Chapter 15
Probability Models

Starter Ch. 15

TODAY
- Turn in all assignments for Ch. 14
- Chapter 14 Test
- Intro to Chapter 15
- Additional HW: Venn Diagrams and Probability

A survey of college students found that 56% live in a campus residence hall, 62% participate in a campus meal, and 42% do both.

1) What is the probability that a randomly selected student either lives or eats on campus?
2) Draw a Venn Diagram.
3) What is the probability that a randomly selected student
   a) lives off campus and doesn't have a meal program?
   b) lives in a residence hall but doesn't have a meal program?

Example

- A survey of college students found that 56% live in a campus residence hall, 62% participate in a campus meal, and 42% do both.

1) Draw a Venn Diagram.
2) What is the probability that a randomly selected student a) lives off campus and doesn't have a meal program? b) lives in a residence hall but doesn't have a meal program?

   ***a)* 0.24**

   ***b)* 0.14**

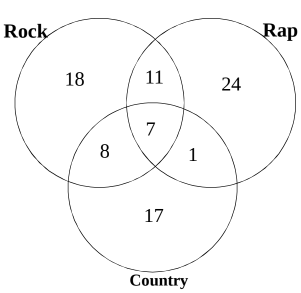
Formal Probability

- A **probability distribution** is a list of all the outcomes in the sample space and their probabilities.
  - ex: Grade of a randomly selected student:

	A	B	C	D	F
Probability:	0.1	0.3	0.4	0.15	?

1. P(F) =
2. P(A or B) =
3. $P(A^C)$ =
4. P(C or better) =
5. $P(A \text{ or } A^C)$ =

Venn Diagram



1) How many people are represented in the diagram?

2) How many people like country?

3) If one person is chosen at random, what is the probability that that person will like rap music?

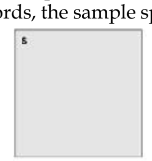
   P (rap) =

Definitions

- The **sample space S** of random phenomenon is the set of all possible outcomes.
- An **event** is any outcome or a set of outcomes of a random phenomenon.
  - That is, an event is a subset of the sample space
- A **probability model** is a mathematical description of a random phenomenon consisting of two parts: a sample space S and a way of assigning probabilities to events.

Formal Probability

- Probabilities must be between 0 and 1, inclusive.
  - A probability of 0 indicates **impossibility**.
  - A probability of 1 indicates **certainty**.
  - For any event A, **$0 \le P(A) \le 1$**.
- The Sample space represents the **"Something has to happen"** rule: Since S represents all the outcomes in an experiment, one of them has to happen. In other words, the sample space contains all possibilities.

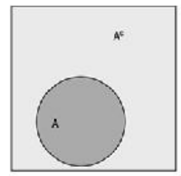


The sample space S.

$$P(S) = 1$$

Formal Probability

- The set of outcomes that are **NOT** in the event A is called the compliment of A, and is denoted **A^C**.



The set A and its complement.

$$P(A^C) = 1 - P(A)$$

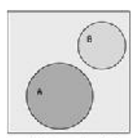
and

$$P(A) + P(A^C) = 1$$

Formal Probability

Addition Rule:

- Events that have no outcomes in common (and, thus, cannot occur together) are called **disjoint** (or **mutually exclusive**).



Two disjoint sets, A and B.

- For two disjoint events **A** and **B**, the probability that one ***or*** the other occurs is the sum of the probabilities of the two events.
- ***P*(A *or* B) =P(A U B) = *P*(A) + *P*(B),** provided that **A** and **B** are disjoint.
  - Example: On a fair six-sided die, what is P(1 or even)?
  - P(1 or even) = P(1) + P(even) = $\frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$

The General Addition Rule

- When two events **A** and **B** are disjoint, we can use the addition rule for disjoint events from Chapter 14:

  ***P*(A Ć B) = *P*(A) + *P*(B)**

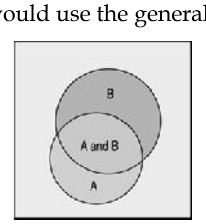
- However, when our events are not disjoint, this earlier addition rule will double count the probability of *both* **A** and **B** occurring. Thus, we need the **General Addition Rule.**
- Let's look at a picture…

The General Addition Rule (cont.)

- **General Addition Rule:**
  - For any two events **A** and **B**,

  ***P*(A Ć B) = *P*(A) + *P*(B) – *P*(A ą B)**

- The following Venn diagram shows a situation in which we would use the general addition rule:



Example

- A survey of college students found that 56% live in a campus residence hall, 62% participate in a campus meal, and 42% do both.
- What is the probability that a randomly selected student either lives or eats on campus?
- Let **L** = (student lives on campus and **M** = {student has campus meal plan}

  ***P*(a student either lives or eats on campus) = *P*(L Ć M)**
  = *P*(L) + *P*(M) – *P*(L ą M)
  = 0.56 + 0.62 – 0.42 = 0.76

- There's a 76% chance that a randomly selected college student either lives or eats on campus.

