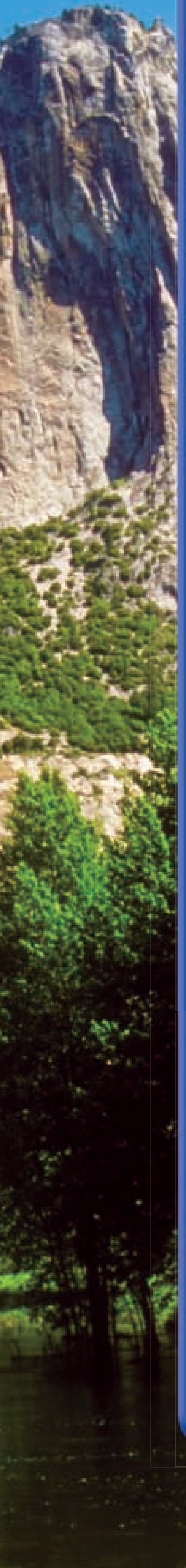


ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. algebraic expression

2. opposites

3. origin

4. variable

A. the point in the coordinate plane where the *x*-axis and the *y*-axis intersect

B. a value that does not change

C. two numbers that are equal distances from zero on a number line

D. a mathematical phrase that contains one or more variables

E. a symbol that represents a quantity that can change

Fractions and Decimals

Write each fraction as a decimal.

5. $\frac{3}{10}$ **6.** $\frac{3}{5}$ **7.** $-\frac{4}{3}$ **8.** $5\frac{3}{4}$

Graph Numbers on a Number Line

Graph each number on the same number line.

9. 3.5 **10.** -4 **11.** $-\frac{12}{4}$ **12.** $3.\overline{3}$

Compare and Order Real Numbers

Compare using < or >.

13. $\frac{5}{6} \square \frac{2}{3}$ **14.** $3\frac{7}{9} \square 3\frac{10}{12}$ **15.** $-0.38 \square -0.3$ **16.** $-\frac{15}{8} \square -2$

Order of Operations

Simplify each expression.

17. $14 \div 2(-3) + 1$

18. $8^2 - (-12) + 15 \div 3$

19. $-2(25 - 21)^2 + 11$

20. $3\left(\frac{21 - 9}{6} - 1\right) \div 2$

Ordered Pairs

Graph each point on the same coordinate plane.

21. $(0, 2)$ **22.** $(-3, 1)$ **23.** $(2, -1)$ **24.** $(-3, -2)$

1-1 Sets of Numbers

Objective
Classify and order real numbers.

Vocabulary
set
element
subset
empty set
roster notation
finite set
infinite set
interval notation
set-builder notation

California Standards

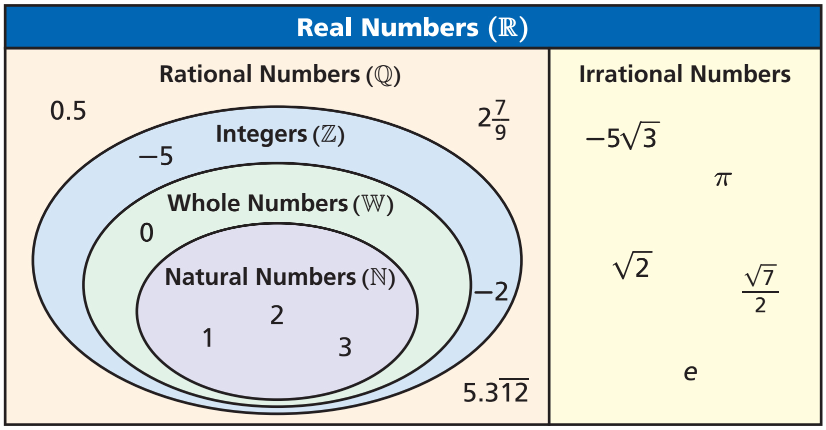
Preparation for 2.0 **Students solve systems of linear equations and inequalities** (in two or three variables) by substitution, with graphs, or with matrices.
Also covered:
Preparation for 1.0

Reading Math

Note the symbols for the sets of numbers.
$\mathbb{R}$: real numbers
$\mathbb{Q}$: rational numbers
$\mathbb{Z}$: integers
$\mathbb{W}$: whole numbers
$\mathbb{N}$: natural numbers

Why learn this?
Sets can be used to organize the balls used in the billiard game 8-ball.

A **set** is a collection of items called **elements**. The rules of 8-ball divide the set of billiard balls into three *subsets:* solids (1 through 7), stripes (9 through 15), and the 8 ball. A **subset** is a set whose elements all belong to another set. The **empty set**, denoted ∅, is a set containing no elements. The diagram shows some important subsets of the real numbers.



Rational numbers can be expressed as a quotient (or *ratio*) of two integers, where the denominator is not zero. The decimal form of a rational number either terminates, such as $\frac{1}{2} = 0.5$, or repeats, such as $-\frac{4}{3} = -1.\overline{3} = -1.333\ldots$.

Irrational numbers, such as $\sqrt{2}$ and π, *cannot* be expressed as a quotient of two integers, and their decimal forms do not terminate or repeat. However, you can approximate these numbers using terminating decimals.

EXAMPLE 1 Ordering and Classifying Real Numbers

Consider the numbers $0.\overline{6}$, $\sqrt{2}$, 0, $-\frac{5}{2}$, and 0.5129.

A **Order the numbers from least to greatest.**

Write each number as a decimal to make it easier to compare them.

$\sqrt{2} \approx 1.414$ *Use a decimal approximation for $\sqrt{2}$.*

$-\frac{5}{2} = -2.5$ *Rewrite $-\frac{5}{2}$ in decimal form.*

$-2.5 < 0 < 0.5129 < 0.666\ldots < 1.414$ *Use < to compare the numbers.*

The numbers in order from least to greatest are $-\frac{5}{2}$, 0, 0.5129, $0.\overline{6}$, and $\sqrt{2}$.

Consider the numbers $0.\overline{6}$, $\sqrt{2}$, 0, $-\frac{5}{2}$, and 0.5129.

B **Classify each number by the subsets of the real numbers to which it belongs. Use a table to classify the numbers.**

Number	Real ($\mathbb{R}$)	Rational ($\mathbb{Q}$)	Integer ($\mathbb{Z}$)	Whole ($\mathbb{W}$)	Natural ($\mathbb{N}$)	Irrational
$-\frac{5}{2}$	✓	✓				
0	✓	✓	✓	✓		
0.5129	✓	✓				
$0.\overline{6}$	✓	✓				
$\sqrt{2}$	✓					✓

CHECK IT OUT! **Consider the numbers -2, π, -0.321, $\frac{3}{2}$, and $-\sqrt{3}$.**

1a. Order the numbers from least to greatest.

1b. Classify each number by the subsets of the real numbers to which it belongs.

There are many ways to represent sets. For instance, you can use words to describe a set. You can also use **roster notation**, in which the elements of a set are listed between braces, $\{\ \ \}$.

Words	Roster Notation
The set of billiard balls is numbered 1 through 15.	$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

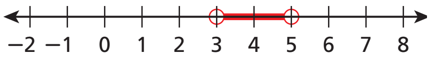
Helpful Hint

The Density Property states that between any two real numbers there is another real number. So any interval that includes more than one point contains infinitely many points.

A set can be *finite* like the set of billiard ball numbers or *infinite* like the natural numbers $\{1, 2, 3, 4\ldots\}$. A **finite set** has a definite, or finite, number of elements. An **infinite set** has an unlimited, or infinite, number of elements.

Many infinite sets, such as the real numbers, cannot be represented in roster notation. There are other methods of representing these sets. For example, the number line represents the set of all real numbers.

The set of real numbers between 3 and 5, which is also an infinite set, can be represented on a number line or by an inequality.

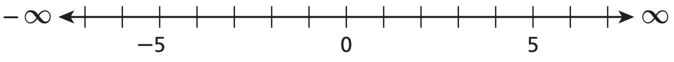


$3 < x < 5$

An interval is the set of all numbers between two endpoints, such as 3 and 5. In **interval notation** the symbols [and] are used to include an endpoint in an interval, and the symbols (and) are used to exclude an endpoint from an interval.

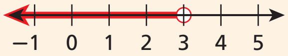
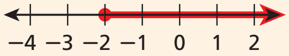
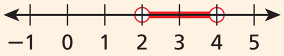
$(3, 5)$ *The set of real numbers between but not including 3 and 5*

An interval that extends forever in the positive direction goes to infinity (∞), and an interval that extends forever in the negative direction goes to negative infinity ($-\infty$).



Because ∞ and $-\infty$ are not numbers, they cannot be included in a set of numbers, so parentheses are used to enclose them in an interval. The table shows the relationship among some methods of representing intervals.

Methods of Representing Intervals

Words	Number Line	Inequality	Interval Notation
Numbers less than 3		$x < 3$	$(-\infty, 3)$
Numbers greater than or equal to −2		$x \geq -2$	$[-2, \infty)$
Numbers between 2 and 4		$2 < x < 4$	$(2, 4)$
Numbers 1 through 3		$1 \leq x \leq 3$	$[1, 3]$

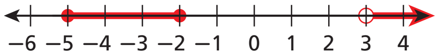
EXAMPLE 2 Interval Notation

Use interval notation to represent each set of numbers.

A $4 \leq x < 6$

$[4, 6)$ *4 is included, but 6 is not.*

B



There are two intervals graphed on the number line.

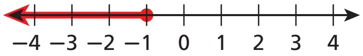
$[-5, -2]$ *−5 and −2 are included.*

$(3, \infty)$ *3 is not included, and the interval continues forever in the positive direction.*

$[-5, -2]$ or $(3, \infty)$ *The word "or" is used to indicate that a set includes more than one interval.*

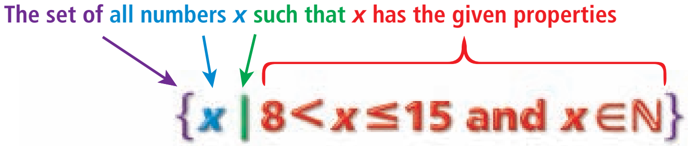
CHECK IT OUT! **Use interval notation to represent each set of numbers.**

2a.



