## STARTER 5.2

Find the reference angle for each angle in standard position.

1) $320^{\circ}$
2) $-545^{0}$
3) $-225^{0}$
4) $89^{\circ}$

Vocabulary of Angles ...
Initial side

Terminal side

Vertex
Standard position


| Quadrant | I | II | III | IV |
| :---: | :---: | :---: | :---: | :---: |
| Reference Angle |  |  |  |  |

## 5.2: Trigonometric Ratios in Right Triangles

## Objective:

- Find the values of trigonometric ratios for acute angles of right triangles.

In a right triangle, one of the angles measures $90^{\circ}$, and the remaining two angles are acute and complementary.
acute angle - an angle that measures less than $90^{\circ}$ complementary angles - two angles that add up to $90^{\circ}$ hypotenuse - the longest side
legs - the two perpendicular sides of a right triangle adjacent side - the leg that is a side of an acute angle
opposite side - the leg opposite an acute angle


## TRIGONOMETRIC RATIOS

|  | Words | Symbol | Definition |  |
| :--- | :---: | :---: | :---: | :---: |
| Trigonometric <br> Ratios | $\operatorname{sine} \theta$ | $\sin \theta$ | $\sin \theta=\frac{\operatorname{side~opposite~}}{\text { hypotenuse }}$ | Side <br> Opposite |
|  | $\operatorname{cosine} \theta$ | $\cos \theta$ | $\cos \theta=\frac{\text { side adjacent }}{\text { hypotenuse }}$ |  |
|  | tangent $\theta$ | $\tan \theta$ | $\tan \theta=\frac{\text { side opposite }}{\text { side adjacent }}$ |  |
| Side Adjacent |  |  |  |  |

## SOH-CAH-TOA

$\operatorname{Sin} \theta=\frac{\text { Opposite }}{\text { Hypotenuse }}$
$\operatorname{Cos} \theta=\frac{\text { Adjacent }}{\text { Hypotenuse }}$

## CAUTION:!!

## $\sin \theta$ is read "the $\sin$ of $\theta$." Writing "sin" by itself is meaningless and must be avoided. NAKED TRIG FUNCTIONS!

Example 1: Find the values of the sine, cosine, and tangent for $\angle \boldsymbol{B}$.
Leave answers to simplest fraction or radical form.


PRACTICE 1: Find the values of the sine, cosine, and tangent for $\angle \boldsymbol{T}$. Leave answers to simplest fraction or radical form.


## RECIPROCAL IDENTITIES

|  | Words | Symbol | Definition | Side Opposite |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Reciprocal Trigonometric Ratios | cosecant $\theta$ | $\csc \theta$ | $\csc \theta=\frac{1}{\sin \theta}$ or $\frac{\text { hypotenuse }}{\text { side opposite }}$ |  |  |
|  | secant $\theta$ | $\sec \theta$ | $\sec \theta=\frac{1}{\cos \theta}$ or $\frac{\text { hypotenuse }}{\text { side adjacent }}$ |  |  |
|  | cotangent $\theta$ | $\cot \theta$ | $\cot \theta=\frac{1}{\tan \theta}$ or $\frac{\text { side adjacent }}{\text { side opposite }}$ |  |  |

$$
\csc \theta=\frac{1}{\sin \theta} \quad \sec \theta=\frac{1}{\cos \theta} \quad \cot \theta=\frac{1}{\tan \theta}
$$

## Example 2:

a) If $\cos \theta=\frac{3}{4}$, find $\sec \boldsymbol{\theta}, \sin \boldsymbol{\theta}$, and $\cot \boldsymbol{\theta}$.
b) If $\csc \theta=1.345$, find $\sin \theta$.

## PRACTICE 2:

a) If $\sin \theta=\frac{2}{5}$, find $\csc \theta, \cos \theta$, and $\cot \theta$.
b) If $\cot \theta=1.5$, find $\sin \theta$.

Example 3: Find the values of the six trigonometric ratios for $\angle \boldsymbol{P}$.
Leave answers to simplest fraction or radical form.


PRACTICE 3: Find the values of the six trigonometric ratios for $\angle \boldsymbol{P}$. Leave answers to simplest fraction or radical form.


RECALL: Special Triangles $30^{\circ}-60^{\circ}-90^{\circ}$ and $45^{\circ}-45^{\circ}-90^{\circ}$


Complete the table:

| $\theta$ | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ | $\csc \theta$ | $\boldsymbol{\operatorname { s e c }} \theta$ | $\boldsymbol{\operatorname { c o t } \theta}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $30^{\circ}$ |  |  |  |  |  |  |
| $45^{\circ}$ |  |  |  |  |  |  |
| $60^{\circ}$ |  |  |  |  |  |  |

Look at the values that are the same in this chart. Do you notice a pattern?

Example 4: Evaluate the following expressions without using a calculator.
a) $\cos 30^{\circ} \sec 30^{\circ}$
b) $\left(\sin 60^{\circ}\right)^{2}+\left(\cos 60^{\circ}\right)^{2}$
c) $\sin 45^{\circ} \cos 45^{\circ}$

## RECALL:

- Two angles are said to be complementary when they add up to $90^{\circ}$.
- The angles $\boldsymbol{\theta}$ and $90^{\circ}-\boldsymbol{\theta}$ are complementary since they add up to $90^{\circ}$.
- $\operatorname{Sin} 30^{\circ}=\operatorname{Cos}\left(90^{\circ}-30^{\circ}\right)=\operatorname{Cos} 30^{\circ}$.

|  | $\sin \theta=\cos \left(90^{\circ}-\boldsymbol{\theta}\right)$ | $\cos \boldsymbol{\theta}=\sin \left(90^{\circ}-\boldsymbol{\theta}\right)$ |
| :--- | :--- | :--- |
| Cofunctions | $\tan \theta=\cot \left(90^{\circ}-\boldsymbol{\theta}\right)$ | $\cot \theta=\tan \left(90^{\circ}-\boldsymbol{\theta}\right)$ |
|  | $\sec \theta=\csc \left(90^{\circ}-\boldsymbol{\theta}\right)$ | $\csc \theta=\sec \left(90^{\circ}-\boldsymbol{\theta}\right)$ |

Example 5: Find the complements of each angle and the required cofunction. Complete the table.

| $\boldsymbol{\theta}$ | Complement | $\sin \boldsymbol{\theta}$ | $\cos \left(90^{0}-\boldsymbol{\theta}\right)$ | $\tan \boldsymbol{\theta}$ | $\cot \left(90^{0}-\boldsymbol{\theta}\right)$ | $\sec \boldsymbol{\theta}$ | $\csc \left(90^{0}-\boldsymbol{\theta}\right)$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $26^{0}$ |  |  |  |  |  |  |  |
| $48^{0}$ |  |  |  |  |  |  |  |
| $72^{0}$ |  |  |  |  |  |  |  |
| $39^{0}$ |  |  |  |  |  |  |  |
| $16^{0}$ |  |  |  |  |  |  |  |

