Section 6.7

Graphs of the Tangent, Cotangent, Cosecant, and Secant Functions

Properties of the Cosecant and Secant Functions

$y = A\csc(k\theta - c) + h$ $y = A\sec(k\theta - c) + h$

- Amplitude $= |A|$
- Period, $T = \dfrac{2\pi}{k}$
- Phase Shift $= \dfrac{c}{k}$
- Subinterval Width $= \dfrac{T}{4}$
- Interval defining ONE Cycle: $\left(\dfrac{c}{k}, \dfrac{c}{k} + T\right)$
- Use 5-keypoints

Properties of the Cosecant and Secant Functions

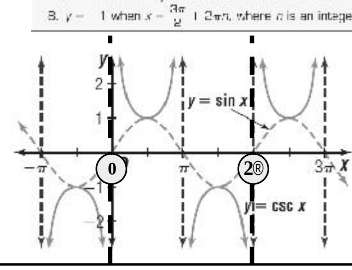
$y = A\csc(k\theta - c) + h$ $y = A\sec(k\theta - c) + h$

Trig. Function	Amplitude	Period	Phase Shift	Sub-Interval Width	Interval defining ONE cycle

Properties of the Trigonometric Functions

Cosecant Function

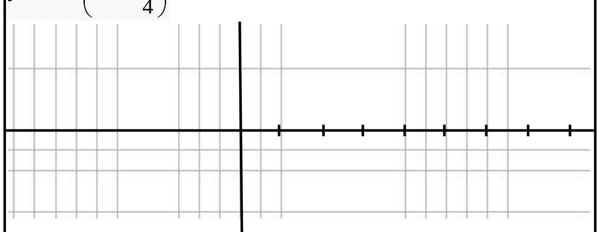
1. The period is 2π.
2. The domain is the set of real numbers except πn, where n is an integer.
3. The range is the set of real numbers greater than or equal to 1 or less than or equal to -1.
4. There are no x-intercepts.
5. There are no y-intercepts.
6. The asymptotes are $x = \pi n$, where n is an integer.
7. $y = 1$ when $x = \frac{\pi}{2} + 2\pi n$, where n is an integer.
8. $y = -1$ when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer.



Graphs of the Cosecant and Secant Functions

Amplitude	Period	Phase Shift	Sub-Interval Width	Interval defining ONE cycle	Key points

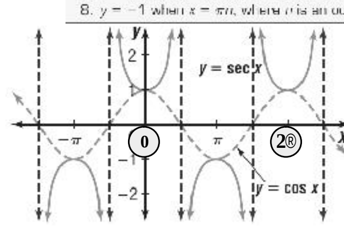
$y = 3\csc\left(2x - \dfrac{\pi}{4}\right)$



Properties of the Trigonometric Functions

Secant Function

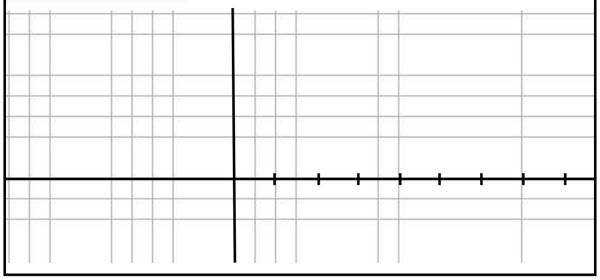
1. The period is 2π.
2. The domain is the set of real numbers except $\frac{\pi}{2}n$, where n is an odd integer.
3. The range is the set of real numbers greater than or equal to 1 or less than or equal to -1.
4. There are no x-intercepts.
5. The y-intercept is 1.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.
7. $y = 1$ when $x = \pi n$, where n is an even integer.
8. $y = -1$ when $x = \pi n$, where n is an odd integer.



