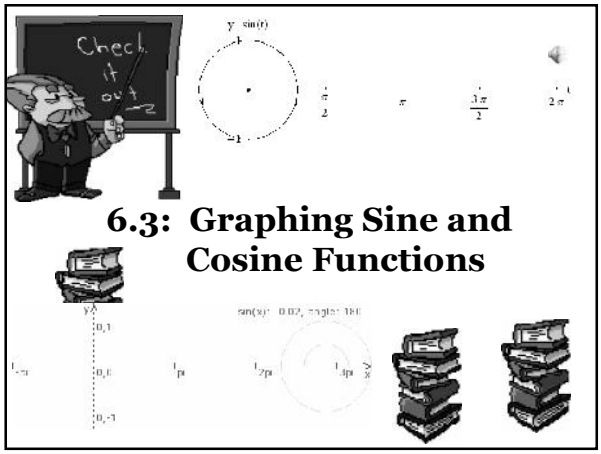


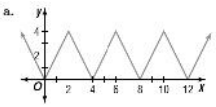
Periodic Function and Period

A function is *periodic* if, for some real number α, $f(x + \alpha) = f(x)$ for each x in the domain of f.

The least positive value of α for which $f(x) = f(x - \alpha)$ is the *period* of the function.

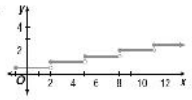
1 Determine if each function is periodic. If so, state the period.

a.



The values of the function repeat for each interval of 4 units. The function is periodic, and the period is 4.

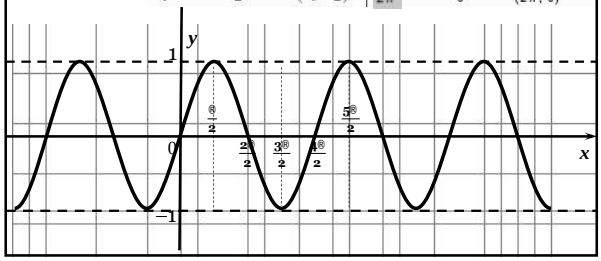
b.



The values of the function do not repeat. The function is not periodic.

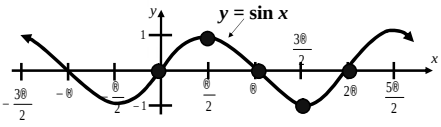
$y = 1 \sin x$

x	$y = \sin x$	(x, y)	x	$y = \sin x$	(x, y)
0	0	$(0, 0)$	π	0	$(\pi, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$	$\frac{7\pi}{6}$	$\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$	$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
			2π	0	$(2\pi, 0)$



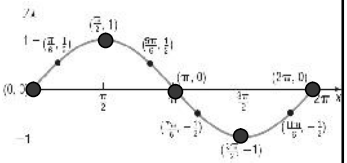
The Graph of the Sine Function $y = \sin x$

x	$y = \sin x$	(x, y)	x	$y = \sin x$	(x, y)
0	0	$(0, 0)$	π	0	$(\pi, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$	$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$	$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
			2π	0	$(2\pi, 0)$



The Graph of the Sine Function $y = \sin x$

x	$y = \sin x$	(x, y)
0	0	$(0, 0)$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	1	$\left(\frac{\pi}{2}, 1\right)$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π	0	$(\pi, 0)$
$\frac{7\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	-1	$\left(\frac{3\pi}{2}, -1\right)$
$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
2π	0	$(2\pi, 0)$

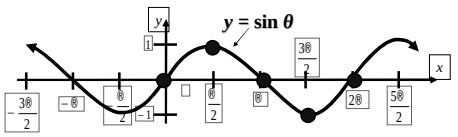


$y = \sin x, 0 \leq x \leq 2\pi$

$y = \sin x, -\infty < x < \infty$

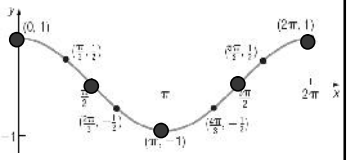
Properties of the Graph of $y = \sin \theta$

1. The period is 2π.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The maximum values are $y = 1$ and occur when $x = \frac{\pi}{2} + 2\pi n$, where n is an integer.
7. The minimum values are y 1 and occur when x $\frac{3\pi}{2}$ $2\pi n$, where n is an integer.