2b. $x \leq 2$ or $3 < x \leq 11$

Another way to represent sets is *set-builder notation.* **Set-builder notation** uses the properties of the elements in the set to define the set. Inequalities and the element symbol ($\in$) are often used in set-builder notation. The set of striped-billiard-ball numbers, or $\{9, 10, 11, 12, 13, 14, 15\}$, is represented below in set-builder notation.



Read the above as "the set of all numbers x such that x is greater than 8 and less than or equal to 15 and x is a natural number."

Reading Math

The symbol $\in$ means "is an element of." So $x \in \mathbb{N}$ is read "x is an element of the set of natural numbers," or "x is a natural number."

Some representations of the same sets of real numbers are shown.

Methods of Set Notation

Words	Roster Notation	Interval Notation	Set-Builder Notation
All real numbers except 1	Cannot be written in roster notation	$(-\infty, 1)$ or $(1, \infty)$	$\{x \mid x \neq 1\}$
Positive odd numbers	$\{1, 3, 5, 7, \dots\}$	Cannot be notated using interval notation	$\{x \mid x = 2n - 1$ and $n \in \mathbb{N}\}$
Numbers within 3 units of 2	Cannot be written in roster notation	$[-1, 5]$	$\{x \mid -1 \leq x \leq 5\}$

EXAMPLE 3 Translating Between Methods of Set Notation

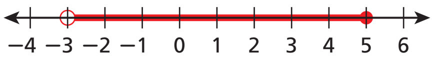
Rewrite each set in the indicated notation.

A $\{x \mid x = 2n$ **and** $n \in \mathbb{N}\}$**; words**

positive even numbers

B **numbers and symbols on a telephone keypad; roster notation**

$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, *, \#\}$ *The order of elements is not important.*

C 

set-builder notation

$\{x \mid -3 < x \leq 5\}$

CHECK IT OUT!

Rewrite each set in the indicated notation.

3a. $\{2, 4, 6, 8\}$; words

3b. $\{x \mid 2 < x < 8$ and $x \in \mathbb{N}\}$; roster notation

3c. $[99, \infty)$; set-builder notation

THINK AND DISCUSS

1. Compare interval notation with roster notation. Is it possible to have a set that can be represented by both methods?

2. Explain whether it is possible to name a number that belongs to both the set of integers and the set of irrational numbers.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, show the correct notation for each set.

Set	Roster Notation	Interval Notation	Set-Builder Notation
1, 2, 3, 4, and 5			
$-2 \leq n \leq 2$			
Whole numbers less than 3			

1-1 Exercises

California Standards

Preparation for 🔑 **1.0**
and 🔑 **2.0**

go.hrw.com
Homework Help Online
KEYWORD: MB7 1-1
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Braces, { }, are used in __?__ . (*interval notation* or *roster notation*)

SEE EXAMPLE 1
p. 6

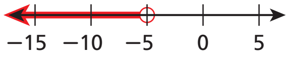
Order the given numbers from least to greatest. Then classify each number by the subsets of the real numbers to which it belongs.

2. $3\sqrt{2}, \sqrt{7}, 5.125, 4\frac{3}{5}, 4.\overline{6}$
3. $-\frac{100}{4}, -6.897, \sqrt{4}, \frac{1}{8}, \sqrt{6}$
4. $\sqrt{5}, \frac{\pi}{2}, -\sqrt{3}, 1.\overline{3}, -1\frac{1}{3}$

SEE EXAMPLE 2
p. 8

Use interval notation to represent each set of numbers.

5. $-10 < x \leq 10$

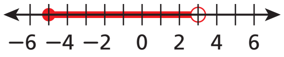
6. 

7. $1 \leq x < 20$ or $x > 30$

SEE EXAMPLE 3
p. 9

Rewrite each set in the indicated notation.

8. $\left\{x \mid x = 1 + \frac{1}{2}(n - n) \text{ and } n \in \mathbb{N}\right\}$; words

9.  set-builder notation

10. $\{0, 5, 10, 15, 20, \ldots\}$; words

11. integers from −5 to 5; roster notation

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–17	2
18–21	3

Extra Practice
Skills Practice p. S4
Application Practice p. S32

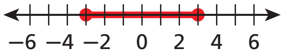
Order the given numbers from least to greatest. Then classify each number by the subsets of the real numbers to which it belongs.

12. $2.33, 5.\overline{5}, 2\sqrt{5}, -\frac{4}{5}, -0.75$
13. $\frac{1}{2}, -2, -\sqrt{2}, \frac{\sqrt{2}}{3}, -1.\overline{25}$
14. $-\sqrt{9}, 2\pi, -1, 5.\overline{12}, -\frac{7}{2}$

Use interval notation to represent each set of numbers.

15. $x \neq 5$

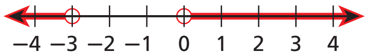
16. $-15 < x < 0$

17. 

Rewrite each set in the indicated notation.

18. $(-\infty, 3]$ or $(5, 11]$; words

19. positive multiples of 11; roster notation

20.  words

21. $\{-9, -7, -5, -3, -1\}$; set-builder notation

Chemistry **Use the table for Exercises 22–25.**

Elements from the Periodic Table

Element	Atomic Mass (amu)	Ionic Charge
Aluminum	26.982	+3
Calcium	40.078	+2
Chlorine	35.4527	−1
Lithium	6.941	+1
Sulfur	32.066	−2

22. Order the given elements from least to greatest atomic mass.

23. Which subset of the real numbers best describes the atomic masses of these elements? Choose from $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{W}$, and $\mathbb{N}$.

24. Which subset of the real numbers best describes the ionic charges of these elements? Choose from $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{W}$, and $\mathbb{N}$.

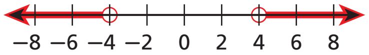
25. Explain why interval notation cannot be used to represent the set of atomic masses given.

Complete the table by writing each set in the indicated notations. If a set cannot be written in a given notation, state this in your answer.

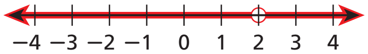
	Words	Roster Notation	Interval Notation	Set-Builder Notation
26.	?	$\{-2, -4, -6, -8, \ldots\}$	?	?
27.	?	?	$[-4, 8)$	?
28.	Even numbers between 27 and 39	?	?	?
29.	?	?	?	$\{x \mid 0 < x < 1\}$

Express each set of numbers using interval notation and set-builder notation.

30.



31.



32. $x \leq 2$ or $3 < x < 5$

33. numbers between 1 and 10

34. numbers more than 2 units from 8

35. $x \neq 5$ and $x \leq 10$

Tell whether each statement is true or false. If false, give a counterexample.

36. Every natural number is an integer.

37. Every real number is irrational.

38. Every integer is a whole number.

39. Every integer is NOT irrational.

Sports **Use the table of soccer ball sizes for Exercises 40–42.**

Soccer Ball Sizes			
Size	3	4	5
Weight (oz)	11–12	12–13	14–16
Circumference (in.)	23–24	25–26	27–28
Age of Player	Under 8	8–12	Over 12

40. Identify the size of each ball:

soccer ball A: 4.36 in. radius

soccer ball B: 7.54 in. diameter

soccer ball C: 276.2 in^3 volume

41. Use set-builder notation to represent the weight range for each soccer ball size.

42. Use interval notation to represent the age range for each soccer ball size.

43. **Critical Thinking** The product of an irrational number and a rational number is an irrational number. Explain why this means that no matter how precisely you measure the diameter of a soccer ball, your calculation for its circumference will NEVER be a rational number.

CONCEPT CONNECTION

44. This problem will prepare you for the Concept Connection on page 42.

Distances in space are often measured in astronomical units (AU). One AU is defined as the average distance between Earth and the Sun.

a. To which subsets of the real numbers do the numbers in the table belong?

b. Order the bodies from least to greatest average distance from Earth.

c. For a given speed, would it take longer to make a round-trip to Venus or a one-way trip to Mars? Explain.

Average Distances from Earth	
Body	**Distance (AU)**
Mars	$\frac{97}{186}$
Mercury	$\frac{117}{310}$
Moon	0.0026
Venus	0.2774

45. Use interval notation to express the set of numbers NOT represented on the number line.

Use a number line to represent each set.

46. $-4 < x \leq 4$ or $x > 5$

47. numbers within 6 units of 5

48. $\{-10, -5, 0, 5, 10\}$

49. $\left\{x \mid x = \frac{1}{2}n \text{ and } n \in \mathbb{N}\right\}$

50. numbers more than 5 units from -3

51. $(-\infty, 2)$ or $[-1.75, 1.75]$ or $(2, \infty)$

52. **Geology** The Mohs scale of hardness gives the increasing order of hardness for minerals. The greater the hardness number, the harder the mineral is. Window glass has a hardness of about 5.5 on the Mohs scale.

a. Use roster notation to represent the set of minerals that are softer than window glass.

b. How many elements does the set of minerals that are harder than window glass have?

c. Explain whether {apatite, diamond, topaz, quartz} is a subset of the set of minerals harder than window glass, the set of minerals softer than window glass, or neither.

Mohs Scale of Hardness	
Talc	1
Gypsum	2
Calcite	3
Fluorite	4
Apatite	5
Orthoclase	6
Quartz	7
Topaz	8
Corundum	9
Diamond	10

Identify which of the real numbers best describes each situation. Choose from $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{W}$, and $\mathbb{N}$.

53. the number of stops a train makes during a trip

54. the cumulative grade point average for a student

55. the squares of the set of integers

56. **Critical Thinking** Are all square roots irrational numbers? Explain.

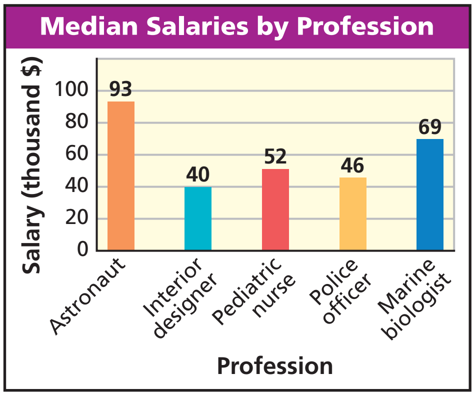
57. **Careers** The graph shows several median salaries by profession.

a. Order the professions by salary from least to greatest.

b. **What if...?** If each salary were increased by $5000, would the order from part **a** change?

c. **What if...?** If each salary were increased by 15%, would the order from part **a** change?

d. Use roster notation to represent the set of salaries from part **c.**



Identify one rational number and one irrational number that belongs to each set. Then explain whether the number 5 is an element of the set.

