STARTER 5.6

Solve each equation if $0^0 \le \theta \le 360^0$.

1) $\tan\theta = \frac{\sqrt{3}}{3}$

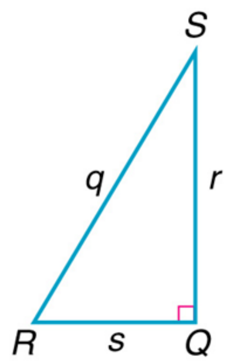
2) $\cos\theta = -\frac{1}{2}$

Evaluate each expression. Assume that all angles are in Quadrant I.

3) $\cos\left(\arccos\left(\frac{2}{5}\right)\right)$

4) $\cos\left(\sin^{-1}\left(\frac{5}{6}\right)\right)$

5) If ***q*** = 38 and ***r*** = 22, find ***S***.



5.6: The Law of Sines

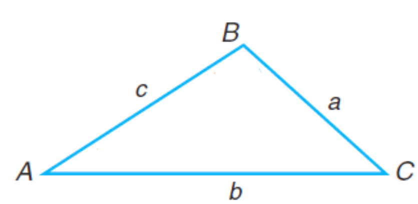
Objective:

- **Solve triangles by using the Law of Sines if the measures of two angles and a side are given.**
- **Find the area of a triangle if the measures of two sides and the included angle or the measures of two angles and a side are given**

Law of Sines

- Is used to solve non-right triangles **(i.e., DO NOT have a right angle)**

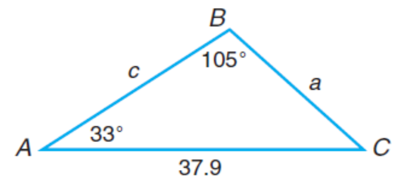
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



where:

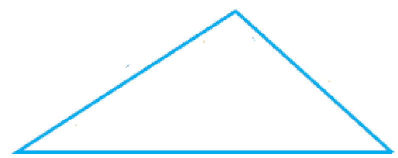
- ***A, B, C*** are angle measures
- ***a, b, c*** represent lengths of the sides opposite their corresponding angles

Example 1: Solve $\triangle ABC$ if $A = 33°$, $B = 105°$, and $b = 37.9$.



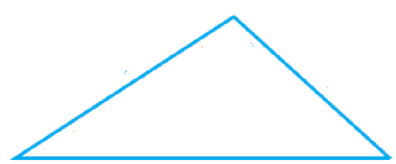
A =		a =	
B =		b =	
C =		c =	

Example 2: Solve $\triangle ABC$ if $A = 35°$, $B = 15°$, and $c = 5$.



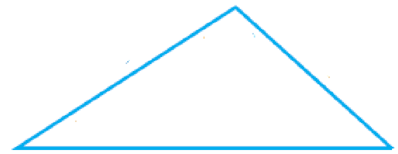
A =		a =	
B =		b =	
C =		c =	

PRACTICE 1: Solve $\triangle ABC$ if $A = 50°$, $B = 60°$, and $a = 3$.



A =		a =	
B =		b =	
C =		c =	

PRACTICE 2: Solve $\triangle ABC$ if $A = 40°$, $B = 59°$, and $c = 14$.



A =		a =	
B =		b =	
C =		c =	

Solve:

1) A baseball fan is sitting directly behind home plate in the last row of the upper deck of U.S. Cellular Field. The angle of depression to home plate is 29°54' and the angle of depression to the pitcher's mound is 24°12'. In major league baseball, the distance between home plate and the pitcher's mound is 60.5 feet. How far is the fan from home plate?

2) From his boat, Matt can see the top of a lighthouse at an angle of elevation of 21°. If he sails 80 meters closer, he sees the top of the lighthouse at an angle of elevation of 33°. How far is Matt's boat from the base of the lighthouse?

Area of a Triangle (SAS)

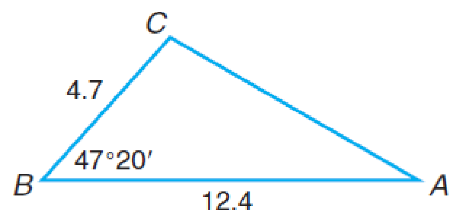
- Let ΔABC be any triangle with ***a, b,*** and ***c*** representing the measures of the sides opposite the angles with measurements ***A, B,*** and ***C,*** respectively.

Then the area ***K*** can be determined using one of the following formulas.

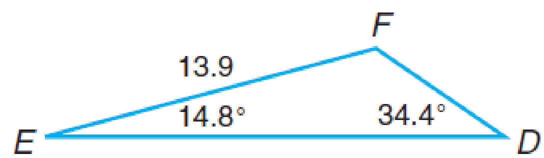
$$K = \frac{1}{2}bc \cdot \sin A, \qquad K = \frac{1}{2}ac \cdot \sin B, \qquad K = \frac{1}{2}ab \cdot \sin C$$

Note: **Area of any triangle could be solve by finding one side first and then use any of the above formulas. (AAS)**

Example 3: **Find the area of ΔABC if a = 4.7, c = 12.4, and B = 47° 20'.**



Example 4: **Find the area of ΔDEF if d = 13.9, D = 34.4°, and E = 14.8°.**



Example 5: **A regular pentagon is inscribed in a circle whose radius measures 9 inches. Find the area of the pentagon.**

Example 6: **A landscaper wants to plant begonias along the edges of a triangular plot of land in Winton Woods Park. Two of the angles of the triangle measure 95° and 40°. The side between these two angles is 80 feet long.**

a) Find the measure of the third angle.

b) Find the length of the other two sides of the triangle.

c) What is the perimeter of this triangular plot of land?

Example 7: **The center of the Pentagon in Arlington, Virginia, is a courtyard in the shape of a regular pentagon. The pentagon could be inscribed in a circle with radius of 300 feet. Find the area of the courtyard.**

Area of a Triangle (SSS): Heron's Formula

- Let ΔABC be any triangle with ***a, b,*** and ***c*** representing the measures of the sides opposite the angles with measurements ***A, B,*** and ***C***, respectively and ***s*** be the semi-perimeter.

Then the area ***K*** can be determined the Heron's formula.

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

***Example* 8**: **Find the area of ΔABC if $a = 3$, $b = 5$, and $c = 7$.**