Graphs of the Cosecant and Secant Functions

Amplitude	Period	Phase Shift	Sub-Interval Width	Interval defining ONE cycle	Key points

$y = 3\sec(® x - 2) + 5$



Properties of the Tangent and Cotangent Functions

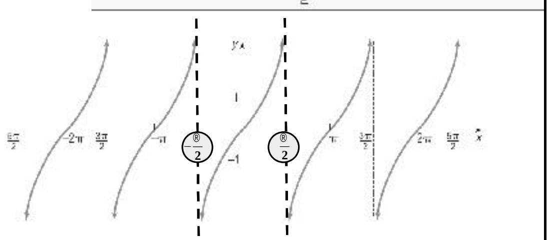
$y = A\tan(k^{-} - c) + h$ $y = A\cot(k^{-} - c) + h$

- NO Amplitude
- Period, $T = \frac{®}{k}$
- Phase Shift $= \frac{c}{k}$
- Interval defining ONE Cycle: $\left(\frac{c}{k}, \frac{c}{k} + T\right)$
- Use 3-keypoints

Properties of the Trigonometric Functions

Tangent Function

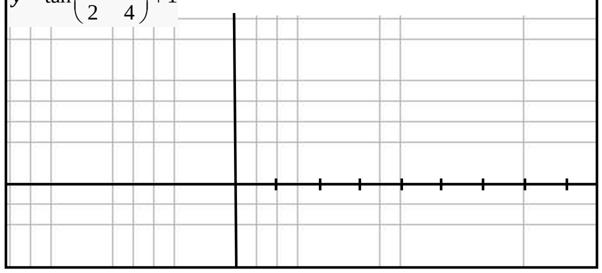
1. The period is π.
2. The domain is the set of real numbers except $\frac{\pi}{2}n$, where n is an odd integer.
3. The range is the set of real numbers
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.



Graphs of the Tangent and Cotangent Functions

Amplitude	Period	Phase Shift	Sub-Interval Width	Interval defining ONE cycle	Key points

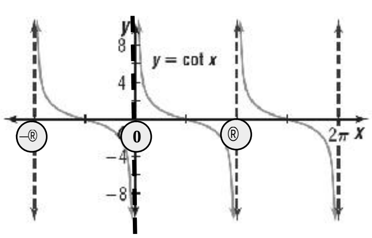
$y = \tan\left(\frac{-}{2} - \frac{®}{4}\right) + 1$



Properties of the Trigonometric Functions

Cotangent Function

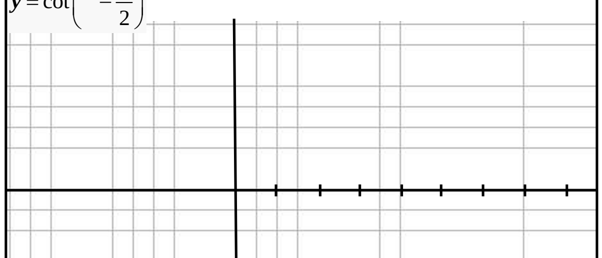
1. The period is π.
2. The domain is the set of real numbers except πn, where n is an intege
3. The range is the set of real numbers.
4. The x-intercepts are located at $\frac{\pi}{2}n$, where n is an odd integer.
5. There is no y-intercept.
6. The asymptotes are $x = \pi n$, where n is an integer.



Graphs of the Tangent and Cotangent Functions

Amplitude	Period	Phase Shift	Sub-Interval Width	Interval defining ONE cycle	Key points

$y = \cot\left(- - \frac{®}{2}\right)$



Tangent and Cotangent Functions

Example 1 Find each value by referring to the graphs of the trigonometric functions.

a. $\tan \frac{9\pi}{2}$ **= undefined**

1. The period is π.
2. The domain is the set of real numbers except $\frac{\pi}{2}n$, where n is an odd integer.
3. The range is the set of real numbers.
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.

b. $\cot \frac{7\pi}{2}$ **= 0**

1. The period is π.
2. The domain is the set of real numbers except πn, where n is an integer.
3. The range is the set of real numbers.
4. The x-intercepts are located at $\frac{\pi}{2}n$, where n is an odd integer.
5. There is no y-intercept.
6. The asymptotes are $x = \pi n$, where n is an integer.

Values of Trigonometric Functions

Example 2 Find the values of θ for which each equation is true.

a. $\csc \theta = 1$

$\theta = \frac{\pi}{2} + 2\pi n$, where n is an integer

1. The period is 2π.
2. The domain is the set of real numbers except πn, where n is an integer.
3. The range is the set of real numbers greater than or equal to 1 or less than or equal to -1.
4. There are no x-intercepts.
5. There are no y-intercepts.
6. The asymptotes are $x = \pi n$, where n is an integer.
7. $y = 1$ when $x = \frac{\pi}{2} + 2\pi n$, where n is an integer.
8. $y = -1$ when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer.

