

State the domain and range of each relation. Then state whether the relation is a function. Write *yes* or *no*.

1. $\{(-1, 2), (0, 5), (3, 4), (2, 4)\}$ **2.** $\{(6, 7), (7, 2), (4, -2), (6, -3), (-5, 0)\}$

Find each value if $f(x) = x - 3x^2$.

- **3**. *f*(4)
- **6. Physics** The equation of the illuminance, *E*, of a small light source is $E = \frac{P}{4\pi d^2}$, where *P* is the luminous flux (in lumens, lm) of the source and *d* is its distance from the surface. Suppose the luminous flux of a desk lamp is 1200 lm.

5. f(a + 2)

a. What is the illuminance (in lm/m^2) on a desktop if the lamp is 0.9 m above it?

4. *f*(−7)

- **b**. Name the values of *d*, if any, that are not in the domain of the given function.
- 7. If $f(x) = x^2 7$ and g(x) = x + 3, find f(x) + g(x), f(x) g(x), $f(x) \cdot g(x)$, and $\left(\frac{f}{g}\right)(x)$.

Find $[f \circ g](x)$ and $[g \circ f](x)$ for each f(x) and g(x).

| 8. | f(x) = x + 1 | 9. | f(x) = 5x |
|----|---------------|----|-------------------|
| | g(x) = 4x - 5 | | $g(x) = 2x^2 + 6$ |

Graph each equation or inequality.

10. y = 3x - 6 **11.** 2x + y = 1 **12.** $y + 4x \le 12$ **13.** y > |x| + 2

Write an equation in slope-intercept form for each line described.

14. slope $=\frac{5}{3}$, passes through (-1, 3) **15.** passes through (0, 4) and (8, -2)

Write the standard forms of the equations of the lines that are parallel and perpendicular to the graph of the given equation and pass through the point with the given coordinates.

16. y = 4x - 1; (0, 2) **17.** x - 5y = 3; (-1, 2)

- **18.** Graph g(x) = [x + 1]**19.** Graph $f(x) = \begin{cases} 2 \text{ if } x \le -3 \\ x + 1 \text{ if } -3 < x \le 1. \\ \frac{1}{2} x \text{ if } x > 1 \end{cases}$
- **20. Grades** The table below shows the statistics grades and the economics grades for a group of college students at the end of a semester.

| Statistics Grade | 95 | 51 | 79 | 47 | 82 | 52 | 67 | 78 | 66 |
|------------------|----|----|----|----|----|----|----|----|----|
| Economics Grade | 88 | 70 | 70 | 67 | 71 | 80 | 68 | 85 | 90 |

- a. Graph the data on a scatter plot.
- **b.** Use two ordered pairs to write the equation of a best-fit line.





1. Solve the system of equations by graphing. Then state whether the system is *consistent and* independent, consistent and dependent, or inconsistent. 2x - y = -1x + y = 4

Solve each system of equations algebraically. **3.** 4x - 5y = -23x + 2y = -13

2. 3x + v = 75x + 2v = 12 4. 4x + 6y - 3z = 20x - 5y + z = -15-7x + y + 2z = 1

14. $\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 4 & 1 & -2 \end{vmatrix}$

17. $\begin{bmatrix} -5 & 4 \\ -15 & 12 \end{bmatrix}$

5. Find the values of *x* and *y* for which $\begin{bmatrix} y \\ 3x - 4 \end{bmatrix} = \begin{bmatrix} 12 - 2x \\ 2y \end{bmatrix}$ is true.

Use matrices A, B, and C to find each of the following. If the matrix does not exist, write impossible.

| $A = \begin{bmatrix} -5\\4 \end{bmatrix}$ | $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ | $B = \begin{bmatrix} -2 & 3\\ -3 & 0 \end{bmatrix}$ | $C = \begin{bmatrix} -1 & 0 & 3\\ 3 & 2 & -4 \end{bmatrix}$ |
|---|--|---|---|
| A + B | 7 3 <i>C</i> | 8 . 2 <i>B</i> – <i>A</i> | 9 . BC |

Use matrices to perform each transformation. Then graph the pre-image and image on the same coordinate grid.

- **10**. Triangle ABC has vertices A(1, -3), B(-4, 3), and C(6, 2). Use scalar multiplication to find the coordinates of the triangle after a dilation of scale factor 1.5.
- **11.** Quadrilateral *FGHJ* has vertices F(3, 2), G(1, -5), H(-6, 1), and J(-3, 5). Find the coordinates of the quadrilateral after a reflection over the *x*-axis.

Find the value of each determinant.

12. $\begin{vmatrix} 5 & 7 \\ 3 & 6 \end{vmatrix}$

6.

Find the inverse of each matrix, if it exists.

- **15**. $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$
- **18**. Solve the system by using a matrix equation. 2

$$2x - y = 7$$
$$3x + y = 3$$

19. Find the maximum and minimum values of f(x, y) = 2x - y for the polygonal convex set determined by the system of inequalities.

