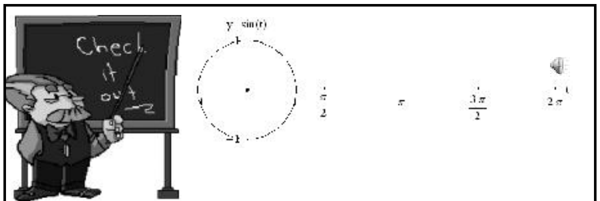
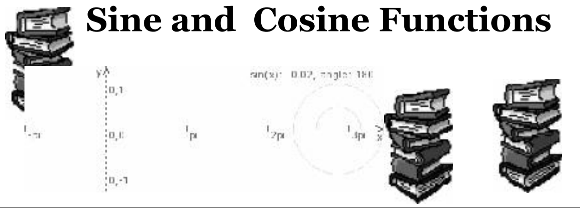


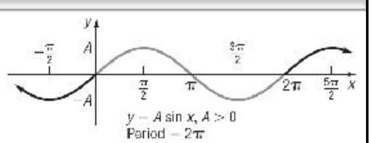
6.4: Amplitude and Period of the Sine and Cosine Functions



Properties of Sinusoidal Functions

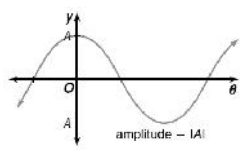
- As a class of functions, we can say that all sinusoidal functions:
  - ✓ are periodic.
  - ✓ have graphs that look like waves.
  - ✓ have a sinusoidal axis
  - ✓ have local maxima and minima.
  - ✓ display symmetry about these local maxima and local minima and these symmetries are equivalent.
  - ✓ can be graphed as transformations of $y = \sin x$ and $y = \cos x$.

Amplitude and Period of Sinusoidal Functions



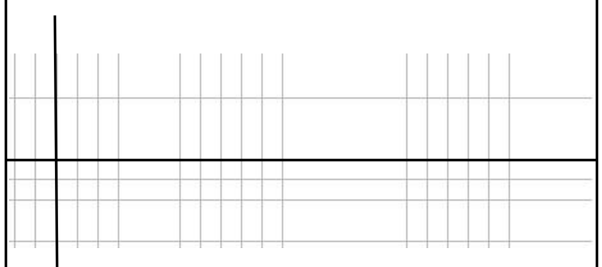
The amplitude of the functions $y = A \sin \theta$ and $y = A \cos \theta$ is the absolute value of A, or $|A|$.

$$|A| = \left|\frac{A-(-A)}{2}\right|$$



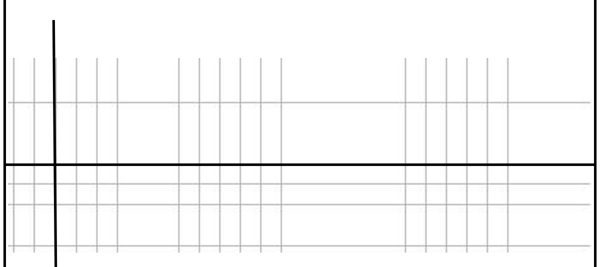
Examples

1 a. State the amplitude for the function $y = 4\cos\theta$.
b. Graph $y = 4\cos\theta$ and $y = \cos\theta$ on the same set of axes.
c. Compare the graphs.

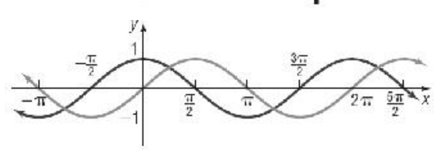


Period of Sine and Cosine Functions

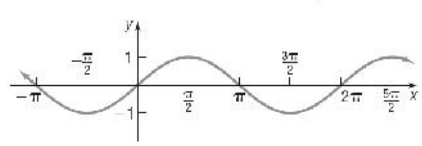
The period of the functions $y = \sin k\theta$ and $y = \cos k\theta$ is $\frac{2\pi}{k}$, where $k > 0$.



Sinusoidal Graphs



(a) $y = \cos x$ $y = \cos\left(x - \frac{\pi}{2}\right)$



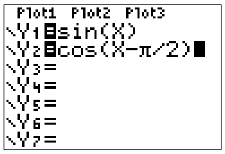
(b) $y = \sin x$

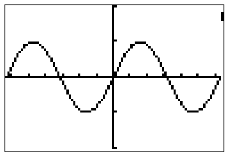
$$\sin x = \cos\left(x - \frac{\pi}{2}\right)$$

— Seeing the Concept —

Graph Y_1 sin x and Y_2 $\cos\left(x - \frac{\pi}{2}\right)$.

How many graphs do you see?