58. $\{x \mid x = 5c \text{ and } 0 < c \leq 1\}$

59. $-1 < x \leq 1$ or $x > 4$

60. numbers within 4 units of 9

61. $(3, 5)$

62. **Write About It** People use sets of tools and eat on sets of dishes. How are mathematical sets similar to and different from such everyday sets?

STANDARDIZED TEST PREP

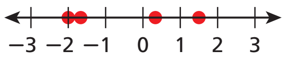
63. Which of the following is NOT equivalent to 4?

Ⓐ $\sqrt{16}$ Ⓑ $3 - (-1)$ Ⓒ $\frac{12}{3}$ Ⓓ $2(-2)$

64. Which list is in order from least to greatest?

Ⓕ $\frac{3}{7}, 0.5, \frac{\sqrt{3}}{2}$ Ⓖ $0.5, \frac{3}{7}, \frac{\sqrt{3}}{2}$ Ⓗ $\frac{3}{7}, \frac{\sqrt{3}}{2}, 0.5$ Ⓙ $\frac{\sqrt{3}}{2}, 0.5, \frac{3}{7}$

65. Which set best describes the numbers graphed on the number line?



Ⓐ $\{-2, -1.5, 0.5, 1.5\}$

Ⓑ $\{-\sqrt{4}, -\frac{5}{3}, 0.\overline{3}, 1\frac{1}{2}\}$

Ⓒ $\{-\frac{6}{3}, -1.\overline{3}, \frac{3}{4}, \sqrt{2}\}$

Ⓓ $\{-1\frac{1}{3}, 0.\overline{3}, 1.5, 2\}$

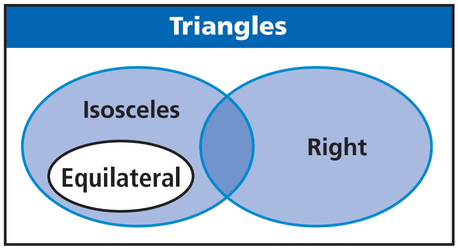
66. Which statement can be determined from the diagram?

Ⓕ Every isosceles triangle is equilateral.

Ⓖ Every triangle is either right or isosceles.

Ⓗ No right triangles are isosceles.

Ⓙ No right triangles are equilateral.



CHALLENGE AND EXTEND

Explain whether each set is finite or infinite. Then identify the subsets of the real numbers to which the set belongs.

67. values in dollars of U.S. coins

68. $\{0.\overline{3}, 0.\overline{6}, 1, 1.\overline{3}, \ldots\}$

69. U.S. Postal Service 5-digit zip codes

70. $\{x \mid x = \frac{c}{4} \text{ and } c \in \mathbb{Z}\}$

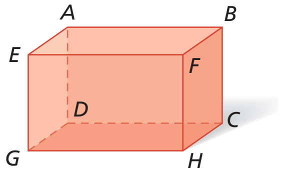
71. The symbol π is used to represent the irrational number 3.14159265358… . The fraction $\frac{22}{7}$ and the decimal 3.14 are approximations of π.

a. Find a rational number between 3.14 and π.

b. Find a rational number between $\frac{22}{7}$ and π.

SPIRAL REVIEW

Use the rectangular prism for Exercises 72–74. *(Previous course)*



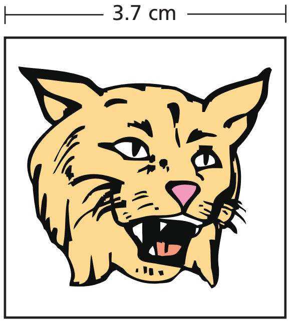
72. Name two edges that intersect to form a right angle.

73. Name two faces that model parallel planes.

74. Name two faces that model perpendicular planes.

75. Debra went shopping with three bills in her wallet. She returned home from shopping with less than \$1.00 in her wallet. She made purchases of \$21.49, \$11.59, and \$12.95, all with 6.5% sales tax. What bills did Debra have in her wallet when she went shopping? *(Previous course)*

76. The Wildcat pep squad is enlarging the Wildcats' team logo to create a square banner. The ratio of the side length of the logo shown to the side length of the banner is 1:120. What is the area of the banner in square centimeters? *(Previous course)*



clipart.com

1-2

Properties of Real Numbers

Objective

Identify and use properties of real numbers.

California Standards

Preparation for 25.0 Students use properties from number systems to justify steps in combining and simplifying functions.

Why learn this?

You can use properties of real numbers to quickly calculate tips in your head. (See Example 3.)

"The tax and tip I understand, but what's this charge for shipping and handling?"

Andrew Toos/CartoonResource.com

The four basic math operations are addition, subtraction, multiplication, and division. Because subtraction is addition of the opposite and division is multiplication by the reciprocal, the properties of real numbers focus on addition and multiplication.

Know it! Note

Properties of Real Numbers

Identities and Inverses

For all real numbers *n*,

WORDS	NUMBERS	ALGEBRA
Additive Identity Property The sum of a number and 0, the additive identity, is the original number.	$3 + 0 = 3$	$n + 0 = 0 + n = n$
Multiplicative Identity Property The product of a number and 1, the multiplicative identity, is the original number.	$\frac{2}{3} \cdot 1 = \frac{2}{3}$	$n \cdot 1 = 1 \cdot n = n$
Additive Inverse Property The sum of a number and its opposite, or additive inverse, is 0.	$5 + (-5) = 0$	$n + (-n) = 0$
Multiplicative Inverse Property The product of a nonzero number and its reciprocal, or multiplicative inverse, is 1.	$8 \cdot \frac{1}{8} = 1$	$n \cdot \frac{1}{n} = 1 \; (n \neq 0)$

Recall from previous courses that the opposite of any number a is $-a$ and the reciprocal of any nonzero number a is $\frac{1}{a}$.

EXAMPLE 1

Finding Inverses

Find the additive and multiplicative inverse of each number.

A -9

additive inverse: 9 — *The opposite of -9 is $-(-9) = 9$.*

Check $-9 + 9 = 0$ ✓ — *The Additive Inverse Property holds.*

multiplicative inverse: $\frac{1}{-9}$ — *The reciprocal of -9 is $\frac{1}{-9}$.*

Check $-9 \cdot \left(\frac{1}{-9}\right) = 1$ ✓ — *The Multiplicative Inverse Property holds.*

Find the additive and multiplicative inverse of each number.

B $\frac{4}{5}$

additive inverse: $-\frac{4}{5}$ — *The opposite of $\frac{4}{5}$ is $-\frac{4}{5}$.*

multiplicative inverse: $\frac{5}{4}$ — *The reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.*

CHECK IT OUT! **Find the additive and multiplicative inverse of each number.**

1a. 500 **1b.** -0.01

Know it! Note

Properties of Real Numbers — Addition and Multiplication

For all real numbers *a* and *b*,

WORDS	NUMBERS	ALGEBRA
Closure Property		
The sum or product of any two real numbers is a real number.	$2 + 3 = 5$ $2(3) = 6$	$a + b \in \mathbb{R}$ $ab \in \mathbb{R}$
Commutative Property		
You can add or multiply real numbers in any order without changing the result.	$7 + 11 = 11 + 7$ $7(11) = 11(7)$	$a + b = b + a$ $ab = ba$
Associative Property		
The sum or product of three or more real numbers is the same regardless of the way the numbers are grouped.	$(5 + 3) + 7 = 5 + (3 + 7)$ $(5 \cdot 3)7 = 5(3 \cdot 7)$	$(a + b) + c = a + (b + c)$ $(ab)c = a(bc)$
Distributive Property		
When you multiply a sum by a number, the result is the same whether you add and then multiply or whether you multiply each term by the number and then add the products.	$5(2 + 8) = 5(2) + 5(8)$ $(2 + 8)5 = (2)5 + (8)5$	$a(b + c) = ab + ac$ $(b + c)a = ba + ca$

Reading Math

Based on the Closure Property, the real numbers are said to be *closed* under addition and *closed* under multiplication.

EXAMPLE 2 Identifying Properties of Real Numbers

Identify the property demonstrated by each equation.

A $\left(3\sqrt{3} + 5\right)2 = \left(3\sqrt{3}\right)2 + (5)2$ — *The 2 has been distributed to each term.*

Distributive Property

B $(3 + 6) + (-6) = 3 + [6 + (-6)]$ — *The numbers have been regrouped.*

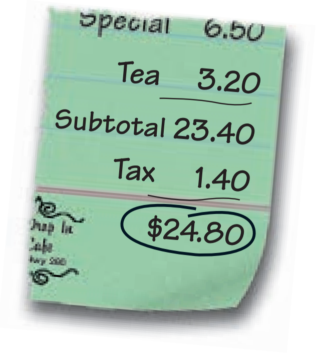
Associative Property of Addition

CHECK IT OUT! **Identify the property demonstrated by each equation.**

2a. $9\sqrt{2} = \left(\sqrt{2}\right)9$ **2b.** $9(12\pi) = (9 \cdot 12)\pi$

You can apply the properties of real numbers to simplify numeric expressions and solve problems mentally.

EXAMPLE 3 Consumer Economics Application



Use mental math to find a 15% tip for the bill shown.

Think: $15\% = 10\% + 5\%$

$(10\% + 5\%)24.80$

$10\%(24.80) + 5\%(24.80)$ *Distributive Property*

Think: Find 10% of $24.80

$10\%(24.80) = 2.480 = 2.48$ *Move the decimal point left 1 place.*

Think: $5\% = \frac{1}{2}(10\%)$

$\frac{1}{2}(2.48) = 1.24$ *5% is half of 10% so find half of 2.48.*

$2.48 + 1.24 = 3.72$ *Add 10% of 24.80 to 5% of 24.80.*

A 15% tip for a meal that totaled $24.80 is $3.72.

