# Bivariate Relationships

- When exploring/describing a bivariate (x,y) relationship:
  - ☑ Determine the Explanatory and Response variables
  - ☑ Plot the data in a scatterplot
  - ☑ Note the Strength, Direction, and Form
  - ☑ Note the mean and standard deviation of x and the mean and standard deviation of y
  - ☑ Calculate and Interpret the Correlation, r
  - ☑ Calculate and Interpret the Least Squares Regression Line in context.
  - ☑ Assess the appropriateness of the LSRL by constructing a Residual Plot.

# Corrosion and Strength

- Consider the following data from the article, "The Carbonation of Concrete Structures in the Tropical Environment of Singapore" (*Magazine of Concrete Research* (1996):293-300):
  - x= carbonation depth in concrete (mm)
  - y= strength of concrete (Mpa)

| x | 8 | 20 | 20 | 30 | 35 | 40 | 50 | 55 | 65 |
|---|---|---|---|---|---|---|---|---|---|
| y | 22.8 | 17.1 | 21.5 | 16.1 | 13.4 | 12.4 | 11.4 | 9.7 | 6.8 |

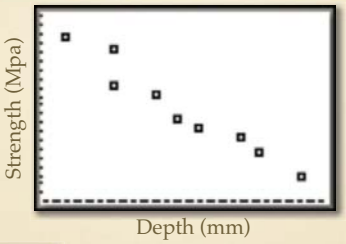
- Define the Explanatory and Response Variables.
- Plot the data and describe the relationship.

# Corrosion and Strength



There is a strong, negative, linear relationship between depth of corrosion and concrete strength. As the depth increases, the strength decreases at a constant rate.

# Corrosion and Strength

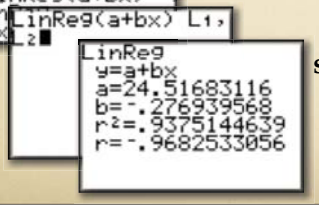


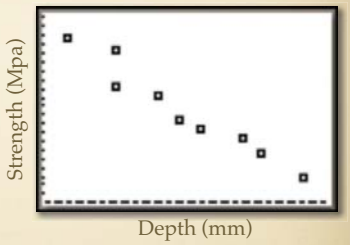


**The mean depth of corrosion is 35.89mm with a standard deviation of 18.53mm. The mean strength is 14.58 Mpa with a standard deviation of 5.29 Mpa.**

# Corrosion and Strength

Find the equation of the Least Squares Regression Line (LSRL) that models the relationship between corrosion and strength.

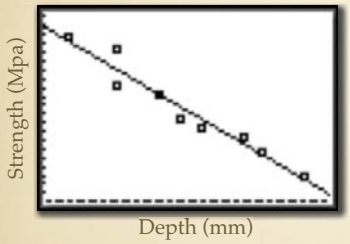




**y=24.52+(-0.28)x**

**strength=24.52+(-0.28)depth**

**r=-0.96**

# Corrosion and Strength




**y=24.52+(-0.28)x**

**strength=24.52+(-0.28)depth**

**r=-0.96**

***What does "r" tell us?***

**There is a Strong, Negative, LINEAR relationship between depth of corrosion and strength of concrete.**

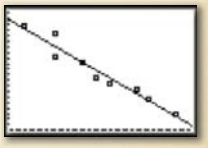
***What does "b=-0.28" tell us?***

**For every increase of 1mm in depth of corrosion, we predict a 0.28 Mpa decrease in strength of the concrete.**

# Corrosion and Strength

- Use the prediction model (LSRL) to determine the following:

- What is the predicted strength of concrete with a corrosion depth of 25mm?
  - strength=24.52+(-0.28)depth
  - strength=24.52+(-0.28)(25)
  - strength=17.59 Mpa



- What is the predicted strength of concrete with a corrosion depth of 40mm?
  - strength=24.52+(-0.28)(40)
  - strength=13.44 Mpa
  - How does this prediction compare with the observed strength at a corrosion depth of 40mm?

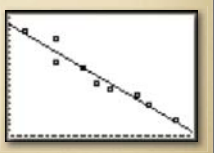
# Residuals

- Note, the predicted strength when corrosion=40mm is:
  - predicted strength=13.44 Mpa
- The observed strength when corrosion=40mm is:
  - observed strength=12.4mm

- The prediction did not match the observation.
  - That is, there was an "error" or "residual" between our prediction and the actual observation.

- RESIDUAL = Observed y - Predicted y

- The residual when corrosion=40mm is:
  - residual = 12.4 - 13.44
  - residual = -1.04

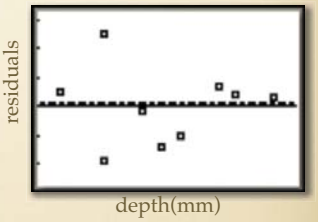


# Assessing the Model

- Is the LSRL the most appropriate prediction model for strength? r suggests it will provide strong predictions...can we do better?
  - To determine this, we need to study the residuals generated by the LSRL.
  - Make a residual plot.
  - Look for a pattern.
    - If no pattern exists, the LSRL may be our best bet for predictions.
    - If a pattern exists, a better prediction model may exist...

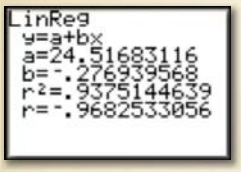
# Residual Plot

- Construct a Residual Plot for the (depth,strength) LSRL.

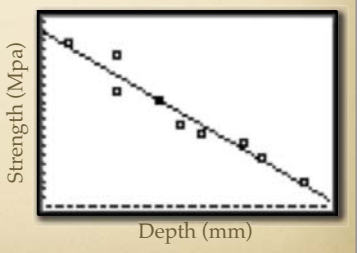

- There appears to be no pattern to the residual plot...therefore, the LSRL may be our best prediction model.

# Coefficient of Determination




We know what "r" tells us about the relationship between depth and strength....what about $r^2$?

93.75% of the variability in predicted strength can be explained by the LSRL on depth.




# Summary

- When exploring a bivariate relationship:
  - Make and interpret a scatterplot:
    - Strength, Direction, Form
  - Describe *x* and *y*:
    - Mean and Standard Deviation in Context
  - Find the Least Squares Regression Line.
    - Write in context.
  - Construct and Interpret a Residual Plot.
  - Interpret *r* and $r^2$ in context.
  - Use the LSRL to make predictions...