13. $\begin{vmatrix} 2 & 4 \\ -10 & 9 \end{vmatrix}$

16. $\begin{vmatrix} 3 & -4 \\ 4 & -2 \end{vmatrix}$

$$x \le 2, y \ge 0, y \le \frac{1}{2}x + 3, y \ge 3x - 4$$

20. Advertising Ruff, a dog food manufacturer, wants to advertise both in a magazine and on radio. The magazine charges \$100 per ad and requires the purchase of at least three ads. The radio station charges \$200 per commercial minute and requires the purchase of at least four minutes. Each magazine ad reaches 12,000 people while each commercial minute reaches 16,000 people. Ruff can spend at most \$1300 on advertising. How many ads and commercial minutes should the manufacturing company purchase to reach the most people?

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Determine whether the graph of each equation is symmetric with respect to the x-axis, y-axis, the line y = x, the line y = -x, or none of these.

1.
$$y = 2x + 1$$
 2. $y = -\frac{2}{x^2}$ **3.** $x = y^2 + 3$ **4.** $xy = 5$

Describe how the graphs of f(x) and g(x) are related.

5. $f(x) = x^2$ and $g(x) = -(x+3)^2$ **6.** f(x) = |x| and g(x) = 4|x| - 2

Graph each inequality.

7. y > |x - 4| **8.** $y \le 2x^2 + 3$

Find $f^{-1}(x)$ and state whether $f^{-1}(x)$ is a function. Then graph the function and its inverse.

9.
$$f(x) = 5x - 4$$
 10. $f(x) = \frac{3}{x - 2}$

Determine whether each function is continuous at the given *x*-value. Justify your answer using the continuity test.

- **11.** $\frac{x^2}{x-2}$; x = 2**12.** $f(x) = \begin{cases} x+1 \text{ if } x > 0 \\ 1 \text{ if } x \le 0 \end{cases}$; x = 0
- **13**. Describe the end behavior of $y = x^4 2x^2 1$.

Determine whether the given critical point is the location of a *maximum*, a *minimum*, or a *point of inflection*.

14. $y = x^2 - 8x + 4$, x = 4**15.** $y = x^3 - 3x^2 + x - 1$, x = 1

Determine the equations of the vertical and horizontal asymptotes, if any, of each function.

16.
$$f(x) = \frac{4x}{x-1}$$
 17. $g(x) = \frac{x}{x^2-4}$

18. Chemistry A chemist pours 4 molar acid solution into 250 mL of *a* 1 molar acid solution. After *x* mL of the 4 molar solution have been added, the concentration of the mixture is given by $C(x) = \frac{0.25 + 0.004x}{0.25 + 0.001x}.$

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- **a**. Find the horizontal asymptote of the graph of C(x).
- b. What is the chemical interpretation of the horizontal asymptote?

Find the constant of variation for each relation and use it to write an equation for each statement. Then solve the equation.

- **19**. If *y* varies directly as *x* and y = 0.5 when x = 2, find *y* when x = 10.
- **20**. If *y* varies inversely as the square of *x* and y = 8 when x = 3, find *x* when y = 18.



1. Write a polynomial equation of least degree with roots 4, i, and -i.

Find the discriminant of each equation and describe the nature of the roots of the equation. Then solve the equation by using the Quadratic Formula.

2. $n^2 - 5n + 4 = 0$ **3.** $z^2 - 7z - 3 = 0$ **4.** $2a^2 - 5a + 4 = 0$

Divide using synthetic division.

5. $(2x^3 - 3x^2 + 3x - 4) \div (x - 2)$ **6.** $(x^4 - 5x^3 - 13x^2 + 53x + 60) \div (x + 1)$

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

7. $(x^3 + 8x^2 + 2x - 11) \div (x + 2)$ **8.** $(4x^4 - 2x^2 + x - 3) \div (x - 1)$

Find the number of possible positive real zeros and the number of possible negative real zeros for each function. Then determine the rational zeros.

9.
$$f(x) = 6x^3 + 11x^2 - 3x - 2$$

10. $f(x) = x^4 + x^3 - 9x^2 - 17x - 8$

Approximate the real zeros of each function to the nearest tenth.

11. $f(x) = x^2 - 3x - 3$ **12.** $f(x) = x^3 - x + 1$

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

13.
$$f(x) = x^3 + 3x^2 - 5x - 9$$

14. $f(x) = 2x^4 + 3x^3 - x^2 + x + 1$

Solve each equation or inequality.

| 15. | $\frac{1}{80} + \frac{1}{a} = \frac{1}{10}$ | 16. | $\frac{4}{x-2} = \frac{3}{x^2 - 4} - \frac{1}{4}$ | 17. | $\frac{5}{x+2} > \frac{5}{x} + \frac{2}{3x}$ |
|-----|---|-----|---|-----|--|
| 18. | $\sqrt{y-2} - 3 = 0$ | 19. | $\sqrt{2x+2} = \sqrt{3x-5}$ | 20. | $\sqrt{11-10m} > 9$ |

Decompose each expression into partial fractions.

