## Translations

- When a constant is added or subtracted from a parent function, the result would be a translation horizontally or vertically.
- Let $g(x)$ be the indicated transformation of $f(x)$ at $(h, k)$.


Ex. 1) Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.
a) $f(x)=2 x+3$; vertical translation 4 units up.

Rule:

$$
\begin{aligned}
x & \rightarrow \text { same } \\
y & \rightarrow y+4 \\
g(x) & =f(x)+4 \\
g(x) & =(2 x+3)+4 \\
g(x) & =2 x+7
\end{aligned}
$$




## Reflections

- Flips a figure over a line called the axis (or line) of symmetry.
- Let $g(x)$ be the indicated transformation of $f(x)$.

|  | Reflections over the $x$-axis | Reflections over the $y$-axis |
| ---: | :---: | :---: |
| Rule | $x \rightarrow$ same | $x \rightarrow-x$ |
|  | $y \rightarrow-y$ | $y \rightarrow$ same |
| As a point | $(x,-y)$ | $(-x, y)$ |

$$
g(x)=-f(x) \quad g(x)=f(-x)
$$

reflection over the $x$-axis
reflection over the $y$-axis
Rule:
$(x, y) \longmapsto(x,-y)$
Rule:
$(x, y) \longmapsto(-x, y)$

Ex. 2) Let $g(x)$ be the indicated transformation of $f(x)$. Write the rule for $g(x)$.
a) linear function defined in the table; reflection across $y$-axis.

- Find the slope:

$$
\boldsymbol{m}=\frac{2-0}{0-(-1)}=2
$$

| $x$ | $f(x)$ |
| ---: | :---: |
| -1 | 0 |
| 0 | 2 |
| 1 | 4 |

- Find the equation in slope-intercept form. Using (0, 2), b=2

$$
f(x)=2 x+2
$$

- Write the rule for $g(x)$. Reflecting $f(x)$ across the $y$-axis replaces each $\boldsymbol{x}$ with $\boldsymbol{- x}$.

$$
\begin{aligned}
& g(x)=2(-x)+2 \\
& g(x)=-2 x+2
\end{aligned}
$$




## Stretches and Compressions

- A transformation that produces an image that is the same shape as the original in which all distances on the coordinate plane are stretch or compressed/shrinked by multiplying either all $\boldsymbol{x}$-coordinates or all $\boldsymbol{y}$-coordinates by a factor.
- Let $g(x)$ be the indicated transformation of $f(x)$.

$0<|a|<1$ vertical compression
by a factor of $a$
Ex. 3) Let $g(x)$ be a horizontal compression of $f(x)=2 x-1$ by a factor of $\frac{1}{3}$. Write the rule for $g(x)$, and graph the function.
- Horizontally compressing $f(x)$ by a factor of $\frac{1}{3}$ replaces each $x$ with $\frac{1}{b}(x)$ where $\boldsymbol{b}=\frac{1}{3}$.

$$
\begin{aligned}
g(x) & =2\left(\frac{1}{\boldsymbol{b}}\right) x-1 \\
& =2\left(\frac{1}{\frac{1}{3}}\right) x-1 \\
& =2(3 x)-1 \\
g(x) & =6 x-1
\end{aligned}
$$




## Summary of Transformations

- Let $g(x)$ be the indicated transformation of $f(x)$ with $a$ and $b$ as factors and translation to $(h, k)$.


Ex. 4) Let $g(x)$ be a vertical shift of $f(x)=x$, down 2 units followed by a vertical stretch by a factor of 5 . Write the rule for $g(x)$.

- Translating $f(x)=x$ down 2 units subtracts 2 from the function. $h(x)=f(x)-2$

$$
\boldsymbol{h}(\boldsymbol{x})=\boldsymbol{x}-2
$$

- Perform the vertical stretch by a factor of $5 \quad g(x)=5 \cdot h(x)$

$$
\begin{aligned}
& g(x)=5(x-2) \\
& g(x)=5 x-10
\end{aligned}
$$

