6.4: Factoring Polynomials

Recall:

- If a number is divided by any of its factors, the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is 0.
- The **Remainder Theorem** states that if a polynomial $P(x)$ is divided by $(x-a)$, the remainder is the value of the function at a. So, if $(x-a)$ is a factor of $P(x)$, then $P(a)=0$.

Factor Theorem

For a polynomial $P(x)$, $x - a$ is a *factor* if and only if $P(a) = 0$.

1) If $P(a) = 0$, then $x - a$ is a *factor* of $P(x)$.
2) If $x - a$ is a *factor* of $P(a)$, then $P(a) = 0$

Example 1) Determining Whether a Linear Binomial is a Factor

Determine whether the given binomial is a factor of the polynomial $P(x)$.

a) $(x-2);\ P(x)=x^2+2x-3$

b) $(x+4);\ P(x)=2x^4+8x^3+2x+8$

Example 2) Factoring by Grouping

Factor each expression completely.

a) $x^3+3x^2-4x-12$

b) $2x^3+x^2+8x+4$

Special Factoring

Difference of Two Squares $a^2-b^2=(a-b)(a+b)$

Sum of Two Cubes $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of Two Cubes $a^3-b^3=(a-b)(a^2+ab+b^2)$

Example 3) Special Factoring

Factor each expression completely.

a) $3x^2-12y^2$

b) $5x^4+40x$

c) $8x^3-27$

d) $2x^5-16x^2$

YOUR TURN! Factor each expression completely.

1) $3x^3+x^2-27x-9$

2) $x^6-14x^4+49x^2$

3) The polynomial ax^3+bx^2+cx+d is factored as $3(x-2)(x+3)(x-4)$. What are the values of a and d?

Use the **Factor Theorem** to verify that each linear binomial is a factor of the given polynomial. Then use **Synthetic Division** to write the polynomial as a product.

4) $(x-2);\ P(x)=x^4-2x^3+5x^2-9x-2$

5) $(x+2);\ P(x)=2x^5+4x^4-6x^2-9x+6$

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