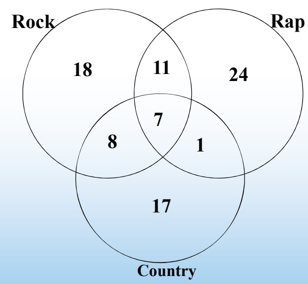
# Chapter 14

**From Randomness to Probability**

**1) Read and study chapter 14.**
**2) Use the internet and do a research on "permutation" and "combination." You are expected to do permutation and combination problems**
**3) Chapter 14 Test on Monday**

# Starter Ch. 14B




1) How many people are represented in the diagram?

2) How many people like country?

3) If one person is chosen at random, what is the probability that that person will like rap music?

P (rap) =

# Chapter 14

**From Randomness to Probability**

## Dealing with Random Phenomena

- A **random phenomenon** is a situation in which we know what outcomes could happen, but we don't know which particular outcome did or will happen.
- In general, each occasion upon which we observe a random phenomenon is called a **trial**.
- At each trial, we note the value of the random phenomenon, and call it an **outcome.**
- When we combine outcomes, the resulting combination is an **event.**
- The collection of *all possible outcomes* is called the **sample space**.



## The Law of Large Numbers

First a definition . . .

- When thinking about what happens with combinations of outcomes, things are simplified if the individual trials are **independent**.
  - Roughly speaking, this means that the outcome of one trial doesn't influence or change the outcome of another.
  - For example, coin flips are independent.



## The Law of Large Numbers (cont.)

- The **Law of Large Numbers (LLN)** says that the long-run *relative frequency* of repeated independent events gets closer and closer to a single value.
- We call the single value the **probability** of the event.
- Because this definition is based on ***repeatedly observing the event's outcome***, this definition of probability is often called **empirical probability**.

## The Nonexistent Law of Averages

- The LLN says nothing about short-run behavior.
- Relative frequencies even out *only in the long run*, and this long run is *really* long (*infinitely* long, in fact).



## Modeling Probability

- When probability was first studied, a group of French mathematicians looked at games of chance in which all the possible outcomes were ***equally likely***. They developed mathematical models of **theoretical probability**.
  - It's equally likely to get any one of six outcomes from the roll of a fair die.
  - It's equally likely to get heads or tails from the toss of a fair coin.
- However, keep in mind that events are *not* always equally likely.
  - A skilled basketball player has a better than 50-50 chance of making a free throw.



## Modeling Probability (cont.)

- The probability of an event is the number of outcomes in the event divided by the total number of possible outcomes.

$$P(\mathbf{A}) = \frac{\text{\# of outcomes in } \mathbf{A}}{\text{\# of possible outcomes}}$$



## Personal Probability

- In everyday speech, when we express a degree of uncertainty *without* basing it on long-run relative frequencies or mathematical models, we are stating **subjective** or **personal probabilities**.
- Personal probabilities don't display the kind of consistency that we will need probabilities to have, so we'll stick with formally defined probabilities.



## The First Three Rules of Working with Probability

- We are dealing with probabilities now, not data, but the three rules don't change.
  - Make a picture.
  - Make a picture.
  - Make a picture.



## The First Three Rules of Working with Probability (cont.)

- The most common kind of picture to make is called a Venn diagram.

- We will see Venn diagrams in practice shortly…

## Formal Probability

1. **Two requirements for a probability:**
   - A probability is a number between 0 and 1.
   - For any event **A**, **$0 \le P(A) \le 1$.**

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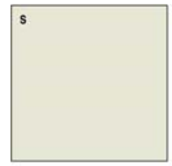
## Formal Probability (cont.)

2. **Probability Assignment Rule:**
   - The probability of the set of all possible outcomes of a trial must be 1.
   - **$P(S) = 1$** (**S** represents the set of all possible outcomes.)



The sample space **S.**

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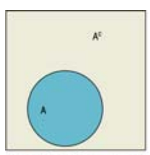
## Formal Probability (cont.)

3. **Complement Rule:**
   - The set of outcomes that are *not* in the event **A** is called the **complement** of **A**, denoted **$A^C$.**
   - The probability of an event occurring is 1 minus the probability that it doesn't occur: **$P(A) = 1 - P(A^C)$**



The set **A** and its complement.

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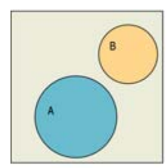
## Formal Probability (cont.)

4. **Addition Rule:**
   - Events that have no outcomes in common (and, thus, cannot occur together) are called **disjoint** (or **mutually exclusive**).



Two disjoint sets, **A** and **B.**

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## Formal Probability (cont.)

4. **Addition Rule (cont.):**
   - For two disjoint events **A** and **B**, the probability that one *or* the other occurs is the sum of the probabilities of the two events.
   - **$P(A \cup B) = P(A) + P(B)$,** provided that **A** and **B** are disjoint.

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## Formal Probability (cont.)

5. **Multiplication Rule:**
   - For two independent events **A** and **B**, the probability that *both* **A** and **B** occur is the product of the probabilities of the two events.
   - **$P(A \cap B) = P(A) \times P(B)$,** provided that **A** and **B** are independent.

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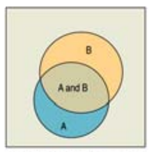
## Formal Probability (cont.)

**5. Multiplication Rule (cont.):**

- Two independent events **A** and **B** are not disjoint, provided the two events have probabilities greater than zero:




Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.



## Formal Probability (cont.)

**5. Multiplication Rule:**

- Many Statistics methods require an **Independence Assumption**, but *assuming* independence doesn't make it true.
- Always *Think* about whether that assumption is reasonable before using the Multiplication Rule.