21. $\frac{5z-11}{2z^2+z-6}$ **22.** $\frac{7x^2+18x-1}{(x^2-1)(x+2)}$

23. What type of polynomial function would be the best model for the set of data?

| x | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|-----------------------|----|----|----|---|---|---|----|
| f (x) | 26 | 16 | 6 | 4 | 3 | 8 | 14 |

24. Manufacturing The volume of a fudge tin must be 120 cubic centimeters. The tin is 7 centimeters longer than it is wide and six times longer than it is tall. Find the dimensions of the tin.

25. Travel A car travels 500 km in the same time that a freight train travels 350 km. The speed of the car is 30 km/h more than the speed of the train. Find the speed of the freight train.

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If each angle is in standard position, determine a coterminal angle that is between 0° and 360°. State the quadrant in which the terminal side lies. **1**. 995° **2**. -234° **3**. 410° **4**. −1245° **5**. Find the values of the six trigonometric R ratios for $\angle R$. 15 in. 12 in. Use the unit circle to find each value. **6**. tan 60° **7**. sec 270° **8**. $\sin(-405^{\circ})$ Find the values of the six trigonometric functions for angle θ in standard position if a point with the given coordinates lies on its terminal side.

9. (3, 5) **10**. (-4, 2)

Solve each problem. Round to the nearest tenth.

- **12.** If b = 42 and $A = 77^{\circ}$, find *c*. **13.** If a = 13 and $B = 27^{\circ}$, find *b*. **14.** If c = 14 and $A = 32^{\circ} 17'$, find *a*. **15**. If a = 23 and c = 37, find *B*.
- **16.** If a = 3 and b = 11, find *A*.
- **17. Recreation** A kite is fastened to the ground by a string that is 65 meters long. If the angle of elevation of the kite is 70° , how far is the kite above the ground?

Find the area of each triangle. Round to the nearest tenth.

18. $A = 36^{\circ}$, a = 24, $C = 87^{\circ}$ **19.** $b = 56.4, c = 92.5, A = 58.4^{\circ}$

20. Geometry An isosceles triangle has a base of 22 centimeters and a vertex angle measuring 36°. Find its perimeter.

Find all solutions for each triangle. If no solutions exist, write *none*. Round to the nearest tenth. **21.** $a = 64, c = 90, C = 98^{\circ}$ **22.** $a = 9, b = 20, A = 31^{\circ}$

Solve each triangle. Round to the nearest tenth.

24. $a = 20, c = 24, B = 47^{\circ}$ **23.** a = 13, b = 7, c = 15

25. Navigation A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The measurement of the angle between signals is 130°. How far apart are the transmitters?







11. (0, -3)



Change each degree measure to radian measure in terms of π .

| 1. 3 | 225° | 2. | 480° |
|------|------|----|---------------|
|------|------|----|---------------|

Evaluate each expression.

3. $\sin \frac{5\pi}{6}$

4. $tan \frac{5\pi}{4}$

5. Determine the linear velocity of a point rotating at an angular velocity of 7.1 radians per second at a distance of 12 centimeters from the center of the rotating object.

Graph each function for the given interval.

6.
$$y = \sin x, -\pi \le x \le \pi$$

7. $y = \cos x, -\pi \le x \le \pi$

State the amplitude and period for each function.

- **8.** $y = 3\cos 4\theta$ **9.** $y = -2\sin \frac{3\pi}{2}\theta$
- **10**. Write an equation of the sine function with amplitude 4, period 4π , phase shift 0, and vertical shift -2.
- **11.** Write an equation of the cosine function with amplitude 0.5, period $\frac{\pi}{2}$, phase shift $\frac{\pi}{4}$, and vertical shift 1.

Graph each function.

12. $y = 3\cos\frac{\theta}{2}$ **13.** $y = \tan\left(2\theta - \frac{\pi}{4}\right)$

Write the equation for the inverse of each function. Then graph the function and its inverse.

14.
$$y = \operatorname{Arccsc} x$$
 15. $y = \tan x$

Find each value.

- **16.** $\sin\left(\operatorname{Arccos}\frac{1}{2}\right)$
- **18**. **Astronomy** The linear velocity of Earth's moon is about 2300 mph. If the average distance from the center of Earth to the center of the Moon is 240,000 miles, how long does it take the Moon to make one revolution about Earth? Assume the orbit is circular.

17. $\tan\left(\pi + \sin^{-1}\frac{1}{2}\right)$

19. **Meteorology** The average monthly temperatures for the city of Chicago, Illinois, are given below.

| Jan. | Feb. | March | April | May | June | July | Aug. | Sept. | Oct. | Nov. | Dec. |
|------|------|-------|-------|-----|------|------|------|-------|------|------|------|
| 21° | 25° | 37° | 49° | 59° | 69° | 73° | 72° | 64° | 53° | 40° | 27° |

Write a sinusoidal function that models the monthly temperatures, using t = 1 to represent January.

