STARTER 5.5

Using your Unit Circle, find the angle(s) θ that fit the following criteria.

1) $\sin\theta = \dfrac{\sqrt{3}}{2}$ **2)** $\cos\theta = 0$ **3)** $\csc\theta = undefined$

4) $\cot\theta = -1$ **5)** $\sec\theta = -2$ **6)** $\tan\theta = 0$

5.5: Solving Right Triangles

Objective:

- **Evaluate inverse trigonometric functions.**
- **Find missing angle measurements.**
- **Solve right triangles.**

If you know a trigonometric value of an angle, but **NOT the ANGLE**, you need to use an **INVERSE** of the trigonometric function to solve for the angle. For example, solving for the angle $\sin\theta = \dfrac{\sqrt{3}}{2}$ can be done using $\theta = arc\sin\left(\dfrac{\sqrt{3}}{2}\right)$ or $\theta = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$.

Inverse of the Trigonometric Functions	Trigonometric Function	Inverse Trigonometric Relation
	$y = \sin\theta$	$\theta = \sin^{-1} y$ *or* $\theta = \arcsin y$
	$y = \cos\theta$	$\theta = \cos^{-1} y$ *or* $\theta = \arccos y$
	$y = \tan\theta$	$\theta = \tan^{-1} y$ *or* $\theta = \arctan y$

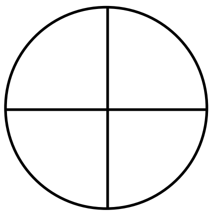
Why is the inverse called a RELATION and NOT a FUNCTION? A relation can have multiple solutions whereas a function has **only one solution**.

IMPORTANT!!!

Inverse trigonometric relations are ANGLE measures.
Thus, $\sin^{-1} y$, $\cos^{-1} y$, *and* $\tan^{-1} y$ are ANGLE measures.

***Example* 1:** Solve each equation for θ.

a) $\sin\theta = \frac{\sqrt{2}}{2}$



RECALL:

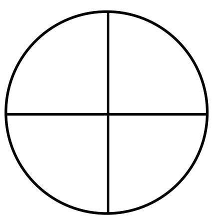
In the unit circle, $y = \sin\theta$.

So, find where the __y__ value is __$\frac{\sqrt{2}}{2}$__ on the unit circle.

Ask yourself, " __***Sine***__ **of what degree(s) is** __$\frac{\sqrt{2}}{2}$__ **?**

θ = ____________________

b) $\cos\theta = -\frac{1}{2}$



RECALL:

In the unit circle, $x = \cos\theta$.

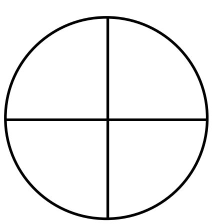
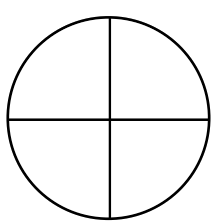
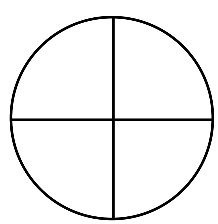
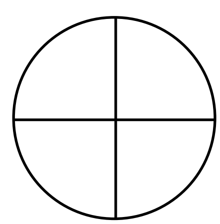
So, find where the ____ value is ____ on the unit circle.

Ask yourself, " ____ **of what degree(s) is** ____ **?**

θ = ____________________

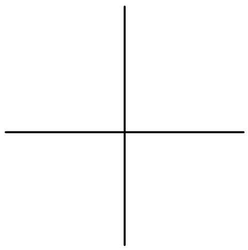
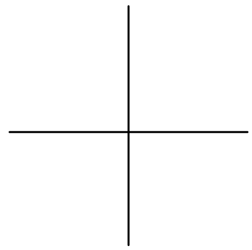
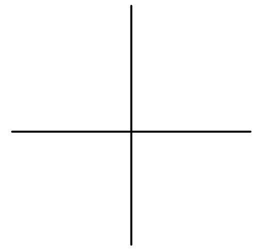
***PRACTICE*:** Solve each equation for θ.

