STARTER 5.3

1) Find two positive and two negative angles that are coterminal with 86^0.

2) Find the values of the six trigonometric ratios for $\angle M$.

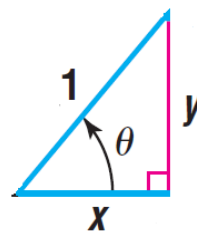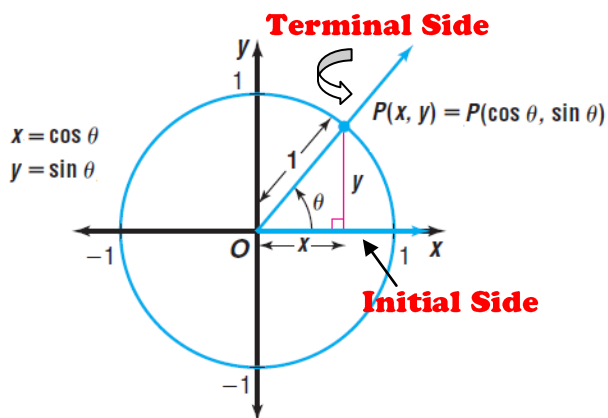**Leave answers to simplest fraction or radical form.**
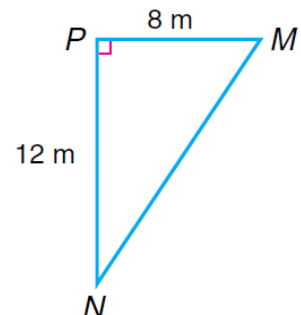
5.3: Trigonometric Functions on the Unit Circle

Objectives:

- Find the values of the six trigonometric functions using the unit circle.
- Find the values of the six trigonometric functions of an angle in standard position given a point on its terminal side.
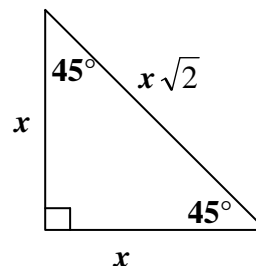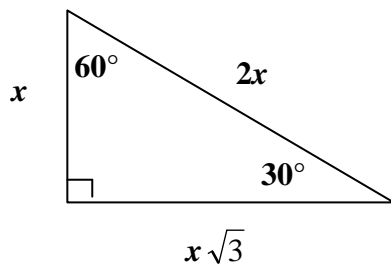
Unit circle – a circle whose **radius = 1**

$x = \cos \theta$
$y = \sin \theta$

$P(x, y) = P(\cos \theta, \sin \theta)$

$$\textbf{S}\text{in } \theta = \frac{\textbf{O}\text{pposite}}{\textbf{H}\text{ypotenuse}} = \frac{y}{1} = y$$

$$\textbf{C}\text{os } \theta = \frac{\textbf{A}\text{djacent}}{\textbf{H}\text{ypotenuse}} = \frac{x}{1} = x$$

RECALL these special right triangles.

RECALL: Fill in the following chart using trig ratios. These are common angles, so you should have these values **memorized** (concentrate on sine, cosine, and tangent).

θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
30^0						
45^0						
60^0						

CIRCULAR (Trigonometric) FUNCTIONS (when the radius = 1)

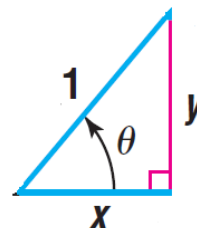$$\sin\theta = y \qquad\qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = x \qquad\qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad\qquad \cot\theta = \frac{x}{y}$$

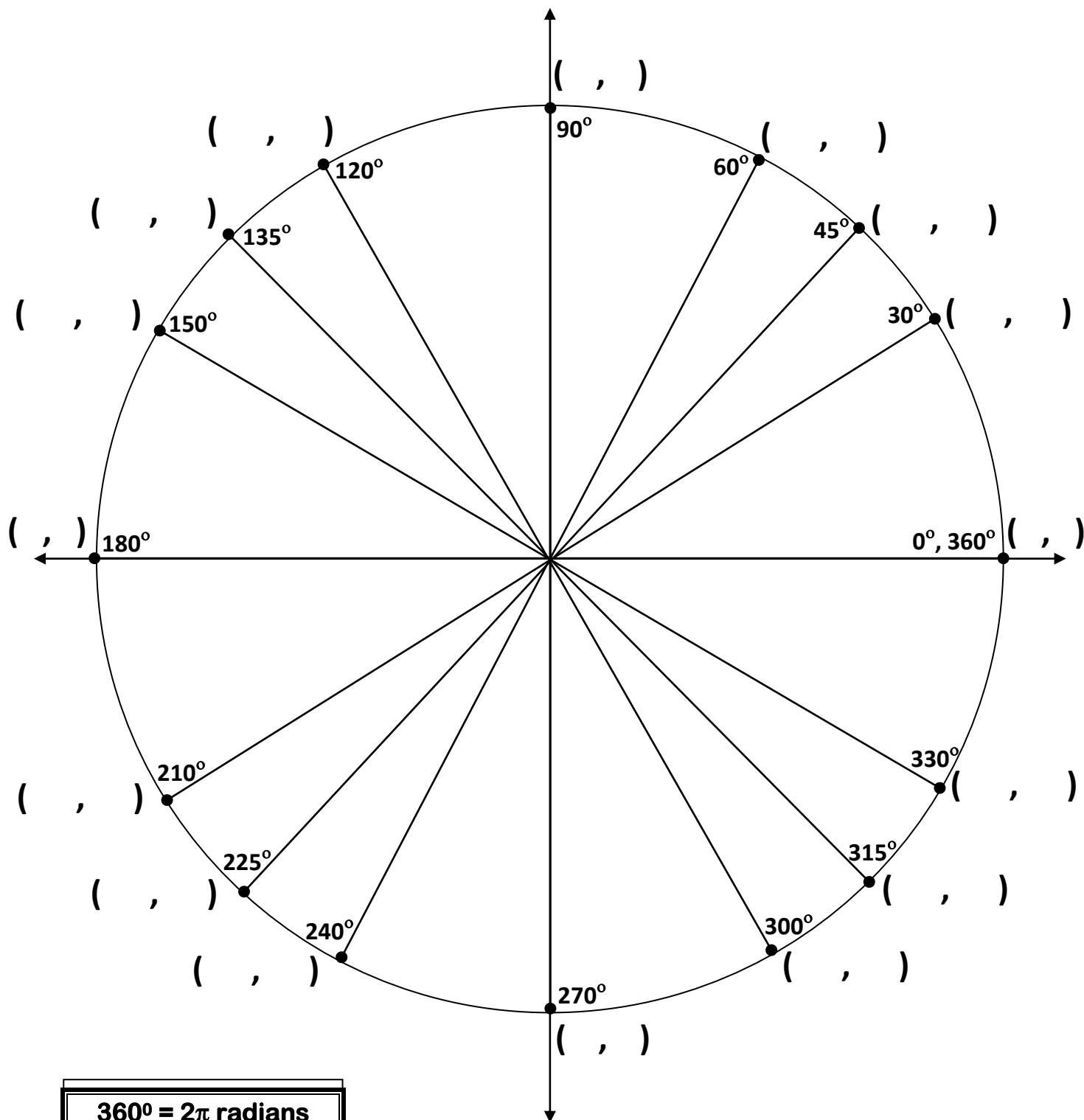Complete the chart for each quadrantal (multiple of 90^0) angle.

θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0^0						
90^0						
180^0						
270^0						

NOTE:

- The unit circle is a *parametric function*. x and y are in terms of a third variable, θ.
- The radius of a circle is defined as a positive value. Therefore, the signs of the six trigonometric functions are determined by the signs of the coordinates of x and y in each quadrant.
- **Due to symmetry, points on quadrants II, III, and IV are mirror images of the points on quadrant I.**

THE UNIT CIRCLE

(,)

90°

(,)

(,)

120°

60°

(,)

(,)

135°

45°

(,)

30°

(,)

150°

(,)

180°

0°, 360°

(,)

210°

330°

(,)

(,)

225°

315°

(,)

240°

300°

(,)

(,)

270°

(,)

360⁰ = 2π radians
180⁰ = π radian

$$360^0 = 2\pi \text{ radians}$$

$$180^0 = \pi \text{ radian}$$

Example 1: Use the unit circle to find each value.
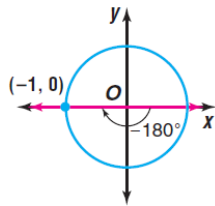
        a) $\cos\,(-180°)$ b) $\sec\,90°$
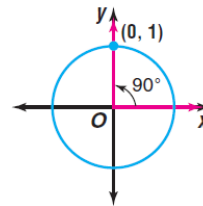
PRACTICE 1: Use the unit circle to find each value.

      a) $\sin\,(-180°)$ b) $\tan\,270°$ c) $\cos\,45°$

Example 2: Using the unit circle, find all six trigonometric values for a 210^0 angle.

PRACTICE 2: Using the unit circle, find all six trigonometric values for a 135^0 angle.

Fill in this chart with the appropriate sign.

Function	Quadrant			
	I	**II**	**III**	**IV**
sin θ _or_ csc θ				
cos θ _or_ sec θ				
tan θ _or_ cot θ				

Trigonometric Functions of an Angle in Standard Position $(r \neq 1)$

For any angle in standard position with measure θ, a point $P(x, y)$ on its terminal side, and $r = \sqrt{x^2 + y^2}$, the trigonometric functions of θ are as follows.
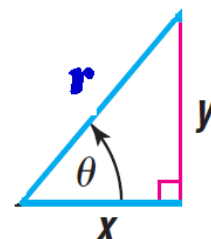
$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

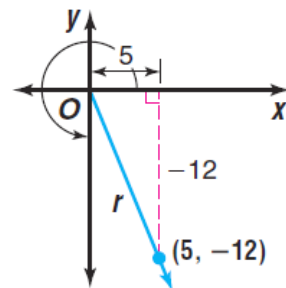$$\csc\theta = \frac{r}{y} \qquad \sec\theta = \frac{r}{x} \qquad \cot\theta = \frac{x}{y}$$

***Example* 3:** Find the values of the six trigonometric functions for angle θ in standard position if a point with coordinates **(5, -12)** lies on its terminal side.

***PRACTICE* 3:** Find the values of the six trigonometric functions for angle θ in standard position if a point with coordinates **(-5, 3)** lies on its terminal side.

***Example* 4:** Suppose θ is an angle in standard position whose terminal side lies in **Quadrant III**. If $\sin\theta = -\dfrac{4}{5}$, find the values of the remaining five trigonometric functions of θ.
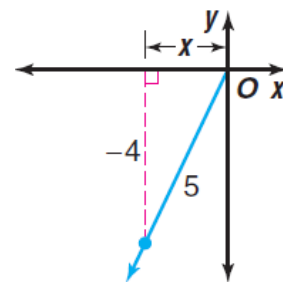
***PRACTICE* 4A:** Suppose θ is an angle in standard position whose terminal side lies in **Quadrant IV**. If $\sec\theta = \dfrac{\sqrt{29}}{5}$, find the values of the remaining five trigonometric functions of θ.

***PRACTICE* 4B:** Find $\csc\theta$ if $\cos\theta = \dfrac{8}{17}$ and the terminal side of θ is in **Quadrant III**.