20. Civil Engineering Jake is designing a curve for a new highway. He uses the equation $\tan \theta = \frac{\nu^2}{rg}$, where the radius *r* will be 1200 feet. The curve will be designed for a maximum velocity ν of 65 mph, and the acceleration *g* due to gravity is 32 ft/s². At what angle θ should the curve be banked? (Express your answer in radians.)

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Use the given information to determine the exact trigonometric value.

- **1.** $\sin \theta = \frac{1}{3}, 0^{\circ} < \theta < 90^{\circ}; \cos \theta$ **2.** $\sec \theta = -2, \frac{\pi}{2} < \theta < \pi; \tan \theta$ **3.** $\sin \theta = -\frac{4}{5}, \pi < \theta < \frac{3\pi}{2}; \sec \theta$ **4.** $\csc \theta = -\frac{5}{3}, 270^{\circ} < \theta < 360^{\circ}; \cos \theta$
- **5**. Express tan (-420°) as a trigonometric function of an angle in Quadrant I.

Verify that each equation is an identity.

6. $\tan \theta (\cot \theta + \tan \theta) = \sec^2 \theta$ 7. $\sin^2 A \cot^2 A = (1 - \sin A)(1 + \sin A)$ 8. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$ 9. $\frac{\cos x}{1 + \sin x} + \frac{\cos x}{1 - \sin x} = 2 \sec x$ 10. $\csc (A - B) = \frac{\sec B}{\sin A - \cos A \tan B}$ 11. $\cot 2\theta = \frac{1}{2} \cot \theta - \frac{1}{2} \tan \theta$

Use sum or difference identities to find the exact value of each trigonometric function.

12. $\sin 255^{\circ}$ **13.** $\tan \frac{5\pi}{12}$ **14.** If $\cos \theta = \frac{3}{4}$ and $270^{\circ} < \theta < 360^{\circ}$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$. **15.** Use a half-angle identity to find the exact value of $\cos 22.5^{\circ}$.

Solve each equation for principal values of x. Express solutions in degrees. 16. $\tan^2 x = \sqrt{3} \tan x$ 17. $\cos 2x - \cos x = 0$

Solve each equation for 0^{\circ} \le x < 360^{\circ}. 18. $\sin x - \cos x = 0$ **19.** $2 \cos^2 x + 3 \sin x = 3$

Write each equation in normal form. Then find the length of the normal and the angle it makes with the positive *x*-axis.

20. y = x + 3 **21.** 5 + 5y = 10x

Find the distance between the point with the given coordinates and the line with the given equation.

- **22.** (-5, 8), 2x + y = 6 **23.** (-6, 8), 3x + 4y + 2 = 0
- **24.** Find an equation of the line that bisects the acute angles formed by the graphs of 5x + 2y = 7 and $y = -\frac{3}{4}x + 1$.

25. Physics The range of a projected object is the distance that it travels from the point where it is released. In the absence of air resistance, a projectile released at an angle of inclination θ with an initial velocity of v_0 has a range of $R = \frac{v_0^2}{g} \sin 2\theta$, where g is the acceleration due to gravity. Find the range of a projectile with an initial velocity of 88 feet per second if $\sin \theta = \frac{3}{5}$ and $\cos \theta = \frac{4}{5}$. The acceleration due to gravity is 32 feet per second squared.



Use a ruler and a protractor to determine the magnitude (in centimeters) and direction of each vector.





Use \overline{a} and \overline{b} above to find the magnitude and direction of each resultant.

 $\mathbf{3.} \ \mathbf{\overline{a}} + \mathbf{\overline{b}}$

4. $2\mathbf{\overline{b}} - \mathbf{\overline{a}}$

Write the order pair or ordered triple that represents \overrightarrow{AB} . Then find the magnitude of \overrightarrow{AB} .

| nine the order pair of order of the pro- | - cp- coonto - 20 - non - non - non - non - non - non |
|---|---|
| 5 . <i>A</i> (3, 6), <i>B</i> (-1, 9) | 6 . <i>A</i> (-2, 7), <i>B</i> (3, 10) |
| 7 . <i>A</i> (2, -4, 5), <i>B</i> (9, -3, 7) | 8 . <i>A</i> (-4, -8, -2), <i>B</i> (-8, -10, 2) |
| | |
| Let $\mathbf{\tilde{r}} = \langle -1, 3, 4 \rangle$ and $\mathbf{\tilde{s}} = \langle 4, 3, -6 \rangle$. | |
| 9 . Find $\vec{\mathbf{r}} = \vec{\mathbf{s}}$. | 10. Find $3\mathbf{\hat{s}} - 2\mathbf{\hat{r}}$. |
| 11. Find $\vec{\mathbf{r}} + 3\hat{\mathbf{s}}$. | 12. Find $ \hat{\mathbf{r}} $. |
| 13. Find $ \widehat{\mathbf{s}} $. | 14 . Write $\vec{\mathbf{r}}$ as the sum of unit vectors. |
| 15 . Write $\hat{\mathbf{s}}$ as the sum of unit vectors. | 16. Find $\mathbf{\hat{r}} \cdot \mathbf{\hat{s}}$. |
| 17. Find $\mathbf{\hat{r}} \times \mathbf{\hat{s}}$. | 18 . Is $\hat{\mathbf{r}}$ perpendicular to $\hat{\mathbf{s}}$? |
| | |

