2.6: Solving Systems of Linear Inequalities

Quick Review

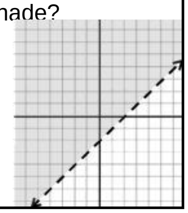
- What is the difference between an equation and an inequality? Which one is shaded? Inequality
  - = (equation), <,> (inequality)
- When is the line solid? ≤, ≥
- When is the line dashed (dotted)? <, >
- How do you figure out where to shade?
  Pick a point to plug in.

Graph this inequality:

$y > x - 2$

$m = 1$

$b = -2$



Check if it's a solution

1. (4, 10)
   $9x - y \geq 23$
   $5x + 0.2y \geq 20$
   YES

Check (4, 10)	
$9x - y \geq 23$	$5x + 0.2y \geq 20$
$9(4) - 10 \geq 23$	$5(4) + 0.2(10) \geq 20$
$36 - 10 \geq 23$	$20 + 2 \geq 20$
$26 \geq 23$ ✓	$22 \geq 20$ ✓

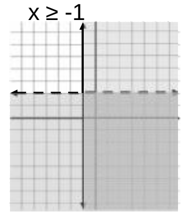
2. (2, -1)
   $y \leq 4x + 1$
   $y > -x + 2$
   NO

Check (2, -1)	
$y \leq 4x + 1$	$y > -x + 2$
$-1 \leq 4(2) + 1$	$-1 > -(2) + 2$
$-1 \leq 8 + 1$	$-1 > 0$ ×
$-1 \leq 9$ ✓	

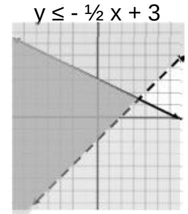
Graphing Systems of Linear Inequalities

Graph each system

3. $y < 2$
   $x \geq -1$



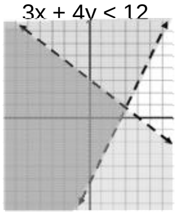
4. $y > x - 2$
   $y \leq -\frac{1}{2}x + 3$



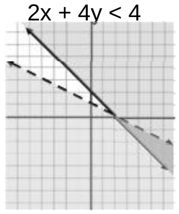
Graphing Systems of Linear Inequalities

Graph each system

5. $y > 2x - 5$
   $3x + 4y < 12$



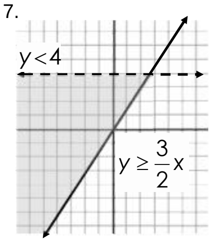
6. $y \geq -x + 2$
   $2x + 4y < 4$



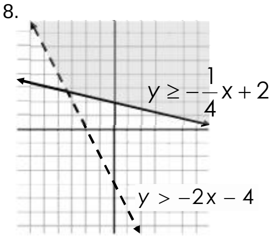
Writing Systems of Linear Inequalities Equation

Write the inequalities for each system

7.

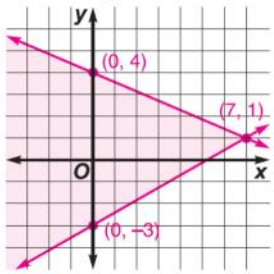


8.



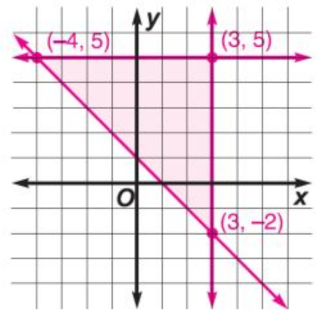
2.6: Solving Systems of Linear Inequalities