## Formal Probability - Notation

Notation alert:

- In this text we use the notation $P(\mathbf{A} \cup \mathbf{B})$ and $P(\mathbf{A} \cap \mathbf{B})$.
- In other situations, you might see the following:
  - *P*(**A** or **B**) instead of $P(\mathbf{A} \cup \mathbf{B})$
  - *P*(**A** and **B**) instead of $P(\mathbf{A} \cap \mathbf{B})$



## Complements

- Suppose the probability of event A is 0.4
- What is the probability of event A NOT occurring?
- 0.6
- The set of outcomes that represent event A NOT occurring is called the **complement of event A**, often denoted as $\mathbf{A^c}$
- The probability of an event and the probability of the complement of that event must add up to 1, so that we can say
- $\mathbf{P(A) = 1 - P(A^c)}$
- Sometimes it is easier to find the probability of $A^c$ than it is to find the probability of A, in which case to find P(A) just subtract $P(A^c)$ from 1!



## Laws of Probability

- Suppose the probability that a randomly selected student is a sophomore (A) is 0.20 and the probability that he or she is a junior (B) is 0.30. What is the probability that the student is either a sophomore OR a junior, written $P(\mathbf{A} \cup \mathbf{B})$
- 50%
- If two events have no outcomes in common (no overlap / both can't be true at once) we say the events are **DISJOINT (or MUTUALLY EXCLUSIVE**)
- For disjoint events A and B, you can find the probability of A OR B, written $P(\mathbf{A} \cup \mathbf{B})$ , by ADDING the separate probabilities



## 2nd example

- Suppose the probability of owning an MP3 player is 0.50. and the probability of owning a computer is 0.90.
- Is the probability of owning an MP3 player OR a computer equal to 1.40?
- No – here the events are NOT disjoint, you CAN own both.. So it is not appropriate to add the separate probabilities together

## $2^{nd}$ law of probability

- Suppose that when you drive to school, the probability that a certain traffic light is red when you reach the intersection is 0.35.
- What is the probability that you will hit the light when it is red on two consecutive days?
- 0.35 * 0.35 = 0.1225 or 12.25%
- **For two INDEPENDENT events A and B, the probability that BOTH A AND B occur is the product of the probabilities of the two events**
- We write ***P*(A ∩ B). = P(A) * P(B)**
- This rule ONLY applies for independent events; we will deal with dependent events in the next chapter!



## Application of complement

- Suppose I want to find the probability that I will hit the traffic light when it is red AT LEAST once this week (that is, at least once in 5 drives to school).
- If you think about it, I can either hit the red light NOT AT ALL or I have to hit it AT LEAST ONCE
- In other words, NOT AT ALL and AT LEAST ONCE are complementary events; one of them must be true when I drive to school 5 times
- So P(AT LEAST ONCE) = 1 – P(NOT AT ALL)
- It is much easier to calculate the probability of AT LEAST ONCE this way
- P(NOT AT ALL) = (0.65)(0.65)(0.65)(0.65)(0.65) = $(0.65)^5$ = 0.116
- P(AT LEAST ONCE) = 1 -0.116 = 0.884
- So there is an 88.4% probability that I will hit the red light at least once this week!



## Mixed example

- In 2001, The manufacturers of M & Ms decided to add a new color of M & M, either purple, pink, or teal. They surveyed kids around the world regarding which color should be added. In Japan, 38% voted pink, 36% voted teal, and 16% voted purple.
- What's the probability a randomly selected respondent preferred either pink or teal?
- If we pick two respondents at random, what is the probability that they BOTH selected purple?
- If we pick three respondents at random, what is the probability that AT LEAST one preferred purple?



### What's the probability a randomly selected respondent preferred either pink or teal?

- P(pink ∪ teal) = P(pink) + P(teal)
  = 0.38 + 0.36
  = 0.74

The probability that a respondent preferred pink or teal is 74%



### If we pick two respondents at random, what is the probability that they BOTH selected purple?

- P( both purple)
- P($1^{st}$ selected purple AND $2^{nd}$ selected purple)
- Are these events independent?
- Yes!
- So multiply: (0.16)(0.16) = $(0.16)^2$ = 0.0256
- The probability both respondents selected purple is 2.56%



### If we pick three respondents at random, what is the probability that AT LEAST one preferred purple?

- P(at least one preferred purple) is:
- 1 – P(none preferred purple)
- = 1 – P($1^{st}$ didn't prefer purple AND $2^{nd}$ didn't prefer purple AND $3^{rd}$ didn't prefer purple)
- = 1 – (0.84)(0.84)(0.84) = 1 –$(0.84)^3$ = 0.407
- There is a 40.7% probability that at least one of the three respondents preferred purple.

## Putting the Rules to Work

- In most situations where we want to find a probability, we'll use the rules in combination.
- A good thing to remember is that it can be easier to work with the *complement* of the event we're really interested in.

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## What Can Go Wrong?

- Beware of probabilities that don't add up to 1.
  - To be a legitimate probability distribution, the sum of the probabilities for all possible outcomes must total 1.
- Don't add probabilities of events if they're not disjoint.
  - Events must be disjoint to use the Addition Rule.

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## What Can Go Wrong? (cont.)

- Don't multiply probabilities of events if they're not independent.
  - The multiplication of probabilities of events that are not independent is one of the most common errors people make in dealing with probabilities.
- Don't confuse disjoint and independent—disjoint events *can't* be independent.

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## What have we learned?

- Probability is based on long-run relative frequencies.
- The Law of Large Numbers speaks only of long-run behavior.
  - Watch out for misinterpreting the LLN.

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## What have we learned? (cont.)

- There are some basic rules for combining probabilities of outcomes to find probabilities of more complex events. We have the:
  - Probability Assignment Rule
  - Complement Rule
  - Addition Rule for disjoint events
  - Multiplication Rule for independent events

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