- **19.** Find the magnitude and direction of the resultant of two forces of 125 pounds and 60 pounds at angles of 40° and 165° with the *x*-axis, respectively.
- **20**. Write parametric equations for the line that passes through the point *P*(3, 11) and is parallel to $\hat{\mathbf{a}} = \langle 2, -5 \rangle$.
- **21**. Write an equation in slope-intercept form of the line with the parametric equations.

$$x = 2t + 3$$
$$y = t + 1$$

- **22**. Find the initial horizontal velocity and vertical velocity of a hockey puck shot at 100 mph at an angle of 2° with the ice.
- **23.** Describe the result of the product of a vertex matrix and the matrix $\begin{bmatrix} -4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.
- **24. Physics** A downward force of 110 pounds is applied to the end of a 1.5-foot lever. What is the resulting torque about the axis of rotation if the angle of the lever to the horizontal is 60°?
- **25. Gardening** Tei uses a sprinkler to water his garden. The sprinkler discharges water with a velocity of 28 feet per second. If the angle of the water with the ground is 35°, how far will the water travel in the horizontal direction? The acceleration due to gravity is 32 feet per second squared.

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Graph each point.

1. A(2.5, 140°)

2. $B\left(-2, \frac{5\pi}{4}\right)$

3.
$$C(3, -\frac{\pi}{6})$$

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Graph each polar equation.

4. $\theta = \frac{3\pi}{2}$ **5.** r = -4 **6.** $r = 6 \cos 3\theta$ **7.** $r = 2 + 2 \sin \theta$

Find the polar coordinates of each point with the given rectangular coordinates. Use $0 \le \theta < 2\pi$ and $r \ge 0$.

8. (2, 2) **9**. (-6, 0) **10**. (-2, -3)

Find the rectangular coordinates of each point with the given polar coordinates. 11. $\left(3, -\frac{5\pi}{4}\right)$ 12. $\left(2, \frac{7\pi}{6}\right)$ 13. (-4, 1.4)

Write each rectangular equation in polar form.

14. y = -3 **15.** $x^2 + y^2 = 3x$

Write each polar equation in rectangular form.

16. r = 7 **17.** $5 = r \cos(\theta - 45^{\circ})$

Write each equation in polar form. Round ϕ to the nearest degree.

18. 5x + 3y = -3 **19.** 2x - 4y = 1

Simplify.

| 20. | i ⁹³ | 21. | (2-5i) + (-2+4i) |
|-----|-------------------------|-----|--------------------|
| 22. | -6i - (-3 + 2i) | 23. | (3+5i)(3-2i) |
| 24. | (1 - 3i)(2 - i)(1 + 2i) | 25. | $\frac{6-2i}{2+i}$ |

Express each complex number in polar form.

26. -4 + 4i **27**. -5

Find each product or quotient. Express the result in rectangular form.

28.
$$4\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) \cdot 3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

29. $2\sqrt{3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) \div \sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
30. Find $(1 - i)^8$.
31. Find $\sqrt[3]{-27i}$

32. Solve the equation $x^3 - i = 0$. Then graph the roots in the complex plane.

33. Electricity The current in a circuit is $8(\cos 307^\circ + \mathbf{j} \sin 307^\circ)$ amps and the impedance is $20(\cos 115^\circ + \mathbf{j} \sin 115^\circ)$ ohms. Find the polar form of the voltage in the circuit. (*Hint:* $E = I \cdot Z$)

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A64 Chapter 9 Test



Find the distance between each pair of points with the given coordinates. Then, find the midpoint of the segment that has endpoints at the given coordinates.