1. $-4x + 7y \geq -21;\ 3x + 7y \leq 28$



2.6: Solving Systems of Linear Inequalities

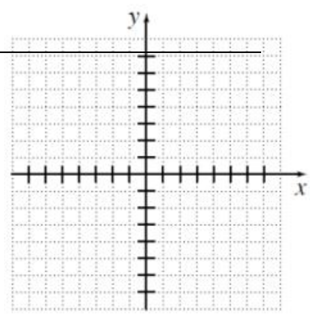
2. $x \leq 3;\ y \leq 5;\ x + y \geq 1$



2.6: Solving Systems of Linear Inequalities

3. $x \geq 0;\ y \geq 0;\ y \geq x - 4;\ 7x + 6y \leq 54$

STARTER 2.6



2.6: Solving Systems of Linear Inequalities

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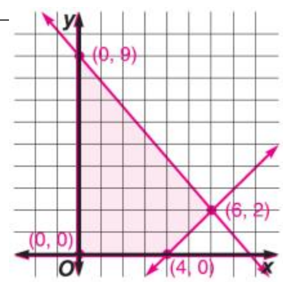
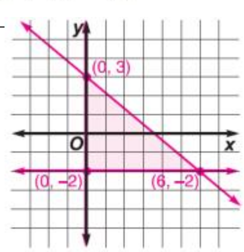


vertices: (0, 0), (0, 9), (6, 2), (4, 0)

2.6: Solving Systems of Linear Inequalities

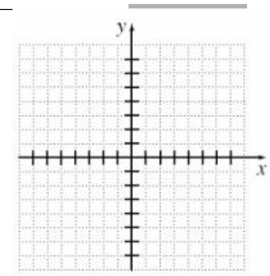
4. $x \geq 0;\ y + 2 \geq 0;\ 5x + 6y \leq 18$



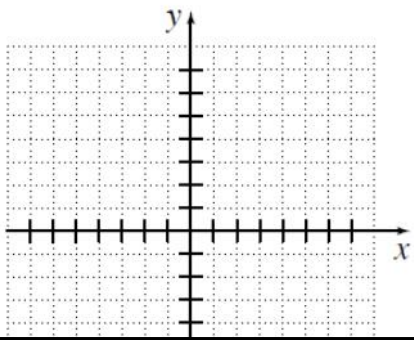
vertices: (0, 3), (0, −2), (6, −2)

2.6: Solving Systems of Linear Inequalities

2. $2x + y \geq -2$
$6x + 3y \leq 6$



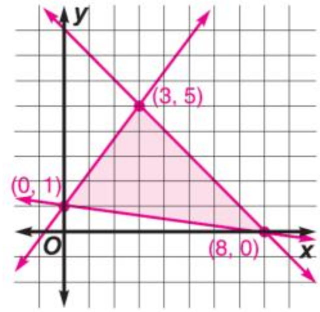
2.6: Solving Systems of Linear Inequalities



Practice 2.6, #6

6. $y \le -x + 8 \quad 4x - 3y \ge -3$
$x + 8y \ge 8 \quad f(x, y) = 4x - 5y$

max = 32
min = −13

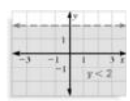


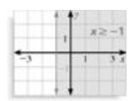
Practice 2.6, #7

7. ***Business*** Henry Jackson, a recent college graduate, plans to start his own business manufacturing bicycle tires. Henry knows that his start-up costs are going to be \$3000 and that each tire will cost him at least \$2 to manufacture. In order to remain competitive, Henry cannot charge more than \$5 per tire. Draw a graph to show when Henry will make a profit.

2.6: Solving Systems of Linear Inequalities

The graph of a linear inequality in two variables is a **half-plane**. The boundary line of the half-plane is dashed if the inequality is < or > and solid if the inequality is ≤ or ≥.





Two or more linear inequalities form a **system of linear inequalities** or simply a *system of inequalities*.

A **solution** of a system of a system of linear inequalities is an ordered pair that is a solution of each inequality in the system.

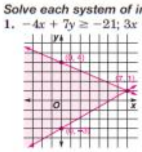
The graph of a system of linear inequalities is the **graph** of ***all solutions*** of the system.

PRACTICE 2.6

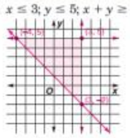
Solving Systems of Linear Inequalities

Solve each system of inequalities by graphing.

1. $-4x + 7y \ge -21$; $3x + 7y \le 28$

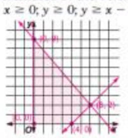


2. $x \le 3$; $y \le 5$; $x + y \ge 1$



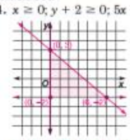
Solve each system of inequalities by graphing. Name the coordinates of the vertices of the polygonal convex set.

