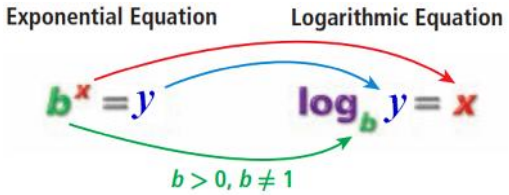
7.3: LOGARITHMIC FUNCTIONS

- A **LOGARITHM** is the exponent to which a specified base is raised to obtain a given value.



Example 1: Write each exponential equation in logarithmic form or vice-versa.

Exponential Equation	*Logarithmic Form*
$2^3 = 8$	
$4^0 = 1$	
$5^{-2} = 0.04$	
$3^x = 8$	
$25 = 5^2$	

Logarithmic Form	*Exponential Equation*
$\log_7 49 = 2$	
$\log_3 81 = 4$	
$\log_8 0.125 = -1$	
$\log_6 6 = 1$	
$\log_5 1 = 0$	

SPECIAL PROPERTIES OF LOGARITHMS

For any base b such that $b > 0$ and $b \neq 1$,

a) Logarithm of Base b: $\log_b b^n = n$ b) Logarithm of 1: $\log_b 1 = 0$

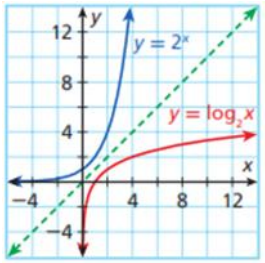
- A logarithm with **base 10** is called a **common logarithm**. If no base is written for a logarithm, the base is assumed to be 10. For example, $\log 5 \Rightarrow \log_{10} 5$.

Example 2: Evaluate by using mental math.

a) $\log 100 =$ b) $\log_4\left(\frac{1}{64}\right) =$ c) $\log_{25}(0.04) =$

Because logarithms are the inverses of exponents, the inverse of an exponential function, such as $y = 2^x$, is a **logarithmic function**, such as $y = \log_2(x)$.

You may notice that the **domain** and **range** of each function are **switched.**



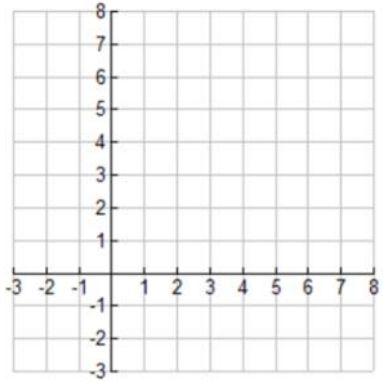
	Exponential *Equation:* $y = 2^x$	*Logarithmic Form:* $y = \log_2(x)$
Domain	all real numbers ($\mathbb{R}$)	$x > 0$
Range	$y > 0$	all real numbers ($\mathbb{R}$)

Example 3: Use the given x-values to graph each function. Then graph its inverse. Describe the domain and range of the inverse function.

a) $f(x) = 3^x$; $x = -2, -1, 0, 1,$ and 2

x	−2	−1	0	1	2
$f(x) = 3^x$					

x					
$f^{-1}(x) = \log_3 x$					



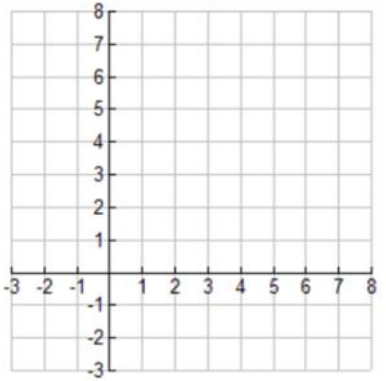
Domain:

Range:

b) $f(x) = 0.8^x$; $x = -3, 0, 1, 4$ and 7

x	−3	0	1	4	7
$f(x) = 0.8^x$					

x					
$f^{-1}(x) = \log_{0.8} x$					



Domain:

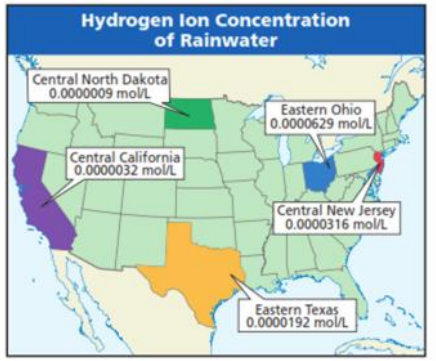
Range:

Example 4: Chemists regularly test rain samples to determine the rain's acidity, or concentration of hydrogen ions (H^+). Acidity is measured in pH, as given by the function $pH = -\log[H^+]$, where $[H^+]$ represents the hydrogen ion concentration in moles per liter.

Find the pH of rainwater from each location.

a) Central New Jersey

b) Central North Dakota



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