3.2: Families of Graphs

Family of graphs
- a group of graphs that displays one or more similar characteristics
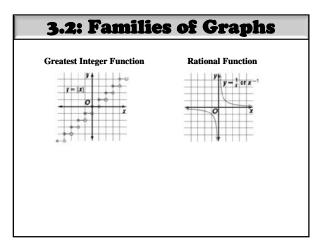
Parent graph
- a basic graph that is transformed to create other members in a family of graphs.

Reflections and translations of the parent function can affect the appearance of the graph. The transformed graph may appear in a different location but **it will resemble the parent graph**.
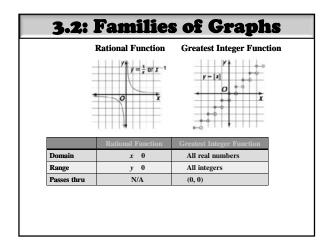
3.2: Families of Graphs

Constant Function   **Linear Function**   **Quadratic Function**

$y = 0$ $y = x$ $y = x^2$

Cubic Function   **Square Root Function**   **Absolute Value Function**

$y = x^3$ $y = \sqrt{x}$ $y = |x|$

3.2: Families of Graphs

Greatest Integer Function   **Rational Function**

$y = [x]$ $y = \frac{1}{x}$ or x^{-1}

3.2: Families of Graphs

Constant Function   **Linear Function**   **Quadratic Function**

$y = 0$ $y = x$ $y = x^2$

	Constant Function	Linear Function	Quadratic Function
Domain	All real numbers	All real numbers	All real numbers
Range	$y = c$ (constant)	All real numbers	$y \geq 0$
Passes thru	c (constant)	(0, 0)	(0, 0)

3.2: Families of Graphs

Cubic Function   **Square Root Function**   **Absolute Value Function**

$y = x^3$ $y = \sqrt{x}$ $y = |x|$

	Cubic Function	Square Root Function	Absolute Value Function
Domain	All real numbers	All real numbers	All real numbers
Range	All real numbers	$y \geq 0$	$y \geq 0$
Passes thru	(0, 0)	(0, 0)	(0, 0)

3.2: Families of Graphs

Rational Function   **Greatest Integer Function**

$y = \frac{1}{x}$ or x^{-1} $y = [x]$

	Rational Function	Greatest Integer Function
Domain	$x \neq 0$	All real numbers
Range	$y \neq 0$	All integers
Passes thru	N/A	(0, 0)

3.2: Families of Graphs

Reflections

- Flips a figure over a line called the **axis** (or **line**) **of symmetry**.
- Let **g(x)** be the transformation of **f(x)**

$$g(x) = -f(x)$$
reflection over the *x*-axis

$$g(x) = f(-x)$$
reflection over the *y*-axis

Rule:
$(x, y) \Longrightarrow (x, -y)$

Rule:
$(x, y) \Longrightarrow (-x, y)$

3.2: Families of Graphs

Ex) Graph $f(x) = |x|$ and $g(x) = -|x|$ on the same axis. Describe how the graphs of *f(x)* and *g(x)* are related.

x	f(x) = |x|	g(x) = -|x|
-2	2	-2
-1	1	-1
0	0	0
1	1	-1
2	2	-2

To graph both equations on the same axis, let *y = f(x)* and *y = g(x)*.

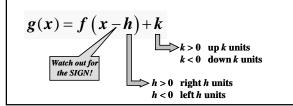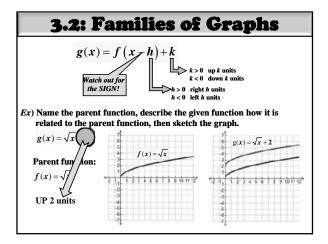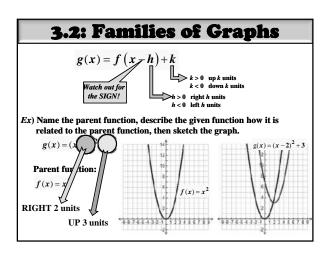The effect of multiplying a function by **-1** is a reflection over the *x*-axis.
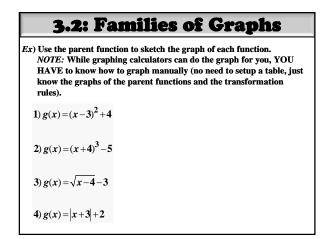
3.2: Families of Graphs

Translations

- When a constant is **added** or **subtracted** from a parent function, the result would be a translation horizontally or vertically.
- Let **g(x)** be the transformation of **f(x)** at **(h, k)**

$$g(x) = f\left(x - h\right) + k$$

Watch out for the SIGN!

