving Systems of Equations in Two Variables

- A set of equations is called a **system of equations**.
- The ***solutions*** must satisfy each equation in the system.
- If all equations in a system are linear, the system is a **system of linear equations**, or a **linear system**.



## 2.1 Linear System in Two Variables

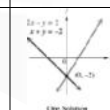
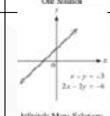
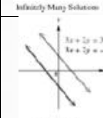



One Solution No Solutions Infinitely Many Solutions

- Three possible solutions to a linear system in two variables:
  1. One solution: coordinates of a point,
  2. No solutions: **inconsistent** case,
  3. Infinitely many solutions: **dependent** case.



## 2.1: Solving Systems of Equations in Two Variables

| Name of system | Slope ($m$) | y-int. ($b$) | # of Solutions | Graph |
|---|---|---|---|---|
| ***Consistent, independent*** | **Different** | **Doesn't matter** | **One** |  |
| ***Consistent, dependent*** | **Same** | **Same** | **Infinitely many** |  |
| ***Inconsistent*** | **Same** | **Different** | **none** |  |



**3.** $y=\frac{2}{3}x-4$
$2x-3y=6$

**4.** $x-4y=-5$
$y=9x-25$

**5.** $5x-6y=8$
$3y=2.5x-4$

**7.** $y=x-4$
$3x+2y=7$

**8.** $y=7x-9$
$y=5x+7$

**9.** $x-3y=1$
$2x+y=-5$

$x+y=-2$
$y=2x-5$



# DO:

12, 14, 16, 22, 28, 32, p. 71

1. Use systems of equations in two variables to solve the following applied problems.

   a) There were a total of 28 students in a philosophy class. The number of men was six more than the number of women. How many men and women were in the class?

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**•2.1: Substitution Method**
**isolate one of the variables then substitute it to the other equation, then solve for the value of variable.**

***Example*:** Solve the system.

$$3x + 2y = 11 \quad \textbf{(1)}$$
$$-x + y = 3 \quad \textbf{(2)}$$

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## 2.1 Substitution Method

**Example** Solve the system.

$$3x + 2y = 11 \quad \textbf{(1)}$$
$$-x + y = 3 \quad \textbf{(2)}$$

**Solution**

| | |
|---|---|
| $y = x + 3$ | Solve **(2)** for $y$. |
| $3x + 2(x+3) = 11$ | Substitute $y = x + 3$ in **(1)**. |
| $3x + 2x + 6 = 11$ | Solve for $x$. |
| $5x = 5$ | |
| $x = 1$ | |
| $y = 1 + 3$ | Substitute $x = 1$ in $y = x + 3$. |
| $y = 4$ | Solution set: {(1, 4)} |

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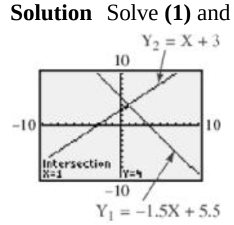
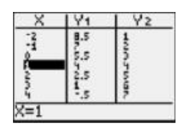
## 2.1 Solving a Linear System in Two Variables Graphically

**Example** Solve the system graphically.

$$3x + 2y = 11 \quad \textbf{(1)}$$
$$-x + y = 3 \quad \textbf{(2)}$$

**Solution** Solve **(1)** and **(2)** for $y$.

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## 2.1 Elimination Method

**Example** Solve the system.

$$3x - 4y = 1 \quad \textbf{(1)}$$
$$2x + 3y = 12 \quad \textbf{(2)}$$

**Solution** To eliminate $x$, multiply **(1)** by –2 and **(2)** by 3 and add the resulting equations.

$$-6x + 8y = -2 \quad \textbf{(3)}$$
$$6x + 9y = 36 \quad \textbf{(4)}$$
$$17y = 34$$
$$y = 2$$

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## 2.1 Elimination Method

Substitute 2 for $y$ in **(1)** or **(2)**.

$$3x - 4(2) = 1$$
$$3x = 9$$
$$x = 3$$

The solution set is {(3, 2)}.

- Check the solution set by substituting 3 in for $x$ and 2 in for $y$ in both of the original equations.

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## 2.1 Solving an Inconsistent System

**Example** Solve the system.

$$\begin{aligned} 3x - 2y &= 4 \quad \textbf{(1)} \\ -6x + 4y &= 7 \quad \textbf{(2)} \end{aligned}$$

**Solution** Eliminate $x$ by multiplying **(1)** by 2 and adding the result to **(2).**

$$\begin{aligned} 6x - 4y &= 8 \\ -6x + 4y &= 7 \\ \hline 0 &= 15 \quad \text{Inconsistent System} \end{aligned}$$

Solution set is $\emptyset$.

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## 2.1 Solving a System with Dependent Equations

**Example** Solve the system.

$$\begin{aligned} -4x + y &= 2 \quad \textbf{(1)} \\ 8x - 2y &= -4 \quad \textbf{(2)} \end{aligned}$$

**Solution** Eliminate $x$ by multiplying **(1)** by 2 and adding the result to **(2)**.

$$\begin{aligned} -8x + 2y &= 4 \\ 8x - 2y &= -4 \\ \hline 0 &= 0 \end{aligned}$$

Each equation is a solution of the other. Choose either equation and solve for $x$.

$$-4x + y = 2 \quad \Rightarrow \quad x = \frac{y-2}{4}$$

The solution set is $\left\{\left(\frac{y-2}{4}, y\right)\right\}$. e.g. $y = -2$: $\{(\frac{-2-2}{4}, -2)\} = (-1, -2)\}$

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## 2.1 Applications of Systems

- To solve problems using a system
  1. Determine the unknown quantities
  2. Let different variables represent those quantities
  3. Write a system of equations – one for each variable

**Example** In a recent year, the national average spent on two varsity athletes, one female and one male, was $6050 for Division I-A schools. However, average expenditures for a male athlete exceeded those for a female athlete by $3900. Determine how much was spent per varsity athlete for each gender.

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## 2.1 Applications of Systems

**Solution** Let $x$ = average expenditures per male
$y$ = average expenditures per female

Average spent on one male and one female $= \frac{x+y}{2} = 6050 \quad \Rightarrow \quad x + y = 12{,}100$

$$\begin{aligned} x + y &= 12100 \quad \textbf{(1)} \\ x - y &= 3900 \quad \textbf{(2)} \\ \hline 2x &= 16000 \\ x &= 8000 \end{aligned}$$

Average Expenditure per male: $8000, and per female: from **(2)** $y = 8000 - 3900 =$ $4100.

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