Formal Probability

- Dice Problems:
  - Suppose you roll two dice and find their sum

1,1 = **2**	1,2 = **3**	1,3 = **4**	1,4 = **5**	1,5 = **6**	1,6 = **7**
2,1 = **3**	2,2 = **4**	2,3 = **5**	2,4 = **6**	2,5 = **7**	2,6 = **8**
3,1 = **4**	3,2 = **5**	3,3 = **6**	3,4 = **7**	3,5 = **8**	3,6 = **9**
4,1 = **5**	4,2 = **6**	4,3 = **7**	4,4 = **8**	4,5 = **9**	4,6 = **10**
5,1 = **6**	5,2 = **7**	5,3 = **8**	5,4 = **9**	5,5 = **10**	5,6 = **11**
6,1 = **7**	6,2 = **8**	6,3 = **9**	6,4 = **10**	6,5 = **11**	6,6 = **12**

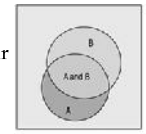
  - What is the probability distribution?

1. P(sum is even) =
2. P(sum ≤ 4) =
3. P(sum < 12) =

Formal Probability

Multiplication Rule:

- For two independent events **A** and **B**, the probability that both **A** and **B** occur is the product of the probabilities of the two events.
- Two independent events **A** and **B** are not disjoint, provided the two events have probabilities greater than zero:



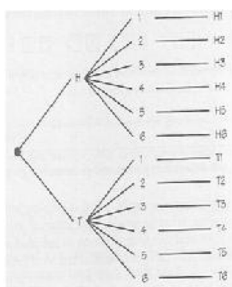
Two sets **A** and **B** that are not disjoint. The event (**A and B**) is their intersection.

- ***P*(A *and* B) = P(A ∩ B) = *P*(A) x *P*(B)**, provided that **A** and **B** are independent.
  - Many Statistics methods require an **Independence Assumption**, but ***assuming*** independence doesn't make it true.
  - Always think about whether the assumption is reasonable before using the multiplication rule.

Formal Probability

- Suppose you flip a coin and roll a die.
  - Draw the tree diagram associated with this scenario:

1. P(Tails) =
2. P(Odd) =
3. P(Tails or Odd) =
4. P(Tails and Odd)=



Common Errors

- Beware of probabilities that don't add up to 1.
  - To be a legitimate probability distribution, the sum of the probabilities for all possible outcomes must total 1.
- Don't add probabilities of events if they're not disjoint.
  - Events must be disjoint to use the Addition Rule.
- Don't multiply probabilities of events if they're not independent.
  - The multiplication of probabilities of events that are not independent is one of the most common errors people make in dealing with probabilities.
- Don't confuse disjoint and independent—**disjoint events *can't* be independent.**

More on Sample Space

- Sometimes it is difficult to find the sample space of certain events. There are several strategies that you can use to find all possible outcomes. To find the sample space for each of the following situations, we used:

1. Rolling two different colored dice (Table Method)
2. Flipping a coin, then rolling a die (Tree Diagram)
3. Flipping a coin three times (???)
   - There's one more common method that we can use…

Flipping 3 Coins – The Systematic Method

3 Heads	**2 Heads**	**1 Heads**	**0 Heads**
H,H,H	H,H,T	H,T,T	T,T,T
	H,T,H	T,H,T	
	T,H,H	T,T,H	

Let's Review

- If the probability of rolling a 3 on a fair six sided die is 1/6, will you get a 3 if you roll the die six times? What if you roll it 60 times?
  - Remember that probability is based on long-run relative frequencies, but anything can happen on the short-run.
- What does the Law of Large Numbers tell you?
  - The **Law of Large Numbers (LLN)** says that the long-run **relative frequency** of repeated independent events gets closer and closer to the **true** relative frequency as the number of trials increases.

Let's Review

- What is the "Something has to happen" rule?
  - The probability of the set of all possible outcomes must be 1. **P(S) = 1**
- What is the complement rule?
  - The probability of an event occurring is 1 minus the probability that it doesn't occur.

$$P(A^C) = 1 - P(A)$$

Let's Review

- What is the addition rule for disjoint events?
  - For two disjoint events, the probability that one or the other occurs is the sum of the probabilities of the two events.

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$$

- What is the multiplication rule for independent events?
  - For two independent events A and B, the probability that both A and B occur is the product of the probabilities of the two events.

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$$