1) $\sin\theta = \frac{\sqrt{3}}{2}$ **2)** $\tan\theta = 1$ **3)** $\sin\theta = -\frac{1}{2}$ **4)** $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

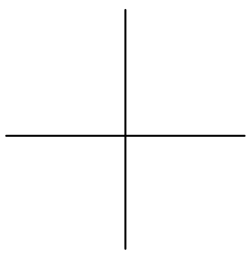
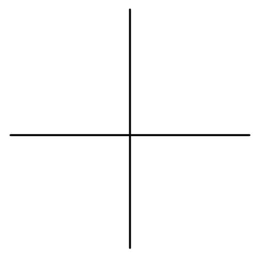
Example 2: **Evaluate each expression. Assume that all angles are in Quadrant I.**

1) $\tan\left(\tan^{-1}\left(\frac{6}{11}\right)\right)$ **2)** $\cos\left(\arcsin\left(\frac{2}{3}\right)\right)$ **3)** $\cos\left(\sec^{-1}\left(\frac{12}{5}\right)\right)$

PRACTICE 2: **Evaluate each expression. Assume that all angles are in Quadrant I.**

1) $\tan\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$ **2)** $\cos\left(\cot^{-1}\left(\frac{2}{5}\right)\right)$

Example 3: **Evaluate each expression. Assume that all angles are in Quadrant I.**

1) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)-\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ **2)** $\tan^{-1}(1)+\cot^{-1}\left(\sqrt{3}\right)$

Example 4: **Determine if the statement is TRUE or FALSE for all values in Quadrant I.**

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\tan^{-1}\left(\sqrt{3}\right)=\sin^{-1}(1)+\cos^{-1}(1)$$

Practice 5.5A

Solve each equation.

1) $\sin\theta = \frac{\sqrt{2}}{2}$

2) $\cos\theta = -\frac{\sqrt{2}}{2}$

3) $\sin\theta = \frac{\sqrt{3}}{2}$

Evaluate each expression. Assume that all angles are in Quadrant I.

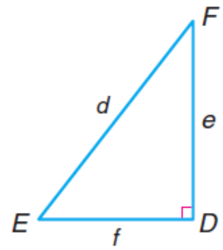
4) $\cos\left(\arctan\left(\frac{3}{7}\right)\right)$

5) $\tan\left(\cos^{-1}\left(\frac{5}{13}\right)\right)$

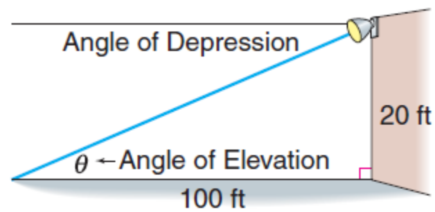
6) $\sec\left(\cot^{-1}\left(\frac{2}{5}\right)\right)$

Inverse trigonometric relations can be used to find the **measure of ANGLES** of right triangles.

Example 5: **If $f = 17$ and $d = 32$, find E.**



Example 6: **A security light is being installed outside a loading dock. The light is mounted 20 feet above the ground. The light must be placed at an angle so that it will illuminate the end of the parking lot. If the end of the parking lot is 100 feet from the loading dock, what should be the angle of depression of the light?**

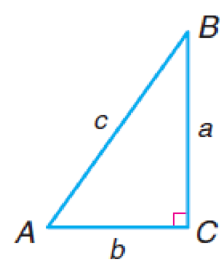


What does it mean to **"solve"** a triangle?

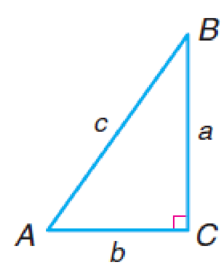
Find **ALL angles** and **ALL side lengths**.

Example 7: **Solve each triangle described, given the triangle at the right.**

a) $A = 33°$, $b = 5.8$



b) $a = 23$, $c = 4$



Practice 5.5B

1) **Tell** whether the solution to each equation is an angle measure or a linear measurement.

   a) $\tan 34^\circ = \frac{x}{12}$

   b) $\tan x = 3.284$

2) **Describe** the relationship of the two acute angles of a right triangle.

3) A highway has a 6% grade, meaning that the highway rises 6 meters for every 100 meters measured horizontally. What is the angle of inclination (elevation)?

4) Jack and Jill fall into the lake off of a dock that is 6 feet above the water. Jack swims out so he is 10 feet further from the dock than Jill. Jill can see the dock at a 40° angle of elevation. At what angle of elevation can Jack see the dock?

Solve for all missing parts given that a and b are the legs of a right triangle. Draw a right triangle and label the parts.

5) B = 42°, b = 4.5

6) b = 18, c = 52