The Graph of the Cosine Function

x	$y = \cos x$	(x, y)
0	1	$(0, 1)$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\left(\frac{\pi}{3}, \frac{1}{2}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
π	-1	$(\pi, -1)$
$\frac{4\pi}{3}$	$-\frac{1}{2}$	$\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$
$\frac{3\pi}{2}$	0	$\left(\frac{3\pi}{2}, 0\right)$
$\frac{5\pi}{3}$	$\frac{1}{2}$	$\left(\frac{5\pi}{3}, \frac{1}{2}\right)$
2π	1	$(2\pi, 1)$

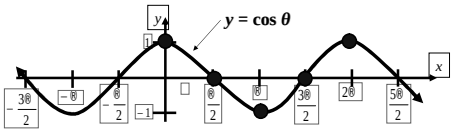


$y = \cos x, 0 \le x \le 2\pi$

$y = \cos x, -\infty < x < \infty$

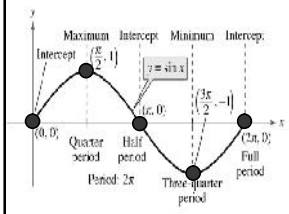
Properties of the Graph of $y = \cos \theta$

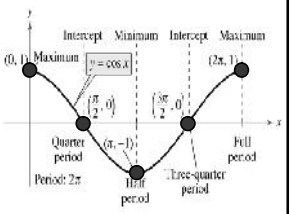
1. The period is 2π.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between 1 and 1, inclusive.
4. The x-intercepts are located at $\frac{\pi}{2}$ + πn, where n is an integer.
5. The y-intercept is 1.
6. The maximum values are y 1 and occur when x πn, where n is an even integer.
7. The minimum values are y 1 and occur when x πn, where n is an odd integer.



Points in the Sine and Cosine Curves

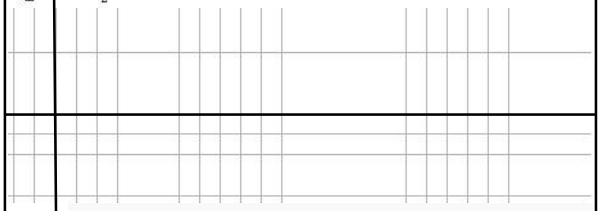
- ***Minimum***
- ***Maximum***
- ***Intercepts***





Examples

2 Find $\sin \frac{9\pi}{2}$ by referring to the graph of the sine function.



1. The period is 2π.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The maximum values are $y = 1$ and occur when $x = \frac{\pi}{2} + 2\pi n$, where n is an integer.
7. The minimum values are y 1 and occur when x $\frac{3\pi}{2}$ + $2\pi n$, where n is an integer.

Examples

2 Find $\sin \frac{9\pi}{2}$ by referring to the graph of the sine function.

1. The period is 2π.
2. The domain is the set of real numbers.
3. The range is the set of real numbers between -1 and 1, inclusive.
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The maximum values are $y = 1$ and occur when $x = \frac{\pi}{2} + 2\pi n$, where n is an integer.
7. The minimum values are y 1 and occur when x $\frac{3\pi}{2}$ + $2\pi n$, where n is an integer.

Examples

2 Find $\sin \frac{9\pi}{2}$ by referring to the graph of the sine function.

3 Find the values of θ for which $\sin \theta = 0$ is true.

4 Graph $y = \sin x$ for $3\pi \le x \le 5\pi$.

Examples

4 Graph $y = \sin x$ for $3\pi \le x \le 5\pi$.

Examples

Graph $y = \sin x, \frac{9\pi}{2} \le x \le \frac{13\pi}{2}$

Examples

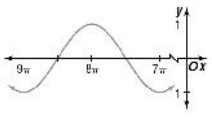
Graph $y = \cos x, 5\pi \le x \le 7\pi$

Examples

Graph $y = \sin x, -4\pi \le x \le -2\pi$

Examples

Determine whether the graph represents $y = \sin x$, $y = \cos x$, or neither.



The maximum value of 1 occurs when x 8π.

The minimum value of -1 occurs at 9π and 7π.

The x-intercepts are $\frac{17\pi}{2}$ and $\frac{15\pi}{2}$.

These are characteristics of the cosine function.