Values of Trigonometric Functions

Example 2 Find the values of θ for which each equation is true.

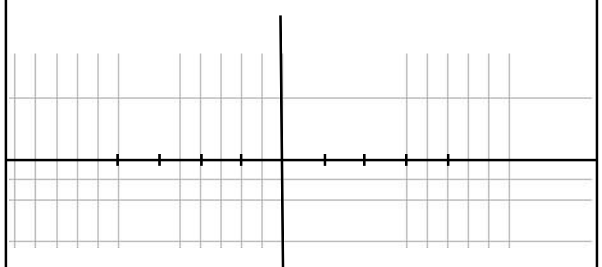
b. $\sec \theta = -1$

$\theta = \pi n$, where n is an odd integer

1. The period is 2π.
2. The domain is the set of real numbers except $\frac{\pi}{2}n$, where n is an odd integer.
3. The range is the set of real numbers greater than or equal to 1 or less than or equal to -1.
4. There are no x-intercepts.
5. The y-intercept is 1.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.
7. $y = 1$ when $x = \pi n$, where n is an even integer.
8. $y = -1$ when $x = \pi n$, where n is an odd integer.

Other Trigonometric Functions

Examples 3 Graph $y = \csc\left(\frac{\theta}{2} - \frac{\pi}{4}\right) + 2$.



Other Trigonometric Functions

Example 5 Write an equation for a secant function with period π, phase shift $\frac{\pi}{3}$, and vertical shift -3.

CW 6.7

Find each value by referring to the graphs of the trigonometric functions.

1) $\tan 4\pi$ **2)** $\csc\left(\frac{7\pi}{2}\right)$

Find the values of θ for which each equation is true.

3) $\sec \theta \quad -1$ **4)** $\cot \theta \quad 1$

Graph each function.

5) $y = \tan\left(\theta + \frac{\pi}{4}\right)$ **6)** $y = \sec(2\theta + \pi) - 1$

Write an equation for the given function given the period, phase shift, and vertical shift.

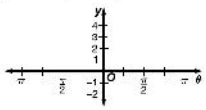
7) cosecant function, period $= 3\pi$, phase shift $= \frac{\pi}{3}$, vertical shift $= \quad 4$

8) cotangent function, period $= 2\pi$, phase shift $= -\frac{\pi}{4}$, vertical shift $= 0$

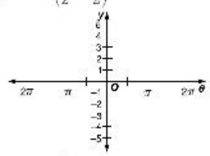
PRACTICE 6.7

Graph each function.

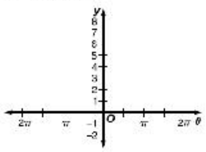
9. $y = \tan(2\theta + \pi) + 1$



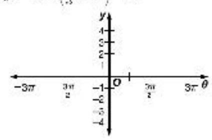
10. $y = \cot\left(\frac{\theta}{2} - \frac{\pi}{2}\right) - 2$



11. $y = \csc\theta + 3$



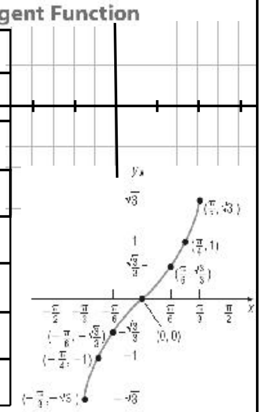
12. $y = \sec\left(\frac{\theta}{3} + \pi\right) - 1$