CHECK IT OUT! 3. Use mental math to find a 20% discount on a $15.60 shirt.

EXAMPLE 4 Classifying Statements as Sometimes, Always, or Never True

Classify each statement as sometimes, always, or never true. Give examples or properties to support your answer.

A $c + d = c$ when $d = 2$

never true

counterexample: $1 + 2 \neq 1$

By the Additive Identity Property, $c + 0 = c$, so $c + d = c$ is only true when $d = 0$, not when $d = 2$.

B $a - c = c - a$

sometimes true

true example: $5 - 5 = 5 - 5$

false example: $5 - 2 \neq 2 - 5$

True and false examples exist. The statement is true when $a = c$ and false when $a \neq c$.

CHECK IT OUT! **Classify each statement as sometimes, always, or never true. Give examples or properties to support your answer.**

4a. $a + (-a) = b + (-b)$ **4b.** $a - (b + c) = (a - b) + (a - c)$

THINK AND DISCUSS

1. Explain whether the Commutative Property applies to subtraction and division.
2. Tell why zero has no multiplicative inverse.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of the property indicated.

Property	Addition	Multiplication
Identity		
Inverse		
Associative		
Commutative		
Distributive		

1-2 Exercises

California Standards

Preparation for 25.0

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Homework Help Online

KEYWORD: MB7 1-2

Parent Resources Online

KEYWORD: MB7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1
p. 14

Find the additive and multiplicative inverse of each number.

1. -36 **2.** -0.05 **3.** $2\sqrt{2}$

4. $\frac{2}{5}$ **5.** $-\frac{1}{500}$ **6.** 0.25

SEE EXAMPLE 2
p. 15

Identify the property demonstrated by each equation.

7. $3(2\sqrt{5}) = (3 \cdot 2)\sqrt{5}$ **8.** $x + 7y = 7y + x$ **9.** $\frac{1}{3}(28)(9) = \frac{1}{3}(9)(28)$

SEE EXAMPLE 3
p. 16

Use mental math to find each value.

10. cost of 3 items at \$2.55 each **11.** a $33\frac{1}{3}\%$ discount on a \$21.99 item

SEE EXAMPLE 4
p. 16

Classify each statement as sometimes, always, or never true. Give examples or properties to support your answer.

12. $20a + 20b = 5(4a + 4b)$ **13.** $a \div b = b \div a$ **14.** $a + (bc) = (a + b)(a + c)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
15–20	1
21–23	2
24–25	3
26–27	4

Extra Practice

Skills Practice p. S4

Application Practice p. S32

Find the additive and multiplicative inverse of each number.

15. -2.5 **16.** 0.75 **17.** 2π

18. $-\frac{2}{3}$ **19.** $\frac{1}{20}$ **20.** 6231

Identify the property demonstrated by each equation.

21. $z(x - y) = zx - zy$ **22.** $4abc = 4acb$ **23.** $(a + 0) + b = a + b$

Use mental math to find each value.

24. 9% sales tax on a \$150 purchase **25.** cost of 5 items at \$1.96 each

Classify each statement as sometimes, always, or never true. Give examples or properties to support your answer.

26. $a - (b - c) = a - b + c$ **27.** $ab\left(\frac{1}{ab}\right) = 0$ for $a \neq 0$ and $b \neq 0$

Shopping **Use the advertisement for Exercises 28–31. Write an expression to represent each total cost and then simplify it.**

28. cost of 2 pencil sets and 3 paintbrush sets

29. cost of 4 acrylic paints minus a refund for 2 pencil sets

30. cost of 4 paintbrush sets at a 15% discount

31. cost of 3 sketch books at a 10% discount and 5 acrylic paints at a 25% discount

ART SUPPLY SALE

Colored Pencils	Sketch Books
\$8.88	\$9.96
Acrylic Paints	Paint Brushes
\$11.99	\$14.99

Estimation **Use the map for Exercises 32–34.**

A San Diego tour van starts at Coronado Island, stops at SeaWorld, then at the Wild Animal Park, and then returns to Coronado Island.

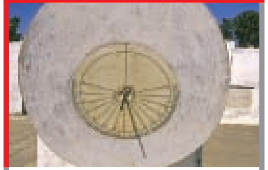
32. Estimate how long it would take the van to make one loop at an average speed of 40 mi/h.

33. **Multi-Step** The tour van gets 8 mi/gal, and the gas tank holds 24 gal. Estimate the number of loops the tour van can make on one tank.

34. **What if...?** The van adds another stop that increases the length of its loop by 20%. Estimate the number of loops the van could make in one 10 h day if it averaged 40 mi/h.

LINK

Math History



Brahmagupta, an Indian mathematician (598–668), was one of the first to use zero as a number. He was also head of the ancient astronomical observatory at Ujjain, India. The photo shows a sundial from the observatory at Ujjain.

Complete each statement, and state the property illustrated.

35. $(10 + \square) + 23 = 10 + (5 + 23)$

36. $12 + \frac{11}{15}x = \square + 12$

37. $j + \square = j$

38. $5 \cdot 4 + 5 \cdot 3 = \square \cdot (4 + 3)$

39. $\frac{4}{5} \cdot \square = 1$

40. $ab = b\square$

41. **Consumer Economics** A store is offering a 25% discount on every item purchased. To find the total discount on a purchase, Gary found the sum of the prices and then multiplied the sum by $\frac{1}{4}$. Maria found the total discount by multiplying each price by $\frac{1}{4}$ and then adding the discounts. Do both methods give the same result? Use properties of real numbers to explain why or why not.

42. **Travel** The base price for an airplane ticket from Austin to Houston is \$185. The final price includes an additional \$16 for airport fees and a \$12 fuel surcharge. José purchased a ticket online for 40% off the base price. Explain how to use mental math to find José's final price to the nearest dollar.

43. **Critical Thinking** Use the terms *additive inverse* and *additive identity* to define the set of integers and the set of rational numbers in terms of the set of natural numbers. For example, the set of whole numbers is made up of the set of natural numbers and the additive identity.

Identify which properties make each statement true for all real values of *c*.

44. $-c + (c + 4) = 4$

45. $(10c) \cdot 1 = 10c$

46. $3(2 + c) = (c + 2)3$

47. $4c + 5 = 4(c + 2) - 3$

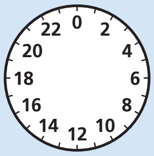
48. $\frac{1}{2}(1 - 5c) = \frac{-5c + 1}{2}$

49. $8 - 16c = 8(1 - 2c)$

CONCEPT CONNECTION

50. This problem will prepare you for the Concept Connection on page 42.

Astronauts use a 24-hour clock to tell time. The 24-hour cycle is an example of *modular arithmetic,* arithmetic performed on a circle. The circle shows mod 24. You can perform arithmetic by moving around the circle. For example, 22 + 8 = 6 in mod 24, because if you move 8 units clockwise from 22, you end up at 6.



a. What is 18 + 13 in mod 24?

b. Is addition commutative in mod 24? Give an example to support your answer.

c. Is addition associative in mod 24? Give an example to support your answer.

51. **Business** A television repair store was offering a 5% discount on parts and a 5% discount on labor. An employee placed a sign in the store window that read "Receive 10% off of your total costs." Use properties of real numbers to explain whether the sign was correct.

52. **Write About It** Explain the difference between the reciprocal of a number and the opposite of a number. Be sure to discuss the relationship between the signs of the numbers.

STANDARDIZED TEST PREP

53. Which equation illustrates the Associative Property of Multiplication?

Ⓐ $12(8 \cdot 9) = 12(9 \cdot 8)$

Ⓑ $12 + (8 + 9) = 12 + (9 + 8)$

Ⓒ $12 + (9 + 8) = (12 + 9) + 8$

Ⓓ $12(9 \cdot 8) = (12 \cdot 9)8$

54. Let a and b be real numbers such that $a \neq b$. Which statement is sometimes true?

Ⓕ $a\left(\frac{1}{b}\right) = 1$

Ⓖ $a - b = 0$

Ⓗ $a(1) = b$

Ⓙ $a = 4 + b$

55. Let c and d represent real numbers such that $c \neq 0$ and $d \neq 0$. Which expression represents the multiplicative inverse of $\frac{2c}{d}$?

Ⓐ $-\frac{2c}{d}$

Ⓑ $-\frac{2d}{c}$

Ⓒ $\frac{d}{2c}$

Ⓓ $\frac{c}{2d}$

56. **Short Response** Show two different methods of simplifying $4(1 + 3)$. Justify each step by using the order of operations or properties of real numbers.

CHALLENGE AND EXTEND

57. A positive real number n is 4 times its multiplicative inverse. What is the value of n?

58. Consider the four pairs of algebraic expressions below.

$a + b$ and $b + a$ $\quad$ $a - b$ and $b - a$ $\quad$ $a \cdot b$ and $b \cdot a$ $\quad$ $a \div b$ and $b \div a$

a. For $a = 3$ and $b = 5$, perform each pair of calculations. Identify which pairs result in a natural number for both calculations.

b. If a and b are natural numbers, which pairs of algebraic expressions always represent natural numbers?

c. Under which operations is the set of natural numbers closed?

d. Under which operations is the set of integers closed?

SPIRAL REVIEW

59. Mr. Connelly planted a 12 ft × 8 ft garden last summer. He wants to increase the size to 16 ft × 10 ft this summer. Find the percent increase in the area of his garden to the nearest tenth. *(Previous course)*

Use the following numbers for Exercises 60–62: 0.89, $\sqrt{9}$, −2, $-\frac{1}{3}$, π, −0.125, $3.\overline{09}$, 0, and $-4\sqrt{2}$. Identify each of the following. *(Lesson 1-1)*

60. greatest value

61. least value

62. irrational numbers

Write the inequality $-10 < x \leq 0$ using the indicated method. If the method is not possible, write "cannot be notated." *(Lesson 1-1)*

63. interval notation

64. set-builder notation

65. roster notation

1-3

Square Roots

Objectives

Estimate square roots.

Simplify, add, subtract, multiply, and divide square roots.