- **1.** (-1, 2), (3, 1) **2.** (3k, k + 1), (2k, k 1)
- **3**. Write the standard form of the equation of the circle that passes through (-6, -4) and has its center at (-8, 3).
- **4**. Find the coordinates of the center, foci, and vertices of the ellipse with equation $\frac{x^2}{6} + \frac{y^2}{10} = 1$.
- **5**. Write an equation of the ellipse centered at the origin that has a horizontal major axis, $e = \frac{1}{2}$, and $c = \frac{1}{2}$.
- **6**. Find the coordinates of the center, foci, and the vertices and the equations of the asymptotes of the graph of the hyperbola with equation $\frac{(y-2)^2}{16} \frac{(x+4)^2}{7} = 1$.
- **7**. Write an equation of the hyperbola that has eccentricity $\frac{3}{2}$ and foci at (-5, -2) and (-5, 4).
- **8**. Find the coordinates of the vertex and focus, and the equations of the directrix and axis of symmetry of the parabola with equation $(y + 3)^2 = 8x$.
- **9**. Write an equation of the parabola that has a focus at (3, -5) and a directrix with equation y = -2.

Identify the conic section represented by each equation. Then write the equation in standard form and graph the equation.

| 10. $x^2 + 6x - 8y - 7 = 0$ | 11. $4x^2 - y^2 = 1$ |
|--|---|
| 12. $x^2 + y^2 + 4x - 12y + 36 = 0$ | 13. $3x^2 - 16y^2 - 18x + 128y - 37 = 0$ |
| $14. \ 9x^2 - y^2 - 90x + 8y + 200 = 0$ | 15. $2x^2 - 13y^2 + 5 = 0$ |
| 16. $y^2 - 2x + 10y + 27 = 0$ | 17. $x^2 + 2y^2 + 2x - 12y + 11 = 0$ |

Find the rectangular equation of the curve whose parametric equations are given.

18. $x = t, y = 2t^2 + t, -\infty < t < \infty$ **19.** $x = 2 \cos t, y = 2 \sin t, 0 \le t \le 2\pi$

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

- **20.** $4(x+1)^2 + (y-3)^2 = 36$ for $T_{(3,-5)}$ **21.** $2x^2 y^2 = 8, \theta = 60^\circ$
- **22.** Solve the system of equations algebraically. Round to the nearest tenth. $x^2 + 4y^2 = 4$ $(x - 1)^2 + y^2 = 1$
- **23**. Graph the system of inequalities.

$$x^{2} + y^{2} - 2x - 4y + 1 \ge 0$$

$$x^{2} - 4y - 2x + 5 \ge 0$$

- **24. Geometry** Write the standard form of the equation of the circle that passes through (0, 1), (-2, 3), and (4, 5). Then identify the center and radius.
- **25. Technology** Mirna bought a motion detector light and installed it in the center of her backyard. It can detect motion within a circle defined by the equation $x^2 + y^2 = 90$. If a person walks northeast through the backyard along a line defined by the equation y = 2x 3, at what point will the person set off the motion detector?

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| Evaluate each expression. 1. $343^{\frac{2}{3}}$ | 2. | $64^{-\frac{1}{3}}$ |
|---|--------------|--|
| Simplify each expression. 3. $((2a)^3)^{-2}$ 5. Express $\sqrt[3]{27a^6b^{12}}$ using rational exponents. 6. Express $m^{\frac{1}{2}}n^{\frac{2}{3}}$ using a radical. | 4. | $\left(x^{\frac{3}{2}}y^{2}z^{\frac{5}{4}}\right)^{4}$ |
| Graph each exponential function. 7. $y = \left(\frac{1}{3}\right)^{x-2}$ | 8. | $y = 5^{x+1}$ |
| Write each equation in exponential form. 9. $\log_4 2 = \frac{1}{2}$ | 10. | $\log_{\frac{1}{6}} 216 = -3$ |
| Write each equation in logarithmic form. 11. $5^4 = 625$ | 12. | $8^5 = m$ |
| Solve each equation. 13. $\log_x 32 = -5$ 15. $3.6^x = 72.4$ | 14. 16. | $log_5 (2x) = log_5 (3x - 4)$ 6 ^{x - 1} = 8 ^{2 - x} |
| Find the value of each logarithm using the chan 17. log ₄ 15 | ge of 18. | base formula. log ₃ 0.9375 |

Evaluate each expression.

| 19. | $\log_{81} 3$ | 20. | log 542 |
|-----|---------------|-----|------------------|
| 21. | ln 0.248 | 22. | antiln (-1.9101) |

- **23**. Find the amount of time required for an amount to double at a rate of 5.4% if the interest is compounded continuously.
- **24. Biology** A certain bacteria will triple in 6 hours. If the final count is 8 times the original count, how much time has passed?
- **25. Electricity** The charge on a discharging capacitor is given by $q(t) = Qe^{-\frac{t}{RC}}$, where *Q* is the initial charge on the capacitor with capacitance *C*, *R* is the resistance in the circuit, and *t* is time, in seconds. If $Q = 5 \times 10^{-6}$ coulomb, R = 3 ohms, and $C = 2 \times 10^{-6}$ farad, find the time required for the charge on the capacitor to decrease to 1×10^{-6} coulomb.

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For Exercises 1–4, assume that each sequence or series is arithmetic.

