Conditional Probability

Permutation

- Consider the possible arrangements of the letters *A*, *B*, and *C*.
- The possible arrangements are: *ABC, ACB, BAC, BCA, CAB, CBA.*
- If the *order of the arrangement is important* then we say that each arrangement is a permutation of the three letters. Thus there are six permutations of the three letters.

Permutation

- An arrangement of ***n*** distinct objects in a specific order is called a **permutation** of the objects.
- ***Note:*** To determine the number of possibilities mathematically, one can use the multiplication rule to get: $3 \times 2 \times 1 = 6$ permutations.

Permutation

- **Permutation Rule :** The arrangement of *n* objects in a specific order using *r* objects at a time is called a permutation of *n* objects taken *r* objects at a time. It is written as $_nP_r$ and the formula is given by

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutation

- How many different ways can a chairperson and an assistant chairperson be selected for a research project if there are seven scientists available?
- ***Solution:*** $= {}_7P_2 = 7! / (7-2)! = 7! / 5! = 42.$

Permutation

- How many different ways can four books be arranged on a shelf if they can be selected from nine books?
- ***Solution:*** number of ways $= {}_9P_4 = 9! / (9-4)! = 9!/5! = 3024.$

Combination

- Consider the possible arrangements of the letters *A*, *B*, and *C*.
- The possible arrangements are: *ABC, ACB, BAC, BCA, CAB, CBA.*
- If the *order of the arrangement is* ***NOT*** *important* then we say that each arrangement is the same. We say there is *one combination of the three letters*.

6-7

Combination

- **Combination Rule :** The number of combinations of of *r* objects from *n* objects is denoted by $_nC_r$ and the formula is given by

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

6-8

Combination

- How many combinations of four objects are there taken two at a time?
- ***Solution:*** Number of combinations: $_4C_2$ **= 4! / [(4 – 2)! 2!] = 4! / [2! 2!] = 6.**

6-9

Combination

- In order to survey the opinions of customers at local malls, a researcher decides to select 5 malls from a total of 12 malls in a specific geographic area. How many different ways can the selection be made?
- ***Solution:*** Number of combinations: $_{12}C_5$ **= 12! / [(12 – 5)! 5!] = 12! / [7!5!] = 792.**

6-10

Combination

- In a club there are 7 women and 5 men. A committee of 3 women and 2 men is to be chosen. How many different possibilities are there?
- ***Solution:*** Number of possibilities: (number of ways of selecting 3 women from 7) × (number of ways of selecting 2 men from 5) = $_7C_3$ ò÷ $_5C_2$ **= (35)(10) = 350.**

6-11

Combination

- A committee of 5 people must be selected from 5 men and 8 women. How many ways can the selection be made if there are at least 3 women on the committee?

6-12

Combination

- *Solution:* The committee can consist of 3 women and 2 men, or 4 women and 1 man, or 5 women. To find the different possibilities, find each separately and then add them:

$${}_8C_3 \times {}_5C_2 + {}_8C_4 \times {}_5C_1 + {}_8C_5 \times {}_5C_0 =$$

$$(56)(10) + (70)(5) + (56)(1) = \mathbf{966}.$$

Probability rules

Coin Toss Example:
S = {Head, Tail}
Probability of heads = 0.5
Probability of tails = 0.5

1) Probabilities range from 0 (*no chance of the event*) to 1 (*the event has to happen*).

For any event A, 0 ≤ *P*(A) ≤ 1

Probability of getting a head = 0.5
We write this as: *P*(head) = 0.5

***P*(neither head nor tail) = 0**
***P*(getting either a head or a tail) = 1**

2) The probability of the complete sample space must equal 1.

***P*(sample space) = 1**

***P*(head) + *P*(tail) = 0.5 + 0.5 = 1**

3) The probability of an event not occurring is 1 minus the probability that is does occur.