3. $x \ge 0$; $y \ge 0$; $y \ge x - 4$; $7x + 6y \le 54$



vertices: (0, 0), (0, 9), (6, 2), (4, 0)

4. $x \ge 0$; $y + 2 \ge 0$; $5x + 6y \le 18$



vertices: (0, 3), (0, −2), (6, −2)

EXAMPLE ***A Triangular Solution Region***

Graph the system of linear inequalities.

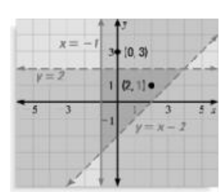
$y < 2$	Inequality 1
$x \ge -1$	Inequality 2
$y > x - 2$	Inequality 3

SOLUTION

Graph all three inequalities in the same coordinate plane. The graph of the system is the overlap, or the *intersection*, of the three half-planes shown.

CHECK You can see from the graph that the point (2, 1) is a solution of the system. To check this, substitute the point into each inequality.

$1 < 2$	True.
$2 \ge -1$	True.
$1 > 2 - 2$	True.



EXAMPLE ***A Triangular Solution Region***

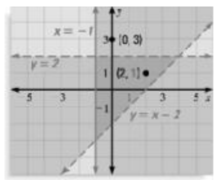
Graph the system of linear inequalities.

$y < 2$	Inequality 1
$x \geq -1$	Inequality 2
$y > x - 2$	Inequality 3

The point $(0, 3)$ is not in the graph of the system. Notice $(0, 3)$ is not a solution of inequality 1. This point is not a solution of the system.

When graphing a system of inequalities, it is helpful to find each corner point (or *vertex*).

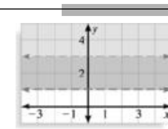
For instance, this graph has three corner points: $(-1, 2)$, $(-1, -3)$, and $(4, 2)$.



EXAMPLE ***Solution Region Between Parallel Lines***

Write a system of inequalities that defines the shaded region shown.

SOLUTION



The graph of one inequality is the half-plane *below* the line $y = 3$.

The graph of the other inequality is the half-plane *above* the line $y = 1$.

The shaded region of the graph is the horizontal band that lies *between* the two horizontal lines, $y = 3$ and $y = 1$, but not on the lines.

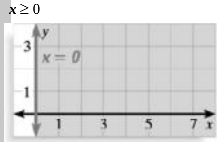
▶ The system of linear inequalities at the right defines the shaded region.

$y < 3$	Inequality 1
$y > 1$	Inequality 2

EXAMPLE ***A Quadrilateral Solution Region***

Graph the system of linear inequalities. Label each vertex of the solution region. Describe the shape of the region.

$x \geq 0$

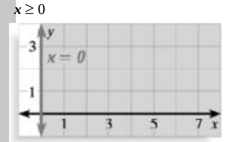


The graph of the first inequality is the half-plane *on and to the right* of the y-axis.

EXAMPLE ***A Quadrilateral Solution Region***

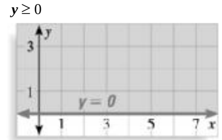
Graph the system of linear inequalities. Label each vertex of the solution region. Describe the shape of the region.

$x \geq 0$



The graph of the first inequality is the half-plane *on and to the right* of the y-axis.

$y \geq 0$

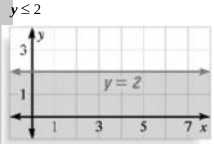


The graph of the second inequality is the half-plane *on and above* of the x-axis.

EXAMPLE ***A Quadrilateral Solution Region***

Graph the system of linear inequalities. Label each vertex of the solution region. Describe the shape of the region.

$y \leq 2$

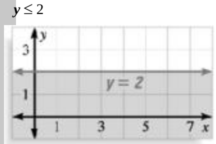


The graph of the third inequality is the half-plane *on and below* the horizontal line $y = 2$.

EXAMPLE ***A Quadrilateral Solution Region***

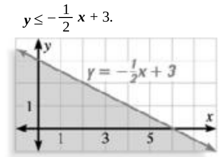
Graph the system of linear inequalities. Label each vertex of the solution region. Describe the shape of the region.