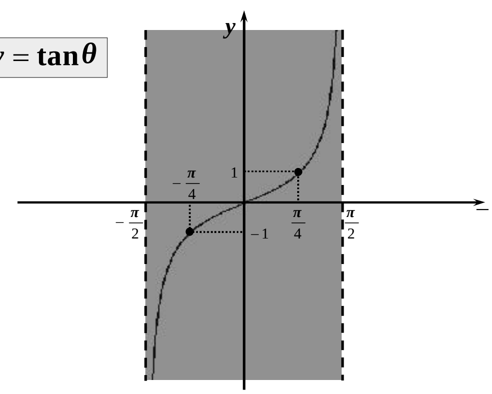
PRACTICE 6.7

The Graph of the Tangent Function

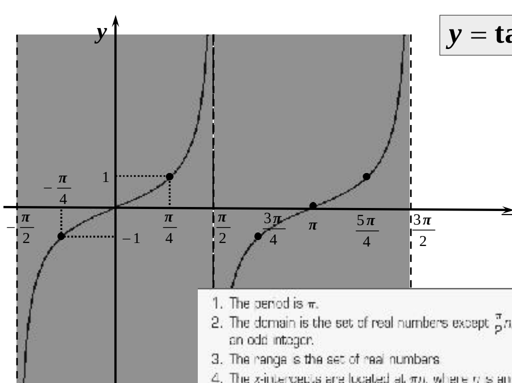
¯	(cos ¯, sin ¯)	tan ¯
$-\frac{®}{3}$	$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$-\sqrt{3} \approx -1.73$
$-\frac{®}{4}$	$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$	-1
$-\frac{®}{6}$	$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$-\frac{\sqrt{3}}{3} \approx 0.58$
0	$(1, 0)$	0
$\frac{®}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\frac{\sqrt{3}}{3} \approx 0.58$
$\frac{®}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	1
$\frac{®}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\sqrt{3} \approx 1.73$



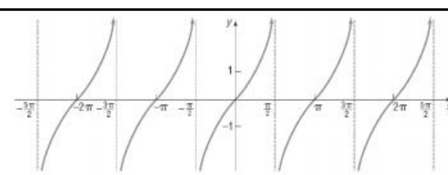
$y = \tan\theta$



$y = \tan\theta$



1. The period is π.
2. The domain is the set of real numbers except $\frac{\pi}{2}n$, where n is an odd integer.
3. The range is the set of real numbers.
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.



$y = \tan x$, $-\infty < x < \infty$, x not equal to odd multiples of $\frac{\pi}{2}$

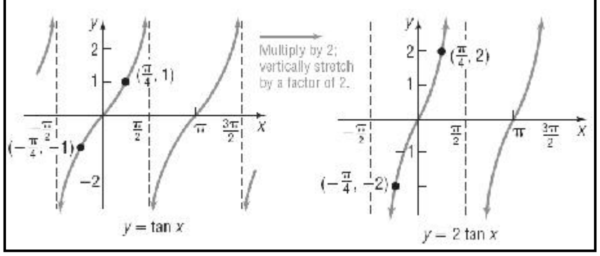
Properties of the Tangent Function

1. The period is π.
2. The domain is the set of real numbers except $\frac{\pi}{2}n$, where n is an odd integer.
3. The range is the set of real numbers.
4. The x-intercepts are located at πn, where n is an integer.
5. The y-intercept is 0.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.

Graphing Variations of y = tan x Using Transformations

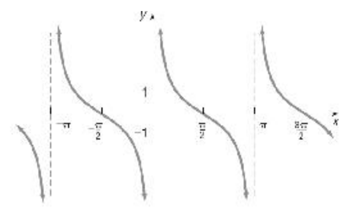
EXAMPLE

Graph: $y = 2\tan x$

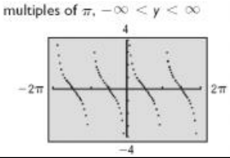


The Graph of the Cotangent Function

x	$y = \cot x$	(x, y)
$\frac{\pi}{6}$	$\sqrt{3}$	$\left(\frac{\pi}{6}, \sqrt{3}\right)$
$\frac{\pi}{4}$	1	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{3}$	$\left(\frac{\pi}{3}, \frac{\sqrt{3}}{3}\right)$
$\frac{\pi}{2}$	0	$\left(\frac{\pi}{2}, 0\right)$
$\frac{2\pi}{3}$	$-\frac{\sqrt{3}}{3}$	$\left(\frac{2\pi}{3}, -\frac{\sqrt{3}}{3}\right)$
$\frac{3\pi}{4}$	-1	$\left(\frac{3\pi}{4}, -1\right)$
$\frac{5\pi}{6}$	$-\sqrt{3}$	$\left(\frac{5\pi}{6}, -\sqrt{3}\right)$

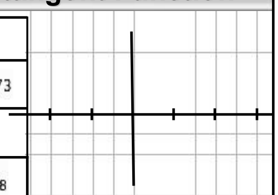


$y = \cot x$, $-\infty < x < \infty$, x not equal to integer multiples of π, $-\infty < y < \infty$