***P*(A) = 1 – *P*(not A)**

***P*(tail) = 1 – *P*(head) = 0.5**

Conditional Probability

1. Event probability ***given*** that another event occurred
2. Revise original sample space to account for ***new*** information
   – Eliminates certain outcomes
3. P(A **|** **B**) = P(A and B) / P(**B**)

Computing Conditional Probability

The Probability of the Event:
Event A given that Event B has occurred

$$\mathbf{P(A\,|\,B)} = \frac{P(A \text{ and } B)}{P(B)}$$

e.g.

P(Red Card *given that it is an* Ace) = $\frac{2\ Red\ Aces}{4\ Aces} = \frac{1}{2}$

Computing Conditional Probability

Conditional Event: Draw 1 Card. Note Kind & Color

Type	Color: Red	Color: Black	Total
Ace	**2**	**2**	**4**
Non-Ace	**24**	**24**	**48**
Total	**26**	**26**	**52**

Revised Sample Space

$$P(Ace\ |\ Red) = \frac{P(Ace\ AND\ Red)}{P(Red)} = \frac{2/52}{26/52} = \frac{2}{26}$$

Computing Conditional Probability

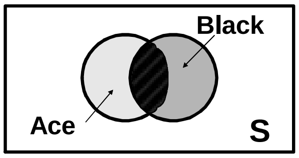
Conditional Probability: $\mathbf{P(A|B)} = \frac{P(A \text{ and } B)}{P(B)}$

Multiplication Rule: **P(A and B) = P(A | B) • P(B)**

Events are Independent: **P(A | B) = P(A)**
Or, P(A and B) = P(A) • P(B)

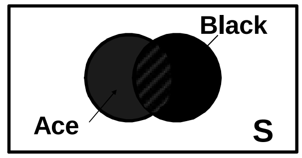
Events A and B are Independent when the probability of one event, A is not affected by another event, B.

Conditional Probability Using Venn Diagram



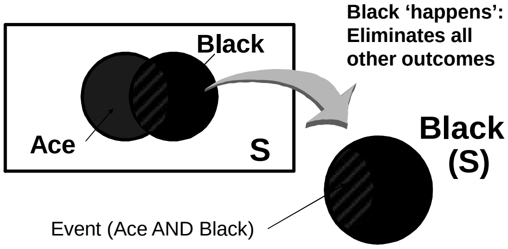
Conditional Probability Using Venn Diagram

Black 'happens': Eliminates all other outcomes



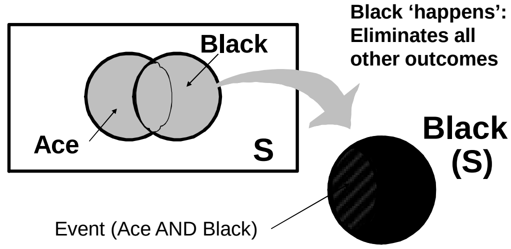
Conditional Probability Using Venn Diagram

Black 'happens': Eliminates all other outcomes



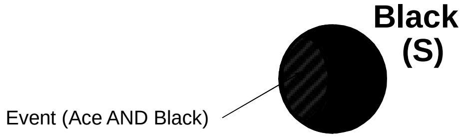
Conditional Probability Using Venn Diagram

Black 'happens': Eliminates all other outcomes



Conditional Probability Using Venn Diagram

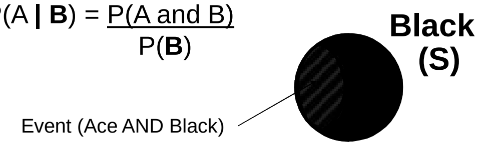
New sample space



Conditional Probability Using Venn Diagram

New sample space

$$P(A \mid \mathbf{B}) = \frac{P(A \text{ and } B)}{P(\mathbf{B})}$$



Conditional Probability Using Contingency Table

Experiment: Draw 1 card. Note kind, color & suit.

Type	Color: Red	Color: Black	Total
Ace	**2**	**2**	**4**
Non-Ace	**24**	**24**	**48**
Total	**26**	**26**	**52**

Revised sample space

$$P(\text{Ace} \mid \text{Black}) = \frac{P(\text{Ace AND Black})}{P(\text{Black})} = \frac{2/52}{26/52} = \frac{2}{26}$$