- **1**. Find the next four terms of the sequence 2, 4.5, 7,
- **2**. Find the 24th term of the sequence $-6, -1, 4, \ldots$.
- **3**. Write a sequence that has three arithmetic means between -4 and 8.
- **4**. Find *n* for the series in which $a_1 = 12$, d = 5, and $S_n = 345$.

For Exercises 5–7, assume that each sequence or series is geometric.

- **5**. Determine the common ratio and find the next three terms of the sequence $\frac{1}{4}, \frac{1}{10}, \frac{1}{25}, \dots$
- **6**. Write a sequence that has three geometric means between 16 and 1.
- **7.** Find the sum of the first 10 terms of the series $\frac{5}{2} + 5 + 10 + \cdots$.

Find each limit, or state that the limit does not exist and explain your reasoning.

8.
$$\lim_{n \to \infty} \frac{n^3 + 3}{3n^2 + 1}$$
 9. $\lim_{n \to \infty} \frac{n^3 + 4}{2n^3 + 3n}$

Determine whether each series is convergent or divergent.

| 10. | 1 1 | 1 | 44 | 1 | 1 | 1 | 2 | |
|-----|---|--------------------------------|-----|---|------------------|------------------|----------------|-----|
| | $\frac{1}{3 \cdot 1^2} + \frac{1}{3 \cdot 2^2}$ | $+\frac{1}{3\cdot 3^2}+\cdots$ | 11. | 6 | $+ \overline{3}$ | $+ \overline{2}$ | $+\frac{-}{3}$ | ••• |

Express each series using sigma notation.

12.
$$5 + 10 + 15 + \dots + 95$$
 13. $6 + 9 + \frac{27}{2} + \frac{81}{4} + \dots$

14. Use the Binomial Theorem to expand $(2a - 3b)^5$.

Find the designated term of each binomial expansion.

15. sixth term of $(a + 2)^{10}$ **16**

16. fifth term of $(3x - y)^8$

- **17**. Write -2 + 2i in exponential form.
- **18**. Find the first three iterates of the function f(z) = 3z + (2 i) for the initial value $z_0 = 2i$.
- **19**. Use mathematical induction to prove that the following proposition is valid for all positive integral values of n.

$$2 \cdot 3 + 4 \cdot 5 + 6 \cdot 7 + \dots + 2n(2n+1) = \frac{n(n+1)(4n+5)}{3}$$

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20. Finance Melissa deposits \$200 into an investment every 3 months. The investment pays an APR of 8% and interest is compounded quarterly. Melissa makes each payment at the beginning of the quarter and the interest is posted at the end of the quarter. What will be the total value of the investment at the end of 10 years?



Find each value.

- **1**. P(6, 2) **2**. P(7, 5) **3**. C(8, 3) **4**. C(5, 4)
- **5.** The letters *r*, *s*, *t*, *u*, and *v* are to be used to form five-letter patterns. How many patterns can be formed if repetitions are not allowed?
- **6. Employment** Five people have applied for three different positions in a store. If each person is qualified for each position, in how many ways can the positions be filled?
- 7. How many ways can 7 people be seated at a round table relative to each other?
- **8. Sports** How many baseball teams can be formed from 15 players if 3 only pitch and the others play any of the remaining 8 positions?

A bag contains 4 red and 6 white marbles.

- **9**. How many ways can 5 marbles be selected if exactly 2 must be red?
- **10**. If two marbles are chosen at random, find P(2 white).
- **11. Sports** The probability that the Pirates will win a game against the Hornets is $\frac{1}{4}$. What are the odds that the Pirates will beat the Hornets?
- **12**. Five cards are dealt from a standard deck of cards. What is the probability that they are all from the same suit?
- **13**. Find the probability of getting a sum of 8 on the first throw of two number cubes and a sum of 4 on the second throw.
- **14. Communication** Callie is having a new phone line installed. What is the probability that the final 3 digits in the telephone number will all be odd?
- **15**. A bag contains 3 red, 4 white, and 5 blue marbles. If 3 marbles are selected at random, what is the probability that all are red or all are blue?
- **16**. A card is drawn from a standard deck of cards. What is the probability of selecting an ace or a black card?
- **17**. A four-digit number is formed from the digits 7, 3, 3, 2, and 2. If the number formed is odd, what is the probability that the 3s are together?
- **18**. Two different numbers are selected at random from the numbers 1 through 9. If their product is even, what is the probability that both numbers are even?
- **19.** Five bent coins are tossed. The probability of heads is $\frac{2}{3}$ for each of them. What is the probability that no more than 2 coins will show heads?
- **20. Archery** While shooting arrows, Akira can hit the center of the target 4 out of 5 times. What is the probability that he will hit it exactly 4 out of the next 7 times?