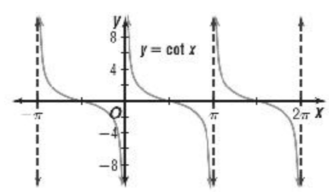
The Graph of the Cotangent Function

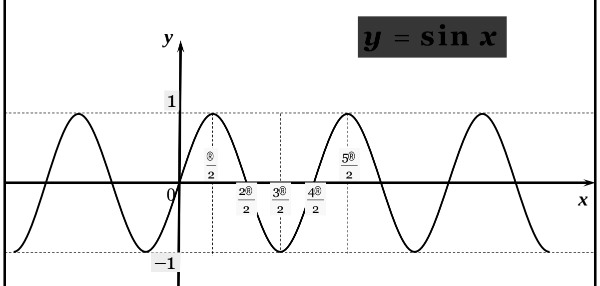
¯	(cos ¯, sin ¯)	tan ¯
$-\frac{®}{3}$	$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$-\sqrt{3} \approx -1.73$
$-\frac{®}{4}$	$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$	-1
$-\frac{®}{6}$	$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$-\frac{\sqrt{3}}{3} \approx 0.58$
0	$(1,0)$	0
$\frac{®}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\frac{\sqrt{3}}{3} \approx 0.58$
$\frac{®}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	1
$\frac{®}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\sqrt{3} \approx 1.73$



Properties of the Cotangent Function

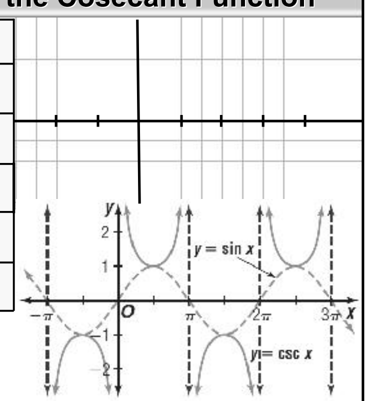
1. The period is π.
2. The domain is the set of real numbers except πn, where n is an integer.
3. The range is the set of real numbers.
4. The x-intercepts are located at $\frac{\pi}{2}n$, where n is an odd integer.
5. There is no y-intercept.
6. The asymptotes are $x = \pi n$, where n is an integer.



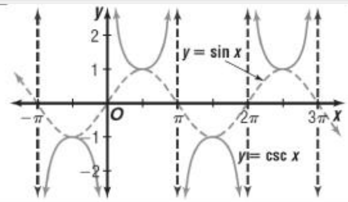


The Graph of the Cosecant Function

¯	sin ¯	csc ¯
0	0	
$\frac{®}{2}$	1	
®	0	
$\frac{3®}{2}$	−1	
2®	0	



Properties of the Cosecant Function



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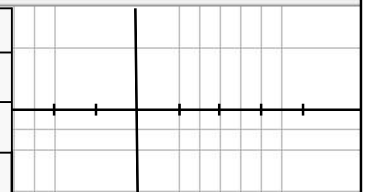
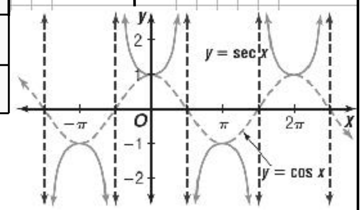
7. $y = 1$ when $x = \frac{\pi}{2} + 2\pi n$, where n is an integer.
8. $y = -1$ when $x = \frac{3\pi}{2} + 2\pi n$, where n is an integer.

The Graph of the Secant Function

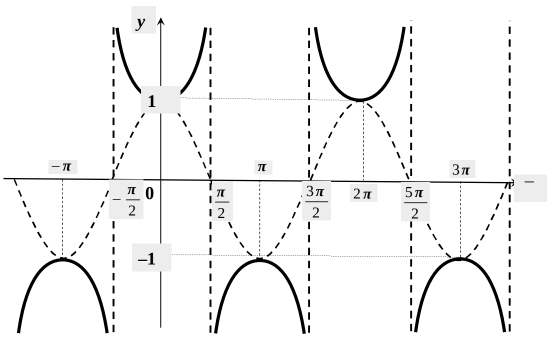
¯	cos ¯	sec ¯
0	1	
®/2	0	
®	−1	
3®/2	0	
2®	1	



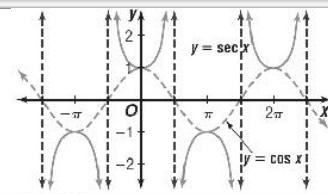


The Graph of the Secant Function

$y = \sec\theta$



Properties of the Secant Function



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3. The range is the set of real numbers greater than or equal to 1 or less than or equal to −1.
4. There are no x-intercepts.
5. The y-intercept is 1.
6. The asymptotes are $x = \frac{\pi}{2}n$, where n is an odd integer.
7. $y = 1$ when $x = \pi n$, where n is an even integer.
8. $y = -1$ when $x = \pi n$, where n is an odd integer.

