6.1: Angles and their Measure

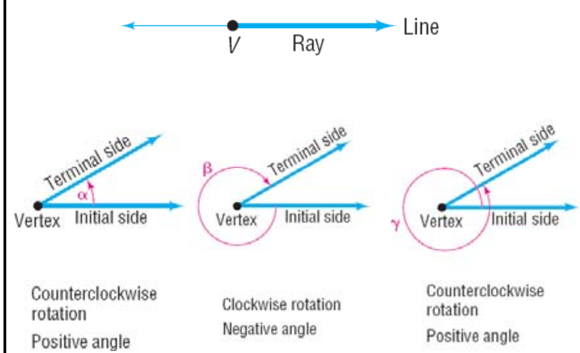
Terms You Need to Know

- **Revolution** – a complete circular motion (360^0).
- **Degree** – a common unit for measuring small angles.
- **Radian measure** – the number of radian units in the length of an arc. $2\pi = 360^0$
- **Standard position** – an angle's position where the vertex is at the origin and its initial ray (side) is along the *x*-axis.
- **Quadrantal Angle**– the terminal ray of an angle lies along an axis (multiples of 90^0 or $\frac{\pi}{2}$).

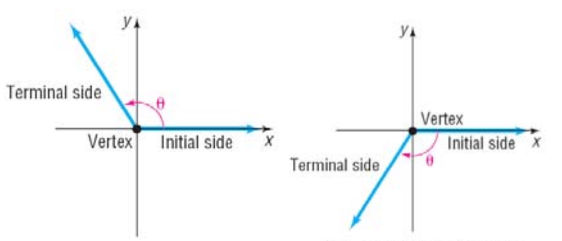
Terms you need to know:

- **Central Angle** – a positive angle whose vertex is at the center of a circle.
- **Co-terminal Angles** – two angles in an extended angle-measurement system that have the same initial side and the same terminal side, yet have different measures.
- **Sector** – the region bounded by a central angle and the intercepted arc.

Terms You Need to Know



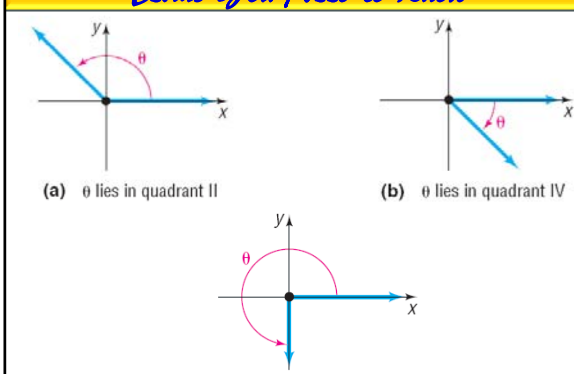
Terms You Need to Know



(a) θ is in standard position; θ is positive

(b) θ is in standard position; θ is negative

Terms You Need to Know

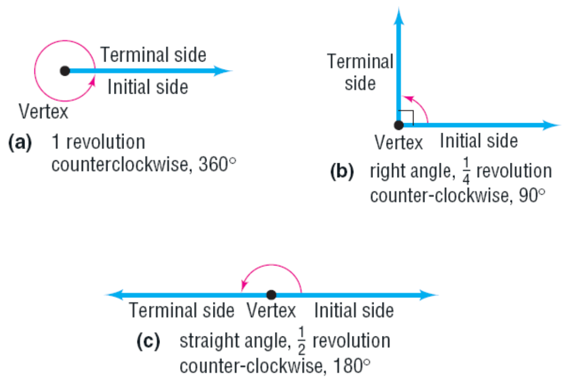


(a) θ lies in quadrant II

(b) θ lies in quadrant IV

(c) θ is a quadrantal angle

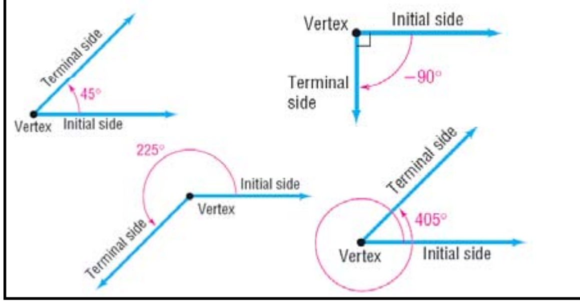
Degrees



EXAMPLE

Draw each angle.

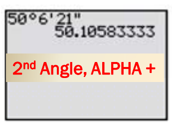
(a) 45° (b) −90° (c) 225° (d) 405°



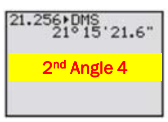
1 counterclockwise revolution = 360°

1° = 60′ 1′ = 60″

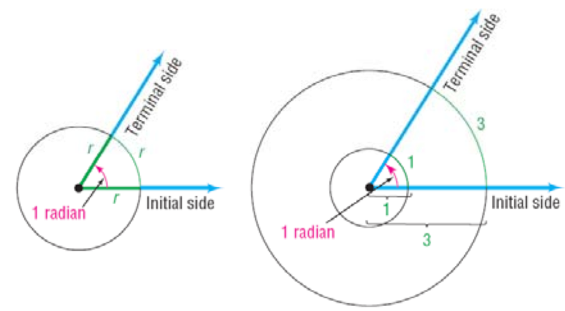
(a) Convert 50°6'21" to decimal in degrees. Round the answer to four decimal places.



