Algebra 2/Trig 3.4: Linear Programming

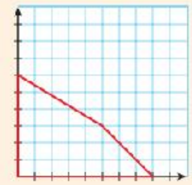
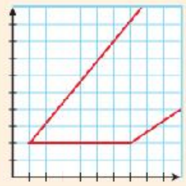
Linear programming

- a method of finding a maximum or minimum value of a function that satisfies a given set of conditions called ***constraints***.

Constraint

- one of the inequalities in a linear programming problem. The solution to the set of constraints can be graphed as a ***feasible region***.

In most linear programming problems, you want to do more than just identifying the feasible region. Often you want to find the best combination of values in order to minimize or maximize a certain function. This function is the **objective function**. The objective function may have a minimum, a maximum, neither, or both depending on the feasible region.

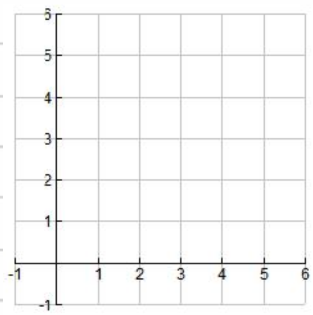
Bounded and Unbounded Regions		
Bounded Feasible Region	Unbounded Feasible Regions	
		
Objective function has both a minimum and a maximum value.	Objective function has either a maximum value or a minimum value but not both.	

The Vertex Principle of Linear Programming

- The maximum or minimum value of the objective function $f(x, y)$ on a polygon convex set occurs at a vertex of the feasible region of the polygon boundary.

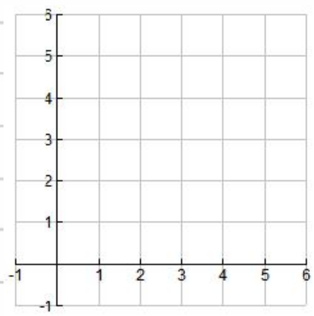
Ex. 1) What is the maximum and minimum of $f(x, y) = 3x - 2y + 5$?

Constraints: $\begin{cases} x + y \le 5 \\ x \ge 0 \\ y \ge 0 \end{cases}$



Ex. 2) Find the maximum and minimum of $f(x, y) = 2x - 3y$ for the polygon set

Constraints: $\begin{cases} x \ge 2 \\ x \le 4 \\ y \ge 1 \\ x - 2y \ge -4 \end{cases}$



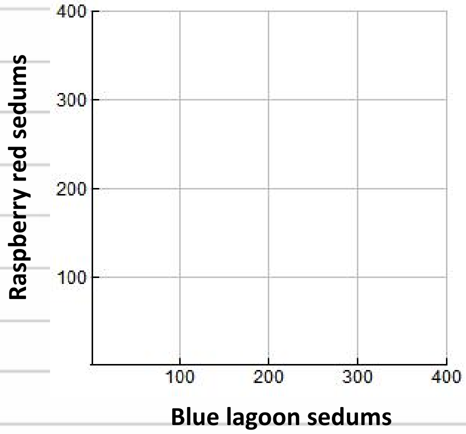
Word Problem Steps:

1) Define the variables.
2) Write the system of inequalities. (the constraints)
3) Graph the system of inequalities.
4) Find the vertices or corner points of the feasible region.
5) Write a function to be maximized or minimized.
6) Use a table to evaluate the function at each vertex.
7) Choose the appropriate value to answer the question, then write the conclusion in context.

Ex. 3) Gillian is planning a green roof that will cover up to 600 square feet. She will use two types of plants: blue lagoon sedum and raspberry red sedum. Each blue lagoon sedum will cover 1.2 square feet. Each raspberry red sedum will cover 2 square feet. Each plant costs $2.50, and Gillian must spend less than $1000. Write the constraints, and graph the feasible region.

Let ***b*** = the number of blue lagoon sedums, and
r = the number of raspberry red sedums.

Constraints: {



Ex. 4) One of Gillian's priorities for the green roof is to help control air pollution. To do this, she wants to maximize the amount of carbon dioxide the plants on the roof absorb. Use the carbon dioxide absorption rates and the data from Example 3 to find the number of each plant Gillian should plant.

Let ***C*** = the number of pounds of carbon dioxide absorbed

1) Write the objective function:
C **(*b*, *r*)** =

2) Evaluate the objective function at the vertices of the feasible region.

(*b*, *r*)	***Objective*: *C* (*b*, *r*)** = 1.4***b*** + 2.1***r***	***C*** **(lb)**

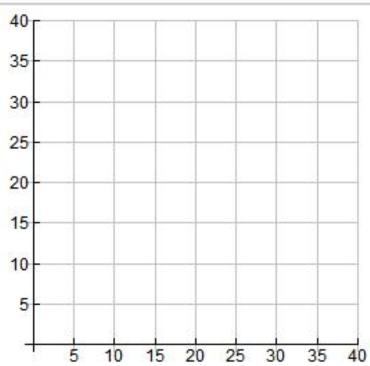
The maximum value of ______ occurs at the vertex (,   ).

3) Write the conclusion in context.
Gillian should plant _____ blue lagoon sedums and ____ raspberry red sedums to maximize the amount of carbon dioxide absorbed.

Ex. 5) Brad is an organizer of the Bolder Boulder 10K race and must hire workers for one day to prepare the race packets. Skilled workers cost $60 a day, and students cost $40 a day. Brad can spend no more than $1440. He needs at least 1 skilled worker for every 3 students, but only 16 skilled workers are available. Skilled workers can prepare 25 packets per hour, and students can prepare 18 packets per hour. Find the number of each type of worker that Brad should hire to maximize the number of packets produced.

Let ***x*** = the number of students, and
y = the number of skilled workers

Constraints: {



Objective function: $P(x, y) = 18x + 25y$

The objective function is maximized at (,   ), so Brad should hire ____ students and ____ skilled workers.