Phase and Vertical Shift of the other Trig. Functions

$$y = \tan(k\theta - c) + h, \quad y = \cot(k\theta - c) + h$$
$$y = \sec(k\theta - c) + h, \quad \text{and } y = \csc(k\theta - c) + h$$

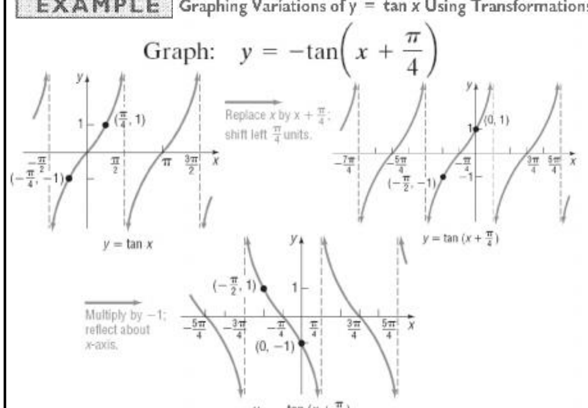
The phase shift of the above functions where $k > 0$ is $\frac{c}{k}$

- **If $c > 0$, the shift is to the right.**
- **If $c < 0$, the shift is to the left.**

The vertical shift of the above functions is h **.**

EXAMPLE Graphing Variations of $y = \tan x$ Using Transformations

Graph: $y = -\tan\left(x + \frac{\pi}{4}\right)$



EXAMPLE

Graphing Functions of the Form $y = A\tan(\omega x) + B$

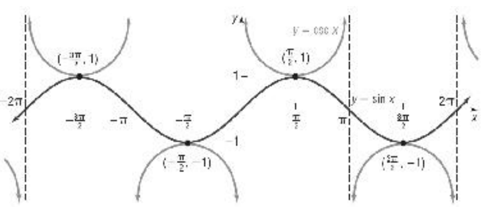
Graph $y = \frac{1}{2}\tan x + 2$. Use the graph to determine the domain and the range of $y = \frac{1}{2}\tan x + 2$.

EXAMPLE

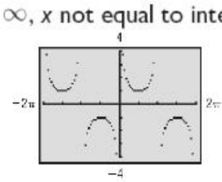
Graphing Functions of the Form $y = A\tan(\omega x) + B$

Graph $y = 3\tan\left(\frac{1}{2}x\right) - 1$. Use the graph to determine the domain and the range of $y = 3\tan\left(\frac{1}{2}x\right) - 1$.

The Graph of the Cosecant Function



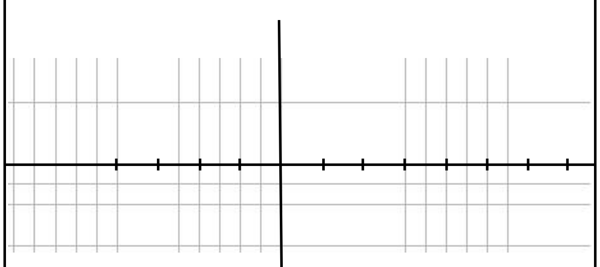
$y = \csc x$, $-\infty < x < \infty$, x not equal to integer multiples of π, $|y| \geq 1$



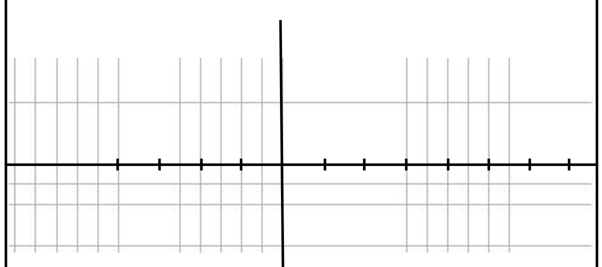
The Graph of the Cotangent Function

Properties of the Graph of $y = \cot x$	1. The period is π. 2. The domain is the set of real numbers except πn, where n is an integer. 3. The range is the set of real numbers. 4. The x-intercepts are located at $\frac{\pi}{2}n$, where n is an odd integer. 5. There is no y-intercept. 6. The asymptotes are $x = \pi n$, where n is an integer.