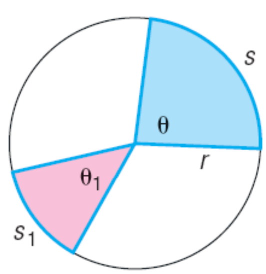
(b) Convert 21.256° to the D°M'S" form. Round the answer to the nearest second.



Radians



Arc Length of a Circle



$$\frac{\theta}{\theta_1} = \frac{s}{s_1}$$

Arc Length of a Circle

Arc Length

For a circle of radius r, a central angle of θ radians subtends an arc whose length s is

$$s = r\theta$$

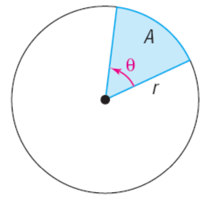
Arc Length of a Circle

EXAMPLE

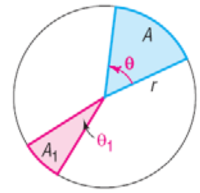
Find the length of the arc of a circle of radius 4 meters subtended by a central angle of 0.5 radian.

$$s = r\theta$$

Area of a Sector



$$\frac{\theta}{\theta_1} = \frac{A}{A_1}$$



The area A of the sector of a circle of radius r formed by a central angle of θ radians is

$$A = \frac{1}{2}r^2\theta$$

Arc Length and Area of a Sector

Let s be an arc length and A be the area of a sector with central angle θ

- **If θ is in degrees, then**

$$s = \frac{\theta}{360}\cdot 2\pi r, \qquad A = \frac{\theta}{360}\cdot \pi r^2$$

- **If θ is in radians, then**

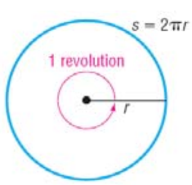
$$s = r\theta, \qquad A = \frac{1}{2}r^2\theta$$

EXAMPLE

Finding the Area of a Sector of a Circle

Find the area of the sector of a circle of radius 5 feet formed by an angle of 40°. Round the answer to two decimal places.

$$A = \frac{1}{2}r^2\theta$$



$$1 \text{ revolution} = 2\pi \text{ radians}$$

$$180° = \pi \text{ radians}$$

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

EXAMPLE Converting from Degrees to Radians

Convert each angle in degrees to radians.

(a) 60° (b) 150° (c) −45° (d) 90° (e) 107°

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian} \qquad 1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

EXAMPLE Converting Radians to Degrees

Convert each angle in radians to degrees.

(a) $\frac{\pi}{6}$ radian (b) $\frac{3\pi}{2}$ radians (c) $-\frac{3\pi}{4}$ radians

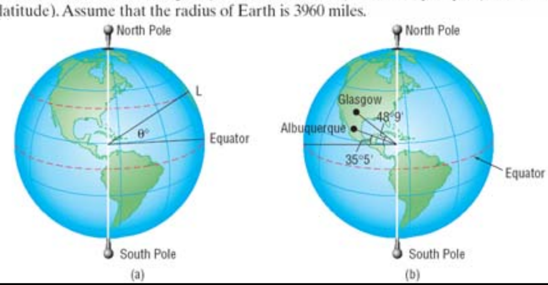
(d) $\frac{7\pi}{3}$ radians (e) 3 radians

Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Degrees		210°	225°	240°	270°	300°	315°	330°	360°
Radians		$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

EXAMPLE

Finding the Distance between Two Cities

See Figure 15(a). The latitude of a location L is the angle formed by a ray drawn from the center of Earth to the Equator and a ray drawn from the center of Earth to L. See Figure 15(b). Glasgow, Montana, is due north of Albuquerque, New Mexico. Find the distance between Glasgow (48°9′ north latitude) and Albuquerque (35°5′ north latitude). Assume that the radius of Earth is 3960 miles.



Angular and Linear Motion

- **Angular speed** is measured in units like revolutions per minute.
- **Linear speed** is measured in units like miles per hour.

Suppose an object moves along a circle of radius r at a constant speed. If s is the distance traveled in time t along this circle, then the **linear speed** v of the object is defined as

$$v = \frac{s}{t}$$

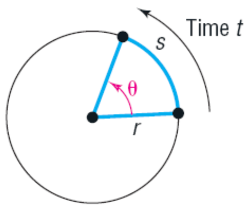
As the object travels along the circle, suppose that θ (measured in radians) is the central angle swept out in time t. Then the **angular speed** ω of this object is the angle (measured in radians) swept out divided by the elapsed time.

$$\omega = \frac{\theta}{t}$$

$$s = r\theta$$

$$\frac{s}{t} = \frac{r\theta}{t}$$

$$v = r\omega$$



$$v = \frac{s}{t}$$

Linear Speed

$$\omega = \frac{\theta}{t}$$

Angular Speed

$$v = r\omega$$