$k > 0$ up *k* units
$k < 0$ down *k* units

$h > 0$ right *h* units
$h < 0$ left *h* units

3.2: Families of Graphs

$$g(x) = f\left(x - h\right) + k$$

Watch out for the SIGN!

$k > 0$ up *k* units
$k < 0$ down *k* units

$h > 0$ right *h* units
$h < 0$ left *h* units

Ex) Name the parent function, describe the given function how it is related to the parent function, then sketch the graph.

$g(x) = \sqrt{x}$

Parent function:
$f(x) = \sqrt{x}$

UP 2 units

$g(x) = \sqrt{x} + 2$

3.2: Families of Graphs

$$g(x) = f\left(x - h\right) + k$$

Watch out for the SIGN!

$k > 0$ up *k* units
$k < 0$ down *k* units

$h > 0$ right *h* units
$h < 0$ left *h* units

Ex) Name the parent function, describe the given function how it is related to the parent function, then sketch the graph.

$g(x) = (x$

Parent function:
$f(x) = x$

RIGHT 2 units

UP 3 units

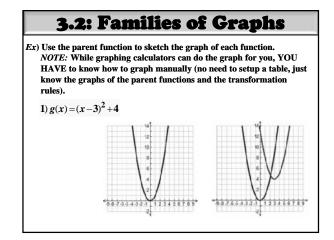$g(x) = (x - 2)^2 + 3$

$f(x) = x^2$

3.2: Families of Graphs

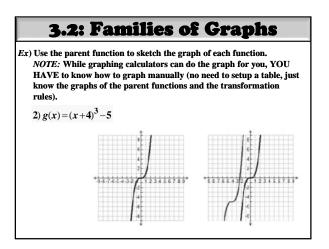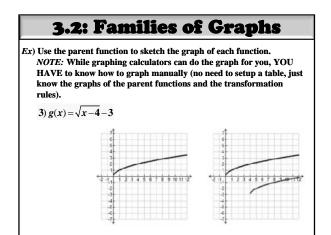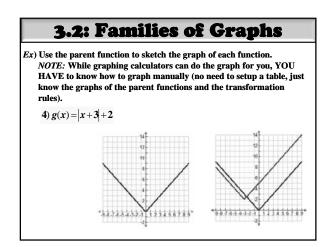*Ex*) Name the parent function, then describe the given function how it is related to the parent function.

Given Function	Parent Function	Description
$g(x) = (x + 4)^3 - 5$		
$g(x) = (x - 4)^2 + 3$		
$g(x) = \sqrt{x + 2} + 5$		
$g(x) = \|x + 2\| - 6$		
$g(x) = (x - 1)^3 - 8$		
$g(x) = (x + 3)^2 + 7$		
$g(x) = \dfrac{3x}{x + 4} - 5$		

3.2: Families of Graphs

Ex) Use the parent function to sketch the graph of each function.
   NOTE: While graphing calculators can do the graph for you, YOU
   HAVE to know how to graph manually (no need to setup a table, just
   know the graphs of the parent functions and the transformation
   rules).

1) $g(x)=(x-3)^2+4$

2) $g(x)=(x+4)^3-5$

3) $g(x)=\sqrt{x-4}-3$

4) $g(x)=|x+3|+2$

3.2: Families of Graphs

Ex) Use the parent function to sketch the graph of each function.
   NOTE: While graphing calculators can do the graph for you, YOU
   HAVE to know how to graph manually (no need to setup a table, just
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1) $g(x)=(x-3)^2+4$

3.2: Families of Graphs

Ex) Use the parent function to sketch the graph of each function.
   NOTE: While graphing calculators can do the graph for you, YOU
   HAVE to know how to graph manually (no need to setup a table, just
   know the graphs of the parent functions and the transformation
   rules).

2) $g(x)=(x+4)^3-5$

3.2: Families of Graphs

Ex) Use the parent function to sketch the graph of each function.
   NOTE: While graphing calculators can do the graph for you, YOU
   HAVE to know how to graph manually (no need to setup a table, just
   know the graphs of the parent functions and the transformation
   rules).

