

5.1: Angles and Degree Measure

Objectives:

- Convert decimal degree measures to degrees, minutes, and seconds and vice versa.
- Find the number of degrees in a given number of rotations.
- Identify angles that are co-terminal with a given angle.

vertex – common endpoint of two rays

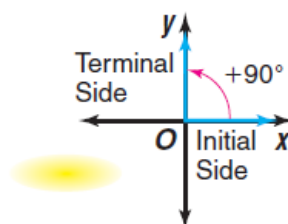
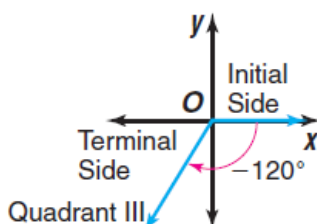
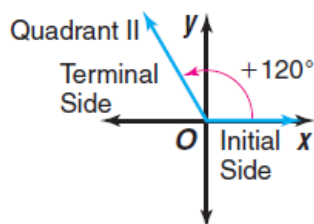
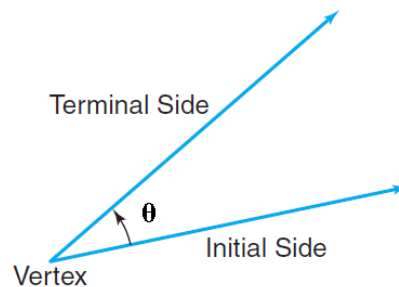
initial side – the fixed, starting ray of an angle

terminal side – the movable, ending side of an angle

θ – Greek symbol *theta* used to stand for angle measure

degree, $^\circ$ – most common unit of measure. The degree is subdivided into 60 equal parts known as **minutes** ($1'$), and the minute is subdivided into 60 equal parts known as **seconds** ($1''$).

standard position – when an angle's vertex is at the origin and its initial side is on the positive x-axis




- If the rotation is in a **counterclockwise** direction, the angle formed is a **positive angle**.
- If the rotation is **clockwise**, it is a **negative angle**.

Example 1:

a) Change 15.735° to degrees, minutes, and seconds.

b) Write north latitude $39^\circ 5' 34''$ as a decimal rounded to the **nearest thousandth**.

**Graphing Calculator Tip**

► DMS on the [ANGLE] menu allows you to convert decimal degree values to degrees, minutes, and seconds.

PRACTICE 1:

1) Change each measure to degrees, minutes, and seconds.

a) 34.95°

b) -72.775°

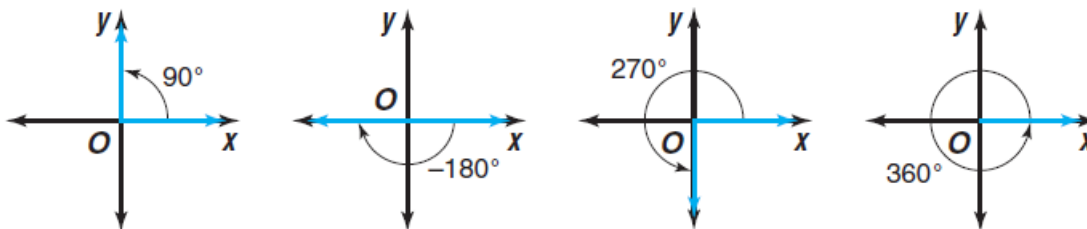
2) Write each measure as a decimal to the **nearest thousandth**.

a) $-128^\circ 30' 45''$

b) $29^\circ 6' 6''$

5.1: Angles and Degree Measure

Quadrantal Angle—an angle in standard position whose terminal side coincides with one of the axes. A **full rotation** around a circle is 360° . Measures of more than 360° represent multiple rotations.



Example 2: Find the angle measure represented by each rotation.

a) 5.5 rotations clockwise.

b) 3.3 rotations counterclockwise

PRACTICE 2: Find the angle measure represented by each rotation.

a) 2 rotations clockwise.

b) 4.5 rotations counterclockwise

coterminal angles – angles in standard position that have the same terminal side; measures will differ by a multiple of 360°
– (informal) two or more angles that end @ the same place

Coterminal Angles

If θ is the degree measure of an angle, then all angles measuring $\theta + 360k^\circ$, where k is an integer, are coterminal with θ .

Example 3A: Identify all angles coterminal with 75° .

Any angle coterminal with an angle of 75° can be written as $75^\circ + 360k^\circ$, where k is the number of rotations around the circle.

- The value of k is a **positive** integer if the rotations are *counterclockwise* and
- a **negative integer** if the rotations are *clockwise*.

Example 3B: Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with the angle.

a) 45°

b) 225°

Example 4: If each angle is in standard position, determine a coterminal angle that is between 0° and 360° . State the quadrant in which the terminal side lies.

a) 775°

b) -1297°

5.1: Angles and Degree Measure

PRACTICE 3: Identify all angles that are coterminal with each angle. Then find one positive angle and one negative angle that are coterminal with the angle.

a) 22°

b) -170°

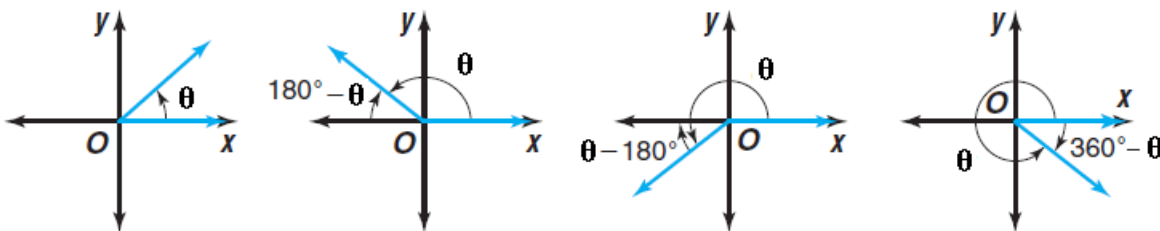
PRACTICE 4: If each angle is in standard position, determine a coterminal angle that is between 0° and 360° . State the quadrant in which the terminal side lies.

a) 453°

b) -798°

Reference Angle

- (formal definition) an acute angle (measures less than 90°) formed between the terminal side of an angle and the x -axis.
- (informal definition) how close an angle is to the x -axis.



For any angle θ , $0^\circ < \theta < 360^\circ$, its reference angle θ' is defined by

Reference

a) θ , when the terminal side is in Quadrant I,

b) $180^\circ - \theta$, when the terminal side is in Quadrant II,

Angle Rule

c) $\theta - 180^\circ$, when the terminal side is in Quadrant III, and

d) $360^\circ - \theta$, when the terminal side is in Quadrant IV.

Example 5: Find the measure of the reference angle for each angle. Convert to a standard angle first ($0^\circ < \theta < 360^\circ$), if necessary, then apply the reference angle rules.

a) 120°

b) -135°

c) 612°

d) -195°

PRACTICE 5: Find the measure of the reference angle for each angle. Convert to a standard angle first ($0^\circ < \theta < 360^\circ$), if necessary, then apply the reference angle rules.

a) 227°

b) -210°

c) 563°

d) -1045°