Vocabulary

radical symbol
radicand
principal root
rationalize the denominator
like radical terms

California Standards

Review of 1A2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, **taking a root,** and raising to a fractional power. They understand and use the rules of exponents.

Who uses this?

Mosaic artists can use square roots to calculate dimensions based on certain areas.

The largest mosaic in the world is located on the exterior walls of the central library of the Universidad Nacional Autónoma de México in Mexico City. It covers an area of 4000 square meters. If it were laid out as a square, you could use square roots to find its dimensions. (See Exercise 42.)

The side length of a square is the square root of its area. This relationship is shown by a **radical symbol** $(\sqrt{\ })$. The number or expression under the radical symbol is called the **radicand**. The radical symbol indicates only the positive square root of a number, called the **principal root**. To indicate both the positive and negative square roots of a number, use the plus or minus sign (±).

$$\sqrt{25} = 5 \qquad -\sqrt{25} = -5 \qquad \pm\sqrt{25} = \pm 5 = 5 \text{ or } -5$$

Numbers such as 25 that have integer square roots are called *perfect squares.* Square roots of integers that are not perfect squares are irrational numbers. You can estimate the value of these square roots by comparing them with perfect squares. For example, $\sqrt{5}$ lies between $\sqrt{4}$ and $\sqrt{9}$, so it lies between 2 and 3.

EXAMPLE 1

Estimating Square Roots

Estimate $\sqrt{34}$ to the nearest tenth.

$\sqrt{25} < \sqrt{34} < \sqrt{36}$ *Find the two perfect squares that 34 lies between.*

$5 < \sqrt{34} < 6$ *Find the two integers that $\sqrt{34}$ lies between.*

Because 34 is closer to 36 than to 25, $\sqrt{34}$ is closer to 6 than to 5.

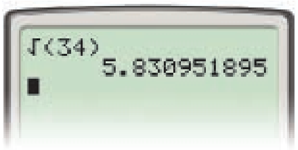
Try 5.8: $5.8^2 = 33.64$ *Too low, try 5.9.*

$5.9^2 = 34.81$ *Too high*

Because 34 is closer to 33.64 than to 34.81, $\sqrt{34}$ is closer to 5.8 than to 5.9.

$\sqrt{34} \approx 5.8$

Check On a calculator $\sqrt{34} \approx 5.830951895 \approx 5.8$ rounded to the nearest tenth. ✓



```
√(34)
         5.830951895
█
```

CHECK IT OUT!

1. Estimate $-\sqrt{55}$ to the nearest tenth.

Square roots have special properties that help you simplify, multiply, and divide them.

Know it! Note

Properties of Square Roots

For $a \geq 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Square Roots The square root of a product is equal to the product of the square roots of the factors.	$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$ $\sqrt{8} \cdot \sqrt{2} = \sqrt{8 \cdot 2} = \sqrt{16} = 4$	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$
Quotient Property of Square Roots The square root of a quotient is equal to the quotient of the square roots of the dividend and the divisor.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$ $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

Notice that these properties can be used to combine quantities under the radical symbol or separate them for the purpose of simplifying square-root expressions. A square-root expression is in simplest form when the radicand has no perfect-square factors (except 1) and there are no radicals in the denominator.

EXAMPLE 2 Simplifying Square-Root Expressions

Simplify each expression.

A $-\sqrt{50}$

$-\sqrt{25 \cdot 2}$ *Find a perfect square factor of 50.*

$-\sqrt{25} \cdot \sqrt{2}$ *Product Property of Square Roots*

$-5\sqrt{2}$

B $\sqrt{\frac{49}{81}}$

$\frac{\sqrt{49}}{\sqrt{81}}$ *Quotient Property of Square Roots*

$\frac{7}{9}$

C $\sqrt{2} \cdot \sqrt{18}$

$\sqrt{2 \cdot 18}$ *Product Property of Square Roots*

$\sqrt{36} = 6$

D $\frac{\sqrt{96}}{\sqrt{6}}$

$\sqrt{\frac{96}{6}}$ *Quotient Property of Square Roots*

$\sqrt{16} = 4$

CHECK IT OUT!

Simplify each expression.

2a. $\sqrt{48}$ **2b.** $\sqrt{\frac{36}{16}}$ **2c.** $\sqrt{5} \cdot \sqrt{20}$ **2d.** $\frac{\sqrt{147}}{\sqrt{3}}$

If a fraction has a denominator that is a square root, you can simplify it by **rationalizing the denominator**. To do this, multiply both the numerator and denominator by a number that produces a perfect square under the radical sign in the denominator.

EXAMPLE 3 Rationalizing the Denominator

Simplify by rationalizing each denominator.

A $\frac{2\sqrt{2}}{\sqrt{3}}$

$\frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ *Multiply by a form of 1.*

$\frac{2\sqrt{2 \cdot 3}}{3}$ *$\sqrt{3} \cdot \sqrt{3} = 3$*

$\frac{2\sqrt{6}}{3}$

B $\frac{\sqrt{8}}{\sqrt{18}}$

$\frac{\sqrt{8}}{\sqrt{18}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ *Multiply by a form of 1.*

$\frac{\sqrt{8 \cdot 2}}{6}$ *$\sqrt{18} \cdot \sqrt{2} = 6$*

$\frac{\sqrt{16}}{6} = \frac{4}{6} = \frac{2}{3}$ *$\sqrt{16} = 4$*

CHECK IT OUT! **Simplify by rationalizing each denominator.**

3a. $\frac{3\sqrt{5}}{\sqrt{7}}$ **3b.** $\frac{5}{\sqrt{10}}$

Square roots that have the same radicand are called **like radical terms**.

Like Radicals	$\sqrt{2}$ and $3\sqrt{2}$	$-6\sqrt{15}$ and $7\sqrt{15}$	$\sqrt{ab^2}$ and $4\sqrt{ab^2}$
Unlike Radicals	$2\sqrt{5}$ and $\sqrt{2}$	$\sqrt{x}$ and $\sqrt{3x}$	$\sqrt{xy^2}$ and $\sqrt{x^2y}$

To add or subtract square roots, first simplify each radical term and then combine like radical terms by adding or subtracting their coefficients.

EXAMPLE 4 Adding and Subtracting Square Roots

Add or subtract.

A $5\sqrt{2} + 3\sqrt{2}$

$(5 + 3)\sqrt{2}$

$8\sqrt{2}$

B $5\sqrt{3} - \sqrt{12}$

$5\sqrt{3} - \sqrt{4 \cdot 3}$ *Simplify radical terms.*

$5\sqrt{3} - 2\sqrt{3}$

$(5 - 2)\sqrt{3}$ *Combine like radical terms.*

$3\sqrt{3}$

CHECK IT OUT! **Add or subtract.**

4a. $3\sqrt{5} + 10\sqrt{5}$ **4b.** $\sqrt{80} - 5\sqrt{5}$

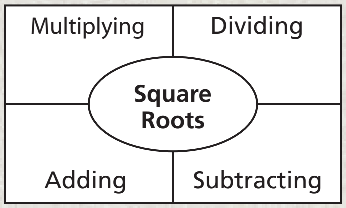
THINK AND DISCUSS

1. Compare $3\sqrt{50}$ with $5\sqrt{18}$.

2. Give two different ways to simplify $\sqrt{16} \cdot \sqrt{4}$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Write examples of each operation with square roots.



1-3 Exercises

California Standards
Review of 🔑 1A2.0;
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GUIDED PRACTICE

1. **Vocabulary** The number under the square root symbol is the $\underline{\ ?\ }$. (*radicand* or *radical*)

SEE EXAMPLE 1 p. 21

Estimate to the nearest tenth.

2. $\sqrt{75}$
3. $\sqrt{20}$
4. $-\sqrt{93}$
5. $\sqrt{13}$

SEE EXAMPLE 2 p. 22

Simplify each expression.

6. $-\sqrt{300}$
7. $\sqrt{24} \cdot \sqrt{6}$
8. $\dfrac{\sqrt{72}}{\sqrt{2}}$
9. $\sqrt{80}$

SEE EXAMPLE 3 p. 23

Simplify by rationalizing each denominator.

10. $\dfrac{1}{\sqrt{2}}$
11. $\dfrac{5\sqrt{6}}{-\sqrt{3}}$
12. $\dfrac{\sqrt{50}}{\sqrt{12}}$
13. $\dfrac{\sqrt{3}}{-\sqrt{21}}$

SEE EXAMPLE 4 p. 23

Add or subtract.

14. $6\sqrt{7} + 7\sqrt{7}$
15. $5\sqrt{32} - 15\sqrt{2}$
16. $4\sqrt{5} + \sqrt{245}$
17. $-\sqrt{50} + 6\sqrt{2}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22–29	2
30–33	3
34–37	4

Extra Practice
Skills Practice p. S4
Application Practice p. S32

Estimate to the nearest tenth.

18. $\sqrt{60}$
19. $-\sqrt{15}$
20. $\sqrt{47}$
21. $\sqrt{99}$

Simplify each expression.

22. $\sqrt{162}$
23. $-\sqrt{\dfrac{1}{121}}$
24. $\sqrt{\dfrac{50}{9}}$
25. $-2\sqrt{10} \cdot \sqrt{8}$
26. $\dfrac{\sqrt{288}}{\sqrt{8}}$
27. $\sqrt{85} \cdot \sqrt{5}$
28. $\dfrac{2\sqrt{126}}{\sqrt{14}}$
29. $-\sqrt{189}$

Simplify by rationalizing each denominator.

30. $\dfrac{2}{\sqrt{3}}$
31. $\dfrac{3\sqrt{27}}{2\sqrt{6}}$
32. $-\dfrac{18}{\sqrt{6}}$
33. $\dfrac{\sqrt{11}}{5\sqrt{132}}$

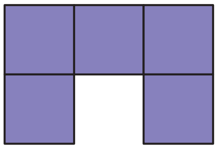
Add or subtract.