Amplitude and Period of Sinusoidal Functions

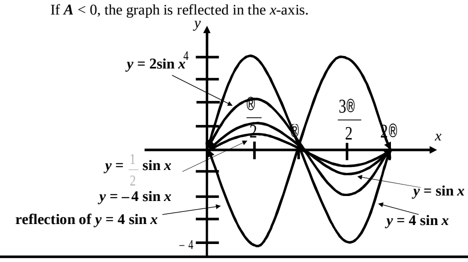
The **amplitude** of ***y* = *A* sin *x*** (or ***y* = *A* cos *x***) is half the distance between the maximum and minimum values of the function.

amplitude = |***A***|

If |***A***| > 1, the amplitude stretches the graph vertically.

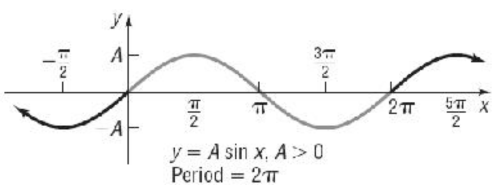
If 0 < |***A***| < 1, the amplitude shrinks the graph vertically.

If ***A*** < 0, the graph is reflected in the *x*-axis.



Amplitude and Period of Sinusoidal Functions

Amplitude



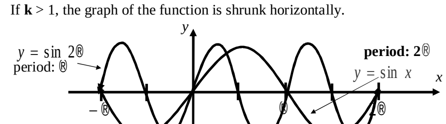
Amplitude and Period of Sinusoidal Functions

The **period** of a function is the *x* interval needed for the function to **complete one cycle**.

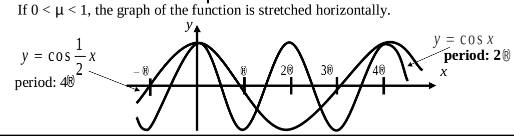
For **k** > 0, the **period** of ***y* = *A* sin** µ*x* is 2®/***k***.

For **k** > 0, the **period** of ***y* = *A* cos** µ*x* is also 2®/***k***.

If **k** > 1, the graph of the function is shrunk horizontally.



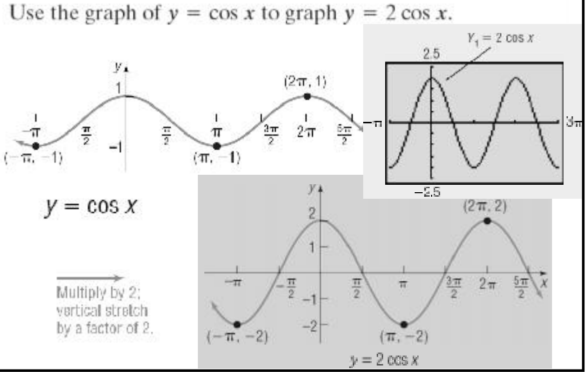
If 0 < µ < 1, the graph of the function is stretched horizontally.



EXAMPLE

Graphing Variations of y = cos x Using Transformations

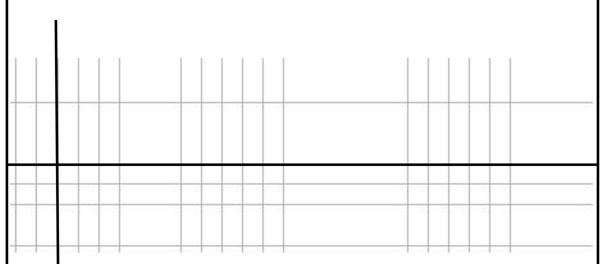
Use the graph of $y = \cos x$ to graph $y = 2\cos x$.



EXAMPLE

Graphing Variations of y = cos x Using Transformations

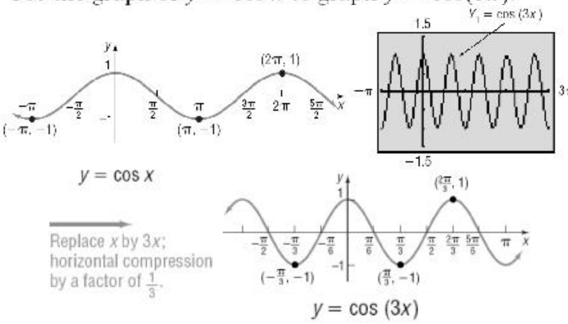
Use the graph of $y = \cos x$ to graph $y = \cos(3x)$.

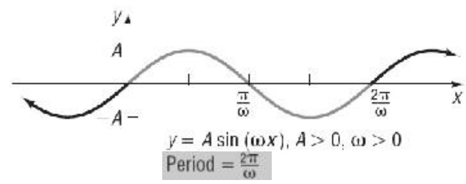


EXAMPLE

Graphing Variations of $y = \cos x$ Using Transformations

Use the graph of $y = \cos x$ to graph $y = \cos(3x)$.