$y \leq 2$



The graph of the third inequality is the half-plane *on and below* the horizontal line $y = 2$.

$y \leq -\frac{1}{2}x + 3.$

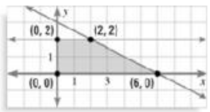


The graph of the fourth inequality is the half-plane *on and below* the line $y = -\frac{1}{2}x + 3.$

EXAMPLE **A Quadrilateral Solution Region**

Graph the system of linear inequalities. Label each vertex of the solution region. Describe the shape of the region.

The region that lies in all four half-planes is a *quadrilateral* with vertices at (0, 2), (0, 0), (6, 0), and (2, 2).



Note that (0, 3) is not a vertex of the solution region even though two boundary lines meet at that point.

Modeling A Real-Life Problem

You are ordering lighting for a theater so the spotlights can follow the performers. The lighting technician needs at least 3 medium-throw spotlights and at least 1 long-throw spotlight. A medium-throw spotlight costs $1000 and a long-throw spotlight costs $3500. The minimum order for free delivery is $10,000.

Write and graph a system of linear inequalities that shows how many medium-throw spotlights and long-throw spotlights should be ordered to get the free delivery.

Verbal Model

Number of medium-throws ≥ 3

Number of long-throws ≥ 1

Number of medium-throws • Price of a medium-throw + Number of long-throws • Price of a long-throw $\geq 10{,}000$

...

GOAL 2 **Modeling A Real-Life Problem**

You are ordering lighting for a theater so the spotlights can follow the performers. The lighting technician needs at least 3 medium-throw spotlights and at least 1 long-throw spotlight. A medium-throw spotlight costs $1000 and a long-throw spotlight costs $3500. The minimum order for free delivery is $10,000.

Write and graph a system of linear inequalities that shows how many medium-throw spotlights and long-throw spotlights should be ordered to get the free delivery.

...

Labels

Number of medium-throws	$= x$	(no units)
Number of long-throws	$= y$	(no units)
Price of a medium-throw	$= 1000$	(dollars)
Price of a long-throw	$= 3500$	(dollars)

...

GOAL 2 **Modeling A Real-Life Problem**

You are ordering lighting for a theater so the spotlights can follow the performers. The lighting technician needs at least 3 medium-throw spotlights and at least 1 long-throw spotlight. A medium-throw spotlight costs $1000 and a long-throw spotlight costs $3500. The minimum order for free delivery is $10,000.

Write and graph a system of linear inequalities that shows how many medium-throw spotlights and long-throw spotlights should be ordered to get the free delivery.

...

Algebraic Model

$x \geq 3$ — Inequality 1

$y \geq 1$ — Inequality 2

$1000x + 3500y \geq 10{,}000$ — Inequality 3

GOAL 2 **Modeling A Real-Life Problem**

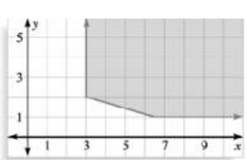
You are ordering lighting for a theater so the spotlights can follow the performers. The lighting technician needs at least 3 medium-throw spotlights and at least 1 long-throw spotlight. A medium-throw spotlight costs $1000 and a long-throw spotlight costs $3500. The minimum order for free delivery is $10,000.

Write and graph a system of linear inequalities that shows how many medium-throw spotlights and long-throw spotlights should be ordered to get the free delivery.

The graph of the system of inequalities is shown.

Any point in the shaded region of the graph is a solution to the system.

A fraction of a spotlight cannot be ordered, so only ordered pairs of integers in the shaded region will correctly answer the problem.



GOAL 2 **Modeling A Real-Life Problem**

You are ordering lighting for a theater so the spotlights can follow the performers. The lighting technician needs at least 3 medium-throw spotlights and at least 1 long-throw spotlight. A medium-throw spotlight costs $1000 and a long-throw spotlight costs $3500. The minimum order for free delivery is $10,000.

Will an order of 4 medium-throw spotlights and 1 long-throw spotlight be delivered free?

The point (4, 1) is outside the solution region, so an order of 4 medium-throw spotlights and 1 long-throw spotlight would not be delivered free.