34. $4\sqrt{3} - 9\sqrt{3}$
35. $\sqrt{112} + \sqrt{63}$
36. $\sqrt{8} - 15\sqrt{2}$
37. $\sqrt{12} + 7\sqrt{27}$
38. $\sqrt{45} + \sqrt{20}$
39. $5\sqrt{28} - 2\sqrt{7}$
40. $2\sqrt{48} + 2\sqrt{12}$
41. $\sqrt{150} - 8\sqrt{6}$

42. **Art** The largest mosaic in the world is on the walls of the central library of the Universidad Nacional Autónoma de México in Mexico City. The mosaic depicts scenes from the nation's history and covers an area of 4000 m^2. If the entire mosaic were on one square wall, what would its dimensions be?

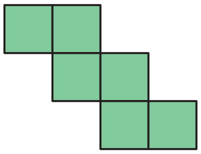
Geometry **Each figure below is made from squares. Given the area of each figure, find its perimeter to the nearest tenth.**

43.



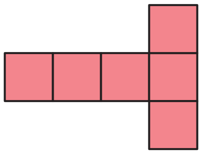
$A = 40\ cm^2$

44.



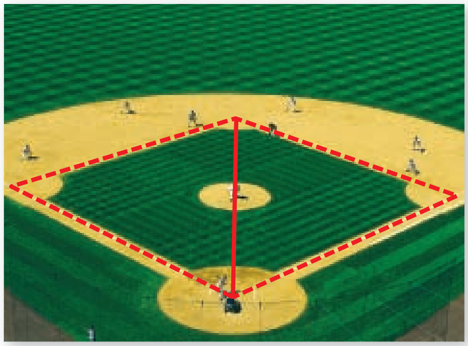
$A = 90\ ft^2$

45.



$A = 300\ in^2$

46. **Sports** A baseball diamond is a square with an area of 8100 square feet. The length of the diagonal of any square is equal to $\sqrt{2}$ times its side length. Find the distance from home plate to second base (the length of the diagonal) to the nearest hundredth of a foot.



47. **Estimation** A painter's canvas will cover 600 square inches. Estimate the dimensions of a square wall mural that is the size of 4 complete canvases. Explain your thinking.

Simplify each expression.

48. $\dfrac{\sqrt{900}}{\sqrt{20}}$

49. $3\sqrt{50} \cdot 3\sqrt{8}$

50. $-3\sqrt{2} + \sqrt{18}$

51. $2\sqrt{5} - 5\sqrt{2}$

52. $\dfrac{4\sqrt{6} + 3\sqrt{2}}{\sqrt{6}}$

53. $\dfrac{3\sqrt{7} + 1}{\sqrt{5}}$

54. $\dfrac{4\sqrt{10} - \sqrt{90}}{\sqrt{2}}$

55. $\dfrac{4\sqrt{32}}{\sqrt{5}}$

56. $\dfrac{3 + 2\sqrt{7}}{\sqrt{7}}$

57. **Geography** The original design for the city of Savannah, Georgia, was based on a gridlike system of wards. At one time the city included a total of 24 wards. Each ward was approximately square, and together the wards covered a total area of about 8,640,000 square feet. Find the approximate dimensions of a ward.

LINK

Geography

Savannah, Georgia, was the first American city to include squares and wards.

Source: www.pps.org/gps

Measurement **Use the table for Exercises 58–61. Find the side length, to the nearest tenth of a foot, of a square with the given area.**

Unit of Area	Square Feet
Acre	43,560
Hectare	107,600
Square kilometer	10,760,000
Square mile	27,880,000

58. 10 acres

59. 2 mi^2

60. 5 hectares

61. 6.2 km^2

Determine whether each statement is sometimes, always, or never true for positive integers *a* and *b*. Give examples to support your conclusion.

62. $\sqrt{a} + \sqrt{b} = \sqrt{ab}$

63. $\dfrac{\sqrt{ab}}{\sqrt{a}} = \sqrt{b}$

64. $a\sqrt{b} + a\sqrt{b} = 2ab$

65. **Critical Thinking** Given that $\sqrt{2 + 2} = 2$, does $\sqrt{a + a} = a$? Explain.

66. **Write About It** Find the value of $\sqrt{2}$ on your calculator. Square this value by entering the number and pressing x^2 and ENTER. Is the result 2? Explain why or why not.

CONCEPT CONNECTION

67. This problem will prepare you for the Concept Connection on page 42.

Gravity on the Moon is much weaker than on Earth. The expression $\sqrt{\frac{h}{0.82}}$ can be used to approximate the time in seconds it takes for an object to reach the surface of the Moon when dropped from an initial height of *h* meters.

a. How long would it take for an object dropped from a height of 50 meters to land on the Moon?

b. The expression $\sqrt{\frac{h}{4.89}}$ can be used to model the time it takes to reach Earth's surface from a height of *h* meters. How long would it take for an object dropped from a height of 50 meters to land on Earth?

STANDARDIZED TEST PREP

68. Which expression is NOT equivalent to the others?

Ⓐ $\sqrt{20}$ Ⓑ $\sqrt{8} \cdot \sqrt{5}$ Ⓒ $2\sqrt{10}$ Ⓓ $\frac{5\sqrt{8}}{\sqrt{5}}$

69. What is the approximate perimeter of a square with an area of 30 square meters?

Ⓕ 5.5 m Ⓖ 11 m Ⓗ 22 m Ⓙ 30 m

70. Which list is in order from least to greatest?

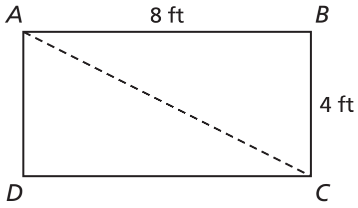
Ⓐ $\sqrt{\frac{9}{4}}, \sqrt{4}, 2\sqrt{2}, 2.5$

Ⓑ $\sqrt{25}, 5.1, 2\sqrt{5}, 6$

Ⓒ $0, \sqrt{\frac{1}{4}}, \frac{1}{4}, \sqrt{1}$

Ⓓ $\frac{1}{\sqrt{2}}, 1, \sqrt{2}, 2$

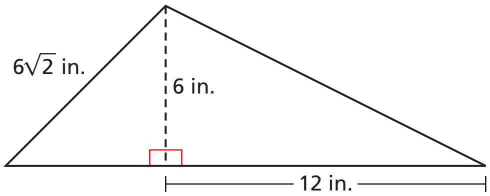
71. **Gridded Response** By the Pythagorean Theorem, the length d of a diagonal of a rectangle is given by $d = \sqrt{\ell^2 + w^2}$. Find the length in feet of diagonal $\overline{AC}$ to the nearest tenth.



CHALLENGE AND EXTEND

72. Evaluate $\frac{a\sqrt{b} - 3a\sqrt{5ab}}{3\sqrt{b}}$ for $a = 5$ and $b = 6$.

73. **Geometry** The Pythagorean Theorem relates the side lengths a and b of a right triangle to the length of its hypotenuse c, with the formula $a^2 + b^2 = c^2$.



a. Use the Pythagorean Theorem to determine the unknown dimensions of the triangle.

b. Find the area of the triangle.

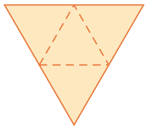
c. Find the perimeter of the triangle.

74. Simplify $\frac{\sqrt{x^3y^5}}{x^2\sqrt{48y^3}}$. Assume all variables are positive.

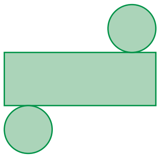
SPIRAL REVIEW

Identify the three-dimensional figure from the net shown. *(Previous course)*

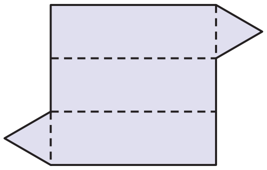
75.



76.



77.



Write an inequality for each set of numbers. *(Lesson 1-1)*

78. $(-7, 1]$ **79.** $(1.5, 8)$ **80.** $[2, 12]$ **81.** $\left(\frac{3}{4}, \frac{5}{2}\right)$

Identify the property demonstrated by each equation. *(Lesson 1-2)*

82. $(a \cdot 1)b = ab$

83. $(x + y) + z = z + (x + y)$

84. $8p(q) = 8(pq)$

85. $st + 3s = s(t + 3)$

1-4

Simplifying Algebraic Expressions

Objective

Simplify and evaluate algebraic expressions.

California Standards

Review of 7AF1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.

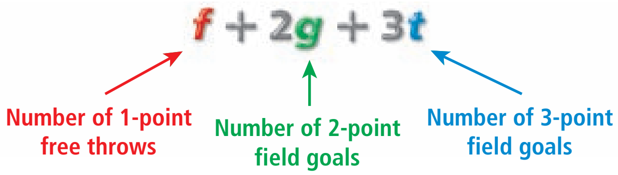
Also covered: **Review of 7AF1.1**

Why learn this?

You can model the total points scored in a basketball game by using an algebraic expression.

There are three different ways in which a basketball player can score points during a game. There are 1-point free throws, 2-point field goals, and 3-point field goals. An algebraic expression can represent the total points scored during a game.

Total Points Scored



To translate a real-world situation into an algebraic expression, you must first determine the action being described. Then choose the operation that is indicated by the type of action and the context clues.

Action	Operation	Possible Context Clues
Combine	Add	How many total?
Combine equal groups	Multiply	How many altogether?
Separate	Subtract	How many more? How many remaining?
Separate into equal groups	Divide	How many in each group?

EXAMPLE 1 Translating Words into Algebraic Expressions

Write an algebraic expression to represent each situation.

A **the distance remaining for a runner after *m* miles of a 26.2-mile marathon**

$26.2 - m$ *Subtract m from 26.2.*

B **the number of hours it takes to fly 1800 miles at an average rate of *n* miles per hour**

$\frac{1800}{n}$ *Divide 1800 by n.*

CHECK IT OUT!

Write an algebraic expression to represent each situation.

1a. Lucy's age *y* years after her 18th birthday

1b. The number of seconds in *h* hours

To evaluate an algebraic expression, substitute a number for each variable and simplify by using the order of operations. One way to remember the order of operations is by using the mnemonic PEMDAS.

Order of Operations

1. **P**arentheses and grouping symbols
2. **E**xponents
3. **M**ultiply and **D**ivide from left to right.
4. **A**dd and **S**ubtract from left to right.

EXAMPLE 2 Evaluating Algebraic Expressions

Evaluate each expression for the given values of the variables.