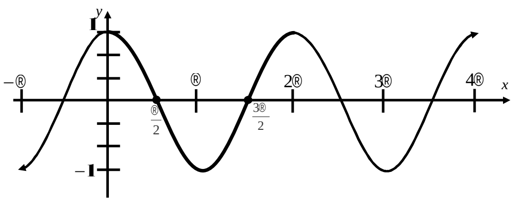
y = csc θ



y = sec θ



Graph of y = sec θ

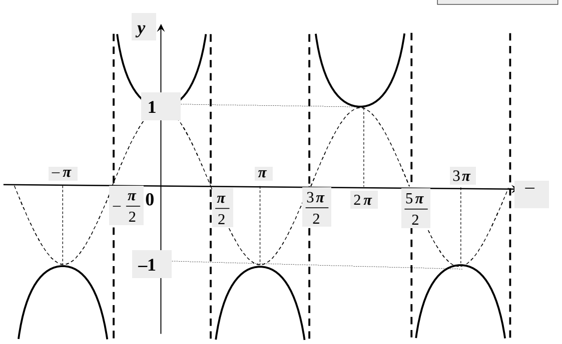


Period of Trigonometric Functions

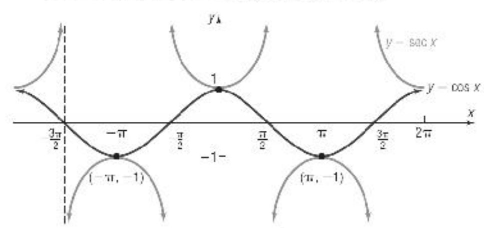
The period of functions $y = \sin k\theta$, $y = \cos k\theta$, $y = \csc k\theta$, and $y = \sec k\theta$ is $\frac{2\pi}{k}$, where $k > 0$.

The period of functions $y = \tan k\theta$ and $y = \cot k\theta$ is $\frac{\pi}{k}$, where $k > 0$.

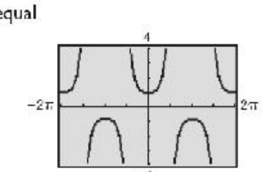
$y = \sec\theta$



The Graph of the Secant Function



$y = \sec x$, $-\infty < x < \infty$, x not equal to odd multiples of $\frac{\pi}{2}$, $|y| \geq 1$



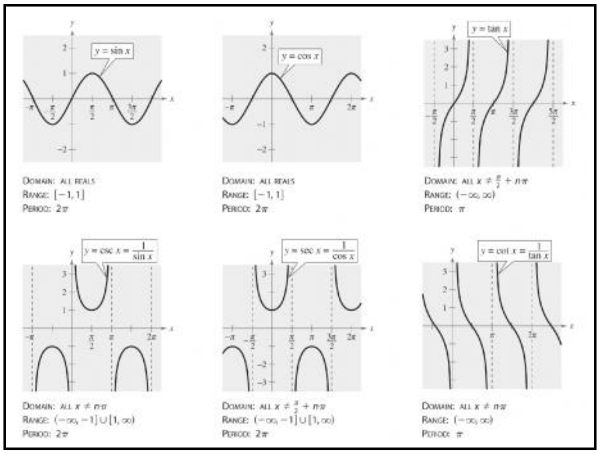
OBJECTIVE 2

2 Graph Functions of the Form $y = A\csc(\omega x) + B$ and $y = A\sec(\omega x) + B$

EXAMPLE

Graphing Functions of the Form $y = A\csc(\omega x) + B$

Graph $y = -\csc(2x) - 1$. Use the graph to determine the domain and the range of $y = -\csc(2x) - 1$.


$y = \sin x$
Domain: all reals
Range: $[-1, 1]$
Period: 2π
$y = \cos x$
Domain: all reals
Range: $[-1, 1]$
Period: 2π
$y = \tan x$
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π
$y = \csc x = \frac{1}{\sin x}$
Domain: all $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
$y = \sec x = \frac{1}{\cos x}$
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
$y = \cot x = \frac{1}{\tan x}$
Domain: all $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π