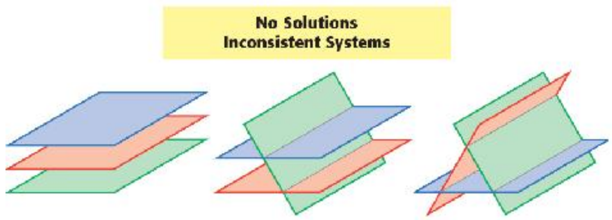
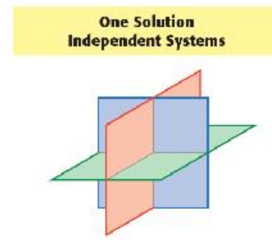
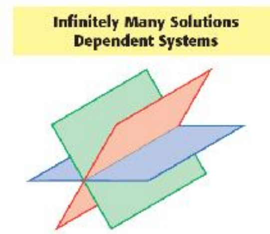
3.6: Solving Linear Systems in Three Variables

- When you graph a system of three linear equations in three dimensions, the result is three planes that may or may not intersect.
- The <u>solution</u> to the system is the set of points where <u>all three planes intersect</u>, i.e, the set of all ordered triples that satisfy all of the equations of the system. These systems may have <u>one</u>, <u>infinitely many,</u> or <u>no solution</u>. A solution to an equation in three variables is an <u>ordered triple</u> of the form (x, y, z) such as (2, 1, 5), where the first coordinate is the value of x, the second coordinate is the value of y, and the third coordinate is the value of z.







Solving a System in Three Variables

1) Name the equations as eq. 1, eq. 2, and eq. 3.
2) Use substitution or addition to eliminate any one of the variables from a pair of equations of the system. Look for the easiest variable to eliminate.
3) Eliminate the same variable from another pair of equations of the system.
4) Solve the resulting system of two equations in two unknowns.
5) After you have found the values of two of the variables, substitute into one of the original equations to find the value of the third variable.
6) Check the three values in all of the original equations.

Solve the given system. Then classify the system as ***consistent*** or ***inconsistent***, and determine the number of solutions.

1)
$$\begin{aligned} x+y-z&=-1 \quad (1)\\ 2x-2y+3z&=8 \quad (2)\\ 2x-y+2z&=9 \quad (3)\end{aligned}$$

2)
$$\begin{aligned} x+2y-3z&=-2 \quad (1)\\ 2x-2y+z&=7 \quad (2)\\ x+y+2z&=-4 \quad (3)\end{aligned}$$

3)
$$\begin{aligned} x+y\qquad&=4 \quad (1)\\ 2x\qquad-3z&=14 \quad (2)\\ 2y+z&=2 \quad (3)\end{aligned}$$

4)
$$\begin{aligned} x+y-z&=5 \quad (1)\\ 3x-2y+z&=8 \quad (2)\\ 2x+2y-2z&=7 \quad (3)\end{aligned}$$

5)
$$2x - 3y - z = 4 \quad (1)$$
$$-6x + 9y + 3z = -12 \quad (2)$$
$$4x - 6y - 2z = 8 \quad (3)$$

6)
$$4x - 2y + 4z = 8 \quad (1)$$
$$-3x + y - z = -4 \quad (2)$$
$$-2x + 2y - 6z = 4 \quad (3)$$

True or False? Explain your answer.

______ 1) The point (1, -2, 3) is in the solution set to the equation $x + y - z = 4$.
______ 2) The point (4, 1, 1) is the only solution to the equation $x + y - z = 4$.
______ 3) The ordered triple (1, -1, 2) satisfies $x + y + z = 2$, $x - y - z = 0$, and $2x + y - z = 1$.
______ 4) Substitution cannot be used on three equations in three variables.
______ 5) Two distinct planes are either parallel or intersect in a single point.
______ 6) The equations $x - y + 2z = 6$and $x - y + 2z = 4$ are inconsistent.
______ 7) The equations $3x + 2y - 6z = 4$ and $-6x - 4y + 12z = -8$ are dependent.
______ 8) The graph of $y = 2x - 3z + 4$ is a straight line.
______ 9) The value of x nickels, y dimes, and z quarters is $0.05x + 0.10y + 0.25z$ cents.
______ 10) If $x = -2$, $z = 3$, and $x + y + z = 6$, then $y = 7$.

Homework 3.6: Reading and Writing

After reading this section, write out the answers to these questions. Use complete sentences.

1) What is a linear equation in three variables?
2) What is an ordered triple?
3) What is a solution to a system of linear equations in three variables?
4) How do we solve systems of linear equations in three variables?
5) What does the graph of a linear equation in three variables look like?
6) How are the planes positioned when a system of linear equations in three variables is inconsistent?

Real-world Applications

Ex. 1) Vince took in a total of $1,240 last week from the rental of three condominiums. He had to pay 10% of the rent from the one-bedroom condo for repairs, 20% of the rent from the two-bedroom condo for repairs, and 30% of the rent from the three-bedroom condo for repairs. His total repair bill was $276. If the three-bedroom condo rents for twice as much as the one-bedroom condo, then what is the rent for each condo?

Let x = rent on the one-bedroom condo
y = rent on two-bedroom condo, and
z = rent on three-bedroom condo

Total rent: $x + y + z = 1240 \quad (1)$
Total repairs: $0.1x + 0.2y + 0.3z = 276 \quad (2)$
Condo rent: $z = 2x \quad (3)$

Ex. 2) **The sum of the measures of the angles of a triangle is 180°. In a given triangle, the measure of the second angle is twice the measure of the first. The measure of the third angle is 30° less than the sum of the measures of the first two. Find the measure of each angle.**

Let x = measure of the first angle
y = measure of the second angle
z = measure of the third angle

Ex. 3) **Monica decided to divide a total of $42,000 into three investments: a savings account paying 5% interest, a time deposit paying 7%, and a bond paying 9%. Her total annual interest from the three investments was $2600, and the interest from the savings account was $200 less than the total interest from the other two investments. How much did she invest at each rate?**

Let x = amount invested at 5%
y = amount invested at 7%
z = amount invested at 9%

Ex. 4) **In 2001, Randy Johnson of the Arizona Diamondbacks won Major League Baseball's Cy Young Award as the best pitcher in the National League. The winner is the pitcher who receives the most points, and a different number of points are given for each first-, second-, and third-place vote. The table shows the votes for the top three finishers. Find the number of points awarded for each vote.**

Player	1st Place	2nd Place	3rd Place	Total Points
Randy Johnson	30	2	0	156
Curt Schilling	2	29	1	98
Matt Morris	0	1	28	31

Let x = number of points for a first-place vote
y = number of points for a second-place vote, and
z = number of points for a third-place vote