Theorem

If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are

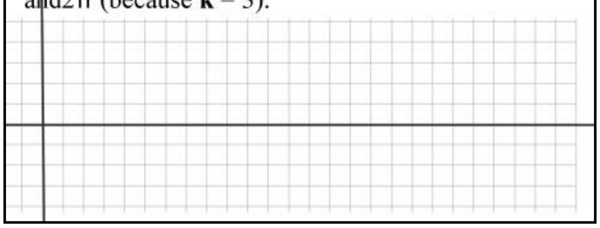
$$\text{Amplitude} = |A| \qquad \text{Period} = T = \frac{2\pi}{\omega}$$

Amplitude and Period of Sinusoidal Functions

k tells us the number of cycles in each **2π**.

•For ***y* = 10cos *x***, there is **one** cycle between 0 and 2π (because **k** = 1).

•For ***y* = 10cos 3*x***, there are **3** cycles between 0 and2π (because **k** = 3).



Amplitude and Period of Sinusoidal Functions

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Amplitude and Period of Sinusoidal Functions

EXAMPLE

Finding the Amplitude and Period of a Sinusoidal Function

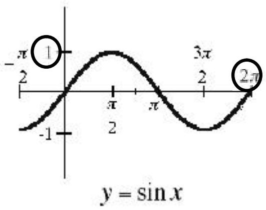
Determine the amplitude and period of $y = -4\cos(3x)$

Amplitude and Period of Sinusoidal Functions

Determine the amplitude and period of $y = -2\sin 2x$, and graph the function.

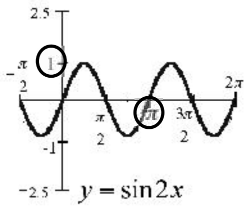
$$y = -2\sin 2x$$
$$y = A\sin \omega x$$
$$A = -2,\ \omega = 2$$
$$\text{Amplitude} = |-2| = 2$$
$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$$

Amplitude and Period of Sinusoidal Functions



Amplitude Z 1

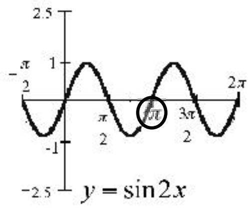
Period Z 2®



Amplitude Z 1

Period Z ®

Amplitude and Period of Sinusoidal Functions

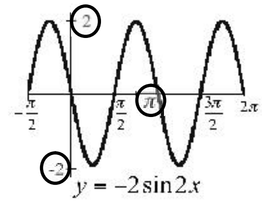


Amplitude Z 1

Period Z ®

Amplitude Z 2

Period Z ®



Amplitude and Period of Sinusoidal Functions

$$y = 3\sin 2x$$

y Z ***A*** *sin(**kx**)*

Amplitude Z |***A***|
Z |3|
Z 3

Period*, *T Z 2®/***k***
Z 2®/2
Z ®

Amplitude and Period of Sinusoidal Functions

$$y = \frac{1}{4}\cos\frac{2x}{3}$$

y Z ***A*** *cos(k **x**)*

Amplitude Z |***A***|
Z |1/4|
Z 1/4

Period*, *T Z 2® / (2/3)
Z 3®

Describe the relationship between the graphs of *f* and *g*. Consider amplitudes, periods, and shifts.

$$f(x) = \sin x$$
$$g(x) = \sin(x - ®)$$

Graph of y = A sin(wx)

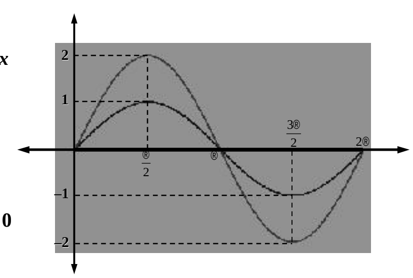
Graph one full period of *y* = 2 sin*x* and *y* = sin*x*

In [0 , π] ,

0≤ sin*x* ≤ 2sin*x*

In [π , 2π] ,

2sin*x* ≤ sin*x* ≤ 0



Graph of y = A sin(wx)

EXAMPLE

Graphing Functions of the Form $y = A \sin(\omega x)$ Using Transformations

Graph $y = 5 \sin x$ using transformations. Use the graph to determine the domain and the range of $y = 5 \sin x$.

Graph of y = A sin(wx)

EXAMPLE

Graphing Functions of the Form $y = A \sin(\omega x)$ Using Transformations

Graph of y = A sin(wx)

EXAMPLE

Graphing Functions of the Form $y = A \sin(\omega x)$ Using Transformations

Graph $y = \frac{1}{2}\sin(-® x)$ using transformations.

Use the graph to determine the domain and the range of

$y = \frac{1}{2}\sin(-® x)$

Graph of y = A cos(wx)

Example: Sketch the graph of **$y = 3 \cos x$** on the interval $[-\pi, 4\pi]$.

Partition the interval $[0, 2\pi]$ into four equal parts. Find the five key points; graph one cycle; then repeat the cycle over the interval.

x	0	$\frac{®}{2}$	π	$\frac{3®}{2}$	2π
$y = 3 \cos x$	**3**	**0**	**-3**	**0**	**3**
	max	***x*-int**	**min**	***x*-int**	**max**