A $x + 3xy - 2y$ for $x = 4$ and $y = 7$

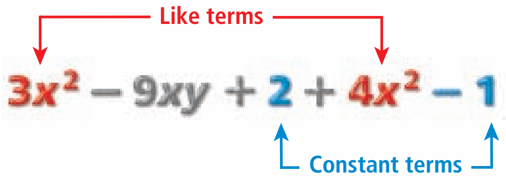
$(4) + 3(4)(7) - 2(7)$	*Substitute 4 for x and 7 for y.*
$4 + 84 - 14$	*Multiply from left to right.*
74	*Add and subtract from left to right.*

B $b^2z - 2bz + z^2$ for $b = 6$ and $z = 2$

$(6)^2(2) - 2(6)(2) + (2)^2$	*Substitute 6 for b and 2 for z.*
$36(2) - 2(6)(2) + 4$	*Evaluate exponential expressions.*
$72 - 24 + 4$	*Multiply from left to right.*
52	*Add and subtract from left to right.*

CHECK IT OUT! **2.** Evaluate $x^2y - xy^2 + 3y$ for $x = 2$ and $y = 5$.

Recall that the terms of an algebraic expression are separated by addition or subtraction symbols. *Like terms* have the same variables raised to the same exponents. Constant terms are like terms that always have the same value.



To simplify an algebraic expression, combine like terms by adding or subtracting their coefficients. Algebraic expressions are equivalent if they contain exactly the same terms when simplified.

EXAMPLE 3 Simplifying Expressions

Remember!

Terms that are written without a coefficient have an understood coefficient of 1.

$x^2 = 1x^2$

Simplify each expression.

A $x^2 + 5x + 2y + 7x^2$

$x^2 + 5x + 2y + 7x^2$	*Identify like terms.*
$8x^2 + 5x + 2y$	*Combine like terms.* $1x^2 + 7x^2 = 8x^2$

B $b(5a^2 - 2a) - 11a^2b + 2ab$

$5a^2b - 2ab - 11a^2b + 2ab$	*Distribute, and identify like terms.*
$-6a^2b$	*Combine like terms.* $-2ab + 2ab = 0$

CHECK IT OUT! **3.** Simplify the expression $-3(2x - xy + 3y) - 11xy$.

Student to Student

Checking Simplified Expressions

Nadia Torres
Madison High School

To check that I simplified an expression correctly, I substitute the same numbers into both expressions. If I get the same value for each expression, my answer is probably correct.

Original Expression		***Simplified Expression***
$3x + 5y - 2x$		$x + 5y$
$3(2) + 5(3) - 2(2)$	*Use $x = 2$ and $y = 3$.*	$2 + 5(3)$
$6 + 15 - 4$	*Multiply.*	$2 + 15$
17	*They are equal.*	17

EXAMPLE 4 *Transportation Application*

Holly's hybrid car gets 45 miles per gallon on the highway and 25 miles per gallon in the city.

A **Write and simplify an expression for the total number of miles she can drive if her fuel tank holds 15 gallons of gas.**

Let h be the number of gallons used on the highway. Then $15 - h$ is the remaining number of gallons used in the city.

$$45h + 25(15 - h) = 45h + 375 - 25h \quad \textit{Distribute 25.}$$
$$= 20h + 375 \quad \textit{Combine like terms.}$$

B **How many total miles can she drive on one tank of gas if she uses 5 gallons on the highway?**

Evaluate $20h + 375$ for $h = 5$.

$20(5) + 375 = 475$

Holly can travel 475 miles if she uses 5 gallons on the highway.

CHECK IT OUT!

4. A travel agent is selling 100 discount packages. He makes \$50 for each Hawaii package and \$80 for each Cancún package.

a. Write an expression to represent the total the agent will make selling a combination of the two packages.

b. How much will he make if he sells 28 Hawaii packages?

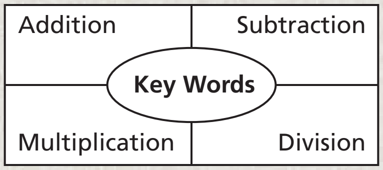
THINK AND DISCUSS

1. Tell how many addition or subtraction symbols an expression with five terms will have. Explain.

2. Explain how adding like terms involves the Distributive Property.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write key words that may indicate each operation.



1-4 Exercises

California Standards

Review of 7AF1.1 and 7AF1.2

go.hrw.com

Homework Help Online

KEYWORD: MB7 1-4

Parent Resources Online

KEYWORD: MB7 Parent

GUIDED PRACTICE

SEE EXAMPLE 1 p. 27

Write an algebraic expression to represent each situation.

1. the cost of c containers of yogurt at \$0.79 each

2. the area of a rectangle with length ℓ meters and width 8 meters

SEE EXAMPLE 2 p. 28

Evaluate each expression for the given values of the variables.

3. $a^2 + b^2 - 2ab$ for $a = 5$ and $b = 8$

4. $\dfrac{3xy}{x^2 - 9y + 2}$ for $x = 2$ and $y = 4$

SEE EXAMPLE 3 p. 28

Simplify each expression.

5. $-8a + 9 - 5a + a$

6. $-2(2x + y) - 7x + 2y$

7. $1 + (ab - 5a)5 - b^2$

SEE EXAMPLE 4 p. 29

8. Athletics Regan runs and bicycles every day for a total of 60 minutes. Her body uses 9 Calories per minute during running and 7 Calories per minute during bicycling.

a. Write and simplify an expression for the total Calories Regan uses running and bicycling each day.

b. How many Calories does she use on a day when she runs for 20 minutes?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–10	1
11–14	2
15–18	3
19	4

Extra Practice

Skills Practice p. S4

Application Practice p. S32

Write an algebraic expression to represent each situation.

9. the measure of the supplement of an angle whose measure is $x°$

10. the number of \$0.60 bagels that can be purchased with d dollars

Evaluate each expression for the given values of the variables.

11. $6c - 3c^2 + d^3$ for $c = 5$ and $d = 3$

12. $y^2 - 2xy^2 - x$ for $x = 2$ and $y = 3$

13. $3a^2b - ab^3 + 5$ for $a = 5$ and $b = 2$

14. $\dfrac{2s - t^2}{st^2}$ for $s = 5$ and $t = 3$

Simplify each expression.

15. $-x - 3y + 4x - 9y + 2$

16. $-4(-a + 3b) - 3(a - 5b)$

17. $5 - (3m + 2n)$

18. $x(4 + y) - 2x(y + 7)$

19. Home Economics Enrique is baking muffins and bread. He wants to bake a total of 10 batches. Each batch of muffins bakes for 30 minutes, and each batch of bread bakes for 50 minutes. Let m represent the number of batches of muffins.

a. Write an expression for the total time required to bake a combination of muffins and bread if each batch is baked separately.

b. If Enrique makes 2 batches of muffins, how long will it take to bake all 10 batches?

Simplify each expression. Then evaluate the expression for the given values of the variables.

20. $-a(a^2 + 2a - 1)$ for $a = 2$

21. $(2g - 1)^2 - 2g + g^2$ for $g = 3$

22. $\dfrac{u^2 - v^2}{uv}$ for $u = 4$ and $v = 2$

23. $\dfrac{a^2 - 2(b^2 - a)}{2 + a}$ for $a = 3$ and $b = 5$

Copy and complete each table. Identify which expressions are equivalent for the given values of *x*.

24.

x	$(x+3)^2$	x^2+9	x^2+6x+9
1	▨	▨	▨
2	▨	▨	▨
3	▨	▨	▨
4	▨	▨	▨

25.

x	$(x-4)^2$	x^2+16	$x^2-8x+16$
1	▨	▨	▨
2	▨	▨	▨
3	▨	▨	▨
4	▨	▨	▨

Super Bowl The cost for a 1-minute commercial during the first Super Bowl was $85,000. The cost per 30-second commercial during Super Bowl XXXVIII was $2,300,000.

a. Write expressions to represent the cost of an m-minute commercial during the first Super Bowl and during Super Bowl XXXVIII.

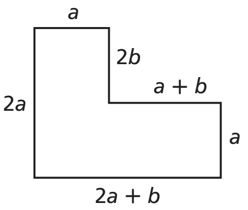
b. If a commercial cost $170,000 during the first Super Bowl, how much would it have cost during Super Bowl XXXVIII? How do the costs compare?

c. About 60 million viewers watched the first Super Bowl, and about 800 million watched Super Bowl XXXVIII. Write expressions to represent how much an m-minute commercial cost per 1000 viewers during each Super Bowl.

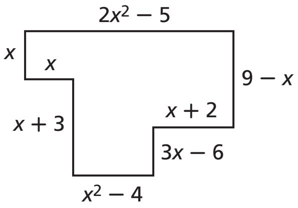
d. What was the cost per 1000 viewers of a 2-minute commercial during each Super Bowl? How do the costs compare?

Geometry **Write and simplify an expression for the perimeter of each figure.**

27.



28.



29. **Travel** The Dane family is going on a 15-day vacation to travel and visit relatives. They budget $100 per day when visiting relatives and $275 per day when traveling.

a. Write an expression for the total budgeted cost of the vacation if they visit relatives for d days.

b. What is the budgeted cost if they stay with relatives for 5 days?

c. How does this cost change for each additional day they stay with relatives?

CONCEPT CONNECTION

30. This problem will prepare you for the Concept Connection on page 42.

While Neil Armstrong and Buzz Aldrin walked on the Moon, the *Apollo 11* command module completed 1 orbit every 119 minutes.

a. Write an expression for the time in minutes needed to complete n orbits.

b. Modify your expression from part **a** so that it represents the time in hours needed to complete n orbits.

c. The *Apollo 11* module made 30 orbits. For how many hours did it orbit the Moon?

d. Estimate the number of orbits the *Apollo 11* module would make in 1 week if it continued at the same rate.

For each equation find the value of y when $x = -3, -2, 0, 2$, and 3.

31. $y = -2x^2 + 5x - 7$ **32.** $y = -\frac{3x + 9}{x^2 - 1}$ **33.** $y = x^3 - 11x + 1$

34. **ERROR ANALYSIS** The expression below was simplified two different ways. Which is incorrect? Explain the error.