3) $g(x)=\sqrt{x-4}-3$

3.2: Families of Graphs

Ex) Use the parent function to sketch the graph of each function.
   NOTE: While graphing calculators can do the graph for you, YOU
   HAVE to know how to graph manually (no need to setup a table, just
   know the graphs of the parent functions and the transformation
   rules).

4) $g(x)=|x+3|+2$

3.2: Families of Graphs

Stretch/Compression

- A transformation that produces an image that is the same
  shape as the original in which all distances on the coordinate
  plane are stretch or compressed/shrinked by multiplying either
  all *x*-coordinates or all *y*-coordinates by a factor.
- Let $g(x)$ be the transformation of $f(x)$

$$g(x)=af\left(bx\right)$$

|*b*| > 1   horizontal compression
by a factor of 1/*b*

0 < |*b*| < 1   horizontal stretch
by a factor of 1/*b*

|*a*| > 1   vertical stretch
by a factor of *a*

0 < |*a*| < 1   vertical compression
by a factor of *a*

3.2: Families of Graphs

Ex) Graph the parent function and the given function on the same axis. Describe how the graphs of $f(x)$ and $g(x)$ are related. $g(x)=2|x|$

x	f(x) = |x|	g(x) =2 |x|
-2	2	4
-1	1	2
0	0	0
1	1	1
2	2	2

The effect of multiplying a function by 2 is a *vertical stretch by a factor of* 2.

3.2: Families of Graphs

Ex) Graph the parent function and the given function on the same axis. Describe how the graphs of $f(x)$ and $g(x)$ are related. $g(x)=0.25|x|$

x	f(x) = |x|	g(x) =0.25 |x|
-8	8	2
-4	4	1
0	0	0
4	4	1
8	8	2

The effect of multiplying a function by 0.25 is a *vertical compression by a factor of* 0.25.

3.2: Families of Graphs

Ex) Graph the parent function and the given function on the same axis. Describe how the graphs of $f(x)$ and $g(x)$ are related. $g(x)=2\sqrt{x}$

x	f(x) = \sqrt{x}	g(x) = 2\sqrt{x}
0	0	0
1	1	2
4	2	4
9	3	6

The effect of multiplying a function by 2 is a *vertical stretch by a factor of* 2.

3.2: Families of Graphs

Ex) Graph the parent function and the given function on the same axis. Describe how the graphs of $f(x)$ and $g(x)$ are related. $g(x)=\sqrt{6x}$

x	g(x) = \sqrt{6x}
0	0
1	2.45
4	4.90
9	7.35

The effect of multiplying x by 6 is a *horizontal compression by a factor of* $1/6$.

3.2: Families of Graphs

Ex) Graph the parent function and the given function on the same axis. Describe how the graphs of $f(x)$ and $g(x)$ are related. $g(x)=\sqrt{0.5x}$

x	g(x) = \sqrt{0.5x}
0	0
2	1
8	2
12	2.45

The effect of multiplying x by 0.5 is a *horizontal stretch by a factor of* 2.

3.2: Families of Graphs

Summary of Transformations

Let $g(x)$ be the transformation of $f(x)$ at (h, k)

$b < 0$ — reflection over the *y*-axis
$|b| > 1$ — horizontal compression by a factor of $1/b$
$0 < |b| < 1$ — horizontal stretch by a factor of $1/b$

Watch out for the SIGN!

$$g(x)=af\left(bx-h\right)+k$$

$a < 0$ — reflection over the *x*-axis
$|a| > 1$ — vertical stretch by a factor of *a*
$0 < |a| < 1$ — vertical compression by a factor of *a*

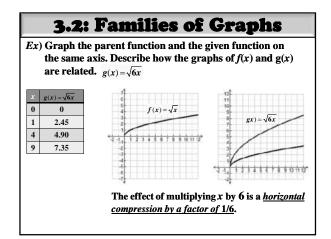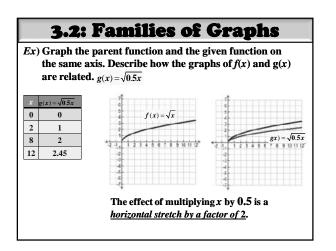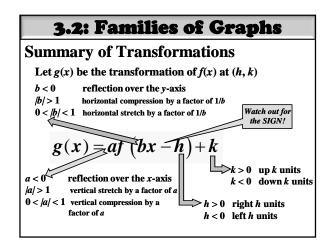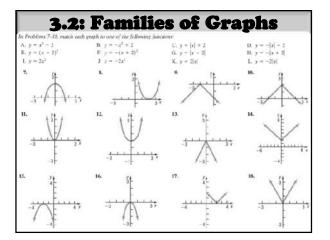$k > 0$ up *k* units
$k < 0$ down *k* units

$h > 0$ right *h* units
$h < 0$ left *h* units

3.2: Families of Graphs

Find the function that is finally graphed after the following three transformations are applied to the graph of $y = |x|$.