The number of absences for a random sample of 80 high school students at Dover High School last school year are given below.

| 6 | 16 | 12 | 7 | 7 | 9 | 13 | 12 | 7 | 7 |
|----|----|----|----|----|----|----|----|----|----|
| 19 | 4 | 9 | 6 | 4 | 11 | 13 | 10 | 16 | 20 |
| 10 | 17 | 11 | 12 | 6 | 9 | 10 | 14 | 3 | 8 |
| 8 | 12 | 13 | 8 | 8 | 11 | 12 | 1 | 11 | 5 |
| 11 | 16 | 13 | 5 | 10 | 1 | 10 | 8 | 15 | 10 |
| 13 | 9 | 20 | 5 | 9 | 15 | 11 | 18 | 12 | 14 |
| 10 | 16 | 8 | 10 | 2 | 11 | 19 | 10 | 12 | 17 |
| 14 | 6 | 9 | 12 | 10 | 14 | 8 | 9 | 7 | 9 |

- 1. What is the range of the data?
- 2. What is an appropriate class interval?
- **3**. What are the class limits?
- 4. What are the class marks?
- **5**. Construct a frequency distribution of the data.
- 6. Draw a histogram of the data.
- 7. Find the mean of the data.
- **8**. Find the median of the data.

A small metal object is weighed on a laboratory balance by each of 15 students in a physics class. The weight of the object in grams is reported as 2.341, 2.347, 2.338, 2.350, 2.344, 2.342, 2.345, 2.348, 2.340, 2.345, 2.343, 2.344, 2.347, 2.341, and 2.344.

- **9**. Find the median of the data.
- **10**. Find the first quartile point and the third quartile point.
- **11**. What is the semi-interquartile range?
- **12**. Make a box-and-whisker plot of the data.
- **13**. What is the mean deviation of the data?
- 14. What is the standard deviation of the data?

The mean of a set of normally distributed data is 24 and the standard deviation is 2.8.

- **15**. Find the interval about the mean that includes 68.3% of the data.
- **16**. Find the interval about the mean that includes 90% of the data.
- **17.** What percent of the data is between 18.4 and 32.4?
- **18**. What percent of the data is between 24 and 29.6?

Suppose a random sample of data has $\sigma = 3.6$, N = 400, and $\overline{X} = 57$.

19. Find the standard error of the mean.

20. Find the interval about the sample mean that has a 5% level of confidence.

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Use the graph of y = f(x) to find each value.

- **1.** $\lim_{x \to -1} f(x)$ and f(-1)
- **2**. $\lim f(x)$ and f(2) $x \rightarrow 2$

Evaluate each limit.

3. $\lim_{x \to 0} \frac{x^2 + 3x}{x}$ $x^2 - 9$...

5.
$$\lim_{x \to 3} \frac{1}{x^3 - 27}$$

7. Use the definition of derivative to find the derivative of $f(x) = x^2 - 2x$.

Use the derivative rules to find the derivative of each function.

8. $f(x) = \frac{1}{2}x^2 - 7x + 1$ **9.** $f(x) = 4x^3 - 4$ **11.** $f(x) = 2x^5 - 4x^3 + \frac{2}{2}x^2 - 6$ **10.** $f(x) = 6x^4 - 2x^2 - 30$ **12.** $f(x) = 2x^4(x^3 + 3x^2)$

Find the antiderivative of each function.

14. f(x) = -2x + 6

- **16.** $f(x) = \frac{1}{2}x^3 \frac{2}{7}x + 5$
- **18**. Use a limit to find the area between the graph of $y = x^3$ and the *x*-axis from x = 0 to x = 2.
- **19**. Find the area of the shaded region in the graph at the right.

4

15.
$$f(x) = -x^{5} + 4x^{2} - x^{5}$$

17. $f(x) = \frac{x^{3} - 4x^{2} + x}{x}$
30 $\psi = 3x^{2}$
30 $\psi = 3x^{2}$
30 $\psi = 3x^{2}$
30 $\psi = 3x^{2}$

Evaluate each definite integral.

20.
$$\int_0^1 (2x+3) dx$$

Evaluate each indefinite integral.

22.
$$(1-2x) dx$$

21.
$$\int_{1}^{5} (-x^2 + x + 6) dx$$

23. $\int (3x^2 + 4x + 7) dx$

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.3

- **24. Baseball** A baseball is hit so that its height, in feet, above the ground after t seconds is given by $h(t) = 3 + 95t - 16t^2$. Find the vertical velocity of the ball after 2 seconds.
- **25.** Geometry The volume of a cone of radius *r* and height *h* can be calculated using the formula $V = 2\pi \int_{0}^{r} \left(-\frac{h}{r}x^{2} + hx\right) dx$. Use this formula to find the volume of a cone with radius 3 units and height 2 units.

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4.
$$\lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1}$$

6.
$$\lim_{x \to 1} \frac{x^2 - 2x + 3}{3x^2 - 5}$$

13.
$$f(x) = (x + 3)^2$$

15. $f(x) = -x^3 + 4x^2 - x + 17$
17. $f(x) = \frac{x^3 - 4x^2 + x}{x}$