

6.3: Dividing Polynomials

Polynomial Long Division is a method for dividing a polynomial by another polynomial of a lower degree. It is very similar to dividing numbers.

Arithmetic Long Division

$$\begin{array}{r} \text{Divisor } 23 \leftarrow \text{Quotient} \\ \text{Dividend } 12 \overline{)277} \\ \underline{24} \\ 37 \\ \underline{36} \\ 1 \leftarrow \text{Remainder} \end{array}$$

Polynomial Long Division

$$\begin{array}{r} \text{Divisor } 2x+3 \leftarrow \text{Quotient} \\ \text{Dividend } x+2 \overline{)2x^2+7x+7} \\ \underline{2x^2+4x} \\ 3x+7 \\ \underline{3x+6} \\ 1 \leftarrow \text{Remainder} \end{array}$$

You can use **Polynomial Long Division** to help find all the zeros (where the function crosses the x-axis) of a polynomial function.

Example 1) Using Long Division to Divide Polynomials

Divide by using long division.

a) $(4x^2 + 3x^3 + 10) \div (x - 2)$

Step 1: Write the dividend in standard form, including terms with a coefficient of 0. $3x^3 + 4x^2 + 0x + 10$

Step 2: Write division in the same way as you would when dividing numbers.

$$x-2 \overline{)3x^3+4x^2+0x+10}$$

Step 3: Divide.

$$x-2 \overline{)3x^3+4x^2+0x+10}$$

Step 4: Write the final answer.

YOUR TURN! Divide by using long division.

b) $(x^3 + 2x^2 - x - 2) \div (x + 2)$

Synthetic Division is a shorthand method of dividing a polynomial by a **linear binomial** $(x - a)$ by using only the coefficients. For synthetic division to work, the polynomial must be written in standard form, using 0 as a coefficient for any missing terms, and the divisor must be in the form $(x - a)$.

Example 2) Using Synthetic Division to Divide by a Linear Binomial Divide by using synthetic division.

a) $(4x^2 - 12x + 9) \div (x + \frac{1}{2})$

Step 1: Find a . Then write the coefficients and a in synthetic division format.

We write $x + \frac{1}{2}$ as $x - (-\frac{1}{2})$. Thus $a = -\frac{1}{2}$.

$$\begin{array}{r|rrr} -\frac{1}{2} & 4 & -12 & 9 \\ & & & \end{array}$$

Step 2: Bring down the first coefficient. Then multiply the first coefficient by the divisor. Write the result under the next coefficient then add for each column. Repeat the steps until the remainder is found.

$$\begin{array}{r|rrr} -\frac{1}{2} & 4 & -12 & 9 \\ & & -2 & 7 \\ \hline & 4 & -14 & \underline{16} \end{array}$$

Step 3: Write the result (quotient and remainder, if any).

$$4x - 14 + \frac{16}{x + \frac{1}{2}}$$

YOUR TURN! Divide by using synthetic division.

b) $(x^4 - 5x + 10) \div (x + 3)$

Remainder Theorem (Synthetic Substitution)

- If the polynomial $P(x)$ is divided by $x - a$, then the **remainder** r is $P(a)$.

Example 2) Using Synthetic Substitution

Use synthetic substitution to evaluate the polynomial for the given value, i.e, find the remainder.

$P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$.