1. Shift left 2 units.
2. Shift up 3 units.
3. Reflect about the y-axis.

1. Shift left 2 units:	Replace x by $x + 2$.	$y = |x - 2|$
2. Shift up 3 units:	Add 3.	$y = |x + 2| + 3$
3. Reflect about the y-axis:	Replace x by $-x$.	$y = |-x + 2| + 3$

3.2: Families of Graphs

In Problems 7–18, match each graph to one of the following functions:

A. $y = x^2 - 2$
B. $y = -x^2 + 2$
C. $y = |x| + 2$
D. $y = -|x| + 2$
E. $y = (x - 2)^2$
F. $y = -(x + 2)^2$
G. $y = |x - 2|$
H. $y = -|x + 2|$
I. $y = 2x^2$
J. $y = -2x^2$
K. $y = 2|x|$
L. $y = -2|x|$

7.   8.   9.   10.

11.   12.   13.   14.

15.   16.   17.   18.

3.2: Families of Graphs

In Problems 19–26, write the function whose graph is the graph of $y = x^3$, but is:

19. Shifted to the right 4 units
20. Shifted to the left 4 units
21. Shifted up 4 units
22. Shifted down 4 units
23. Reflected about the y-axis
24. Reflected about the x-axis
25. Vertically stretched by a factor of 4
26. Horizontally stretched by a factor of 4

In Problems 27–30, find the function that is finally graphed after the following transformations are applied to the graph of $y = \sqrt{x}$.

27. (1) Shift up 2 units
    (2) Reflect about the x-axis
    (3) Reflect about the y-axis

28. (1) Reflect about the x-axis
    (2) Shift right 3 units
    (3) Shift down 2 units

29. (1) Reflect about the x-axis
    (2) Shift up 2 units
    (3) Shift left 3 units

30. (1) Shift up 2 units
    (2) Reflect about the y-axis
    (3) Shift left 3 units

31. If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = -f(x)$?
    (a) $(6, 3)$
    (b) $(6, -3)$
    (c) $(3, -6)$
    (d) $(-3, 6)$

32. If $(3, 6)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(-x)$?
    (a) $(6, 3)$
    (b) $(6, -3)$
    (c) $(3, 6)$
    (d) $(-3, 6)$

3.2: Families of Graphs

33. If $(1, 3)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = 2f(x)$?
    (a) $(1, 3)$
    (b) $(2, 3)$
    (c) $(1, 6)$
    (d) $\left(\frac{1}{2}, 3\right)$

34. If $(4, 2)$ is a point on the graph of $y = f(x)$, which of the following points must be on the graph of $y = f(2x)$?
    (a) $(4, 1)$
    (b) $(8, 2)$
    (c) $(2, 2)$
    (d) $(4, 4)$

35. Suppose that the x-intercepts of the graph of $y = f(x)$ are -5 and 3.
    (a) What are the x-intercepts of the graph of $y = f(x + 2)$?
    (b) What are the x-intercepts of the graph of $y = f(x - 2)$?
    (c) What are the x-intercepts of the graph of $y = 4f(x)$?
    (d) What are the x-intercepts of the graph of $y = f(-x)$?

36. Suppose that the x-intercepts of the graph of $y = f(x)$ are -8 and 1.
    (a) What are the x-intercepts of the graph of $y = f(x + 4)$?
    (b) What are the x-intercepts of the graph of $y = f(x - 3)$?
    (c) What are the x-intercepts of the graph of $y = 2f(x)$?
    (d) What are the x-intercepts of the graph of $y = f(-x)$?

3.2: Families of Graphs

Ex) Name the parent function, then describe the transformation.

Given Function	Parent Function	Description
1) $g(x) = 2(-3x - 2)^3 + 5$		
2) $g(x) = 0.3(2x + 4)^2 - 3$		
3) $g(x) = -3\sqrt{-0.5x - 2} - 6$		
4) $g(x) = -\frac{2}{3}\left|-\frac{4}{5}x + 2\right| - 6$		
5) $g(x) = 5(0.4x + 3)^2 + 7$		