Ⓐ
$-2x(4x - y)$
$= (-2x)(4x) + 2x(-y)$
$= -8x^2 - 2xy$

Ⓑ
$-2x(4x - y)$
$= (-2x)(4x) + (-2x)(-y)$
$= -8x^2 + 2xy$

35. **Write About It** What property of real numbers relates addition and multiplication, and how does it relate them?

STANDARDIZED TEST PREP

36. Which expression is NOT equivalent to the others?

Ⓐ $-2x(1 - 3x)$ Ⓑ $2(3x - 1)x$ Ⓒ $(3x - 1)2x$ Ⓓ $6x^2 + 2x$

37. Which expression is greatest when $s = 10$?

Ⓕ Number of inches in s feet
Ⓗ Number of days in s weeks
Ⓖ Number of minutes in s hours
Ⓙ Number of inches in s yards

38. What is the value of $3x(y - 1)^2$ when $x = 4$ and $y = 3$?

Ⓐ 576 Ⓑ 81 Ⓒ 48 Ⓓ 24

CHALLENGE AND EXTEND

Find the value of a for which the expression $2a - 5$ has the given value.

39. 11 **40.** -5 **41.** 39 **42.** 225

43. Consider the expression $\frac{3(x + 2)^2}{(x - 1)(x - 3)}$.

a. Evaluate the expression for $x = 0, 1, 2, 3, 4$, and 5.

b. Identify the values of x for which the expression cannot be evaluated.

c. Use your results from part **b** to identify the set of reasonable values for x.

SPIRAL REVIEW

Name the three-dimensional figure that has the given shapes as its faces. *(Previous course)*

44. three rectangles and two triangles **45.** one square and four triangles

Classify each number by the subsets of the real numbers to which it belongs. *(Lesson 1-1)*

46. 0 **47.** $\frac{5}{16}$ **48.** -6.5 **49.** $3\sqrt{2}$

Simplify each expression. *(Lesson 1-3)*

50. $\sqrt{\frac{52}{25}}$ **51.** $\sqrt{24} + \sqrt{6}$ **52.** $\frac{4\sqrt{27}}{18}$ **53.** $\sqrt{28} \cdot \sqrt{7}$

1-5 Properties of Exponents

Objectives

Simplify expressions involving exponents.

Use scientific notation.

Vocabulary

scientific notation

California Standards

Review of 1A2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. **They understand and use the rules of exponents.**

Also covered: **Review of 7AF2.1**

Who uses this?

Astronomers use exponents when working with large distances such as that between Earth and the Eagle Nebula. (See Example 5.)

In an expression of the form a^n, a is the base, n is the exponent, and the quantity a^n is called a power. The exponent indicates the number of times that the base is used as a factor.

$$\overset{\text{Base}}{a}{}^{\overset{\text{Exponent}}{n}} = \underbrace{a \cdot a \cdot a \cdot \ldots \cdot a \cdot a \cdot a}_{a \text{ is a factor } n \text{ times}}$$

Reading Math

A *power* includes a base and an exponent. The expression 2^3 is a power of 2. It is read "2 to the third power" or "2 cubed."

When the base includes more than one symbol, it is written in parentheses.

Exponential Form	Base	Expanded Form
$-2x^3$	x	$-2(x \cdot x \cdot x)$
$-(2x)^3$	$2x$	$-(2x)(2x)(2x)$
$(-2x)^3$	$-2x$	$(-2x)(-2x)(-2x)$

EXAMPLE 1 Writing Exponential Expressions in Expanded Form

Write each expression in expanded form.

A $(4y)^3$

$(4y)^3$ *The base is 4y, and the exponent is 3.*

$(4y)(4y)(4y)$ *4y is a factor 3 times.*

B $-a^2$

$-a^2$ *The base is a, and the exponent is 2.*

$-(a \cdot a) = -a \cdot a$ *a is a factor 2 times.*

C $2y^2(x-3)^3$

$2y^2(x-3)^3$ *There are two bases: y and x − 3.*

$2(y)(y)(x-3)(x-3)(x-3)$ *y is a factor 2 times, and x − 3 is a factor 3 times.*

CHECK IT OUT! **Write each expression in expanded form.**

1a. $(2a)^5$ **1b.** $3b^4$ **1c.** $-(2x-1)^3y^2$

Zero and Negative Exponents

For all nonzero real numbers *a* and *b* and integers *n*,

WORDS	NUMBERS	ALGEBRA
Zero Exponent Property A nonzero quantity raised to the zero power is equal to 1.	$100^0 = 1$	$a^0 = 1$
Negative Exponent Property A nonzero base raised to a negative exponent is equal to the reciprocal of the base raised to the opposite, positive exponent.	$7^{-2} = \left(\frac{1}{7}\right)^2 = \frac{1}{7^2}$ $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4$	$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

EXAMPLE 2 Simplifying Expressions with Negative Exponents

Simplify each expression.

A 2^{-3}

$$\frac{1}{2^3}$$ *The reciprocal of 2 is $\frac{1}{2}$.*

$$\frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$$

B $-\left(\frac{3}{4}\right)^{-4}$

$$-\left(\frac{4}{3}\right)^4$$ *The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.*

$$-\frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3} \cdot \frac{4}{3} = -\frac{256}{81}, \text{ or } -3\frac{13}{81}$$

Caution!

Do not confuse a negative exponent with a negative expression.

$$a^{-n} \neq -a^n \neq \frac{1}{-a^n}$$

CHECK IT OUT! Simplify each expression.

2a. $\left(\frac{1}{3}\right)^{-2}$

2b. $(-5)^{-5}$

You can use the properties of exponents to simplify powers.

Properties of Exponents

For all nonzero real numbers *a* and *b* and integers *m* and *n*,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$4^3 \cdot 4^2 = 4^{3+2} = 4^5$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{3^7}{3^2} = 3^{7-2} = 3^5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$(4^3)^2 = 4^{3 \cdot 2} = 4^6$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, apply the exponent to each factor.	$(3 \cdot 4)^2 = 3^2 \cdot 4^2$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, apply the exponent to the numerator and denominator.	$\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

An algebraic expression is *simplified* when it contains no negative exponents, no grouping symbols, and no like terms.

EXAMPLE 3

Using Properties of Exponents to Simplify Expressions

Simplify each expression. Assume all variables are nonzero.

A $2x^3(-5x)$

$2 \cdot (-5) \cdot x^3 \cdot x^1$

$-10x^{3+1}$ *Product of Powers*

$-10x^4$ *Simplify.*

B $\left(\frac{ab^4}{b^7}\right)^2$

$(ab^{4-7})^2 = (ab^{-3})^2$ *Quotient of Powers*

$a^2(b^{-3})^2$ *Power of a Product*

$a^2b^{(-3)(2)}$ *Power of a Power*

$a^2b^{-6} = \frac{a^2}{b^6}$ *Negative Exponent Property*

CHECK IT OUT! **Simplify each expression. Assume all variables are nonzero.**

3a. $(5x^6)^3$ **3b.** $(-2a^3b)^{-3}$

Remember!

When you multiply by a power of 10, move the decimal to the right if the exponent is positive. Move the decimal to the left if the exponent is negative.

Scientific notation is a method of writing numbers by using powers of 10. In scientific notation, a number takes the form $m \times 10^n$, where $1 \le m < 10$ and n is an integer.

Scientific Notation	Move the decimal	Standard Notation
1.275×10^7	Right 7 places	12,750,000
3.5×10^{-7}	Left 7 places	0.00000035

You can use the properties of exponents to calculate with numbers expressed in scientific notation.

EXAMPLE 4

Simplifying Expressions Involving Scientific Notation

Simplify each expression. Write the answer in scientific notation.

A $\frac{9.1 \times 10^{-3}}{1.3 \times 10^8}$

$\left(\frac{9.1}{1.3}\right) \times \left(\frac{10^{-3}}{10^8}\right)$ $\frac{a \cdot b}{c \cdot d} = \frac{a}{c} \cdot \frac{b}{d}$

7.0×10^{-11} *Divide 9.1 by 1.3 and subtract exponents: $-3 - 8 = -11$.*

B $(3.5 \times 10^8)(5.2 \times 10^5)$

$(3.5)(5.2) \times (10^8)(10^5)$

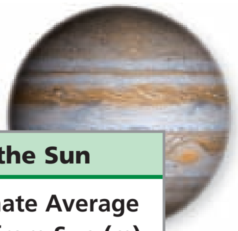
18.2×10^{13} *Multiply 3.5 and 5.2 and add exponents: $8 + 5 = 13$.*

1.82×10^{14} *Because $18.2 > 10$, move the decimal point left 1 place and add 1 to the exponent.*

CHECK IT OUT! **Simplify each expression. Write the answer in scientific notation.**

4a. $\frac{2.325 \times 10^6}{9.3 \times 10^9}$ **4b.** $(4 \times 10^{-6})(3.1 \times 10^{-4})$

EXAMPLE 5 Problem-Solving Application



Light travels through space at a speed of about 3×10^5 kilometers per second. How many minutes does it take light to travel from the Sun to Jupiter?

Distances from the Sun	
Object	**Approximate Average Distance from Sun (m)**
Mercury	5.8×10^{10}
Venus	1.1×10^{11}
Earth	1.5×10^{11}
Mars	2.3×10^{11}
Jupiter	7.8×10^{11}
Saturn	1.4×10^{12}
Uranus	2.9×10^{12}
Neptune	4.5×10^{12}
Pluto	5.9×10^{12}

1 Understand the Problem

The **answer** will be the time it takes for light to travel from the Sun to Jupiter.

List the important information:

- The speed of light in space is 3×10^5 kilometers per second.
- The distance from the Sun to Jupiter is 7.8×10^{11} meters.

2 Make a Plan

Use the relationship: rate, or speed, equals distance divided by time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \text{ so time} = \frac{\text{distance}}{\text{speed}}$$

3 Solve

First, convert the speed of light from $\frac{\text{kilometers}}{\text{second}}$ to $\frac{\text{meters}}{\text{minute}}$.

$$3 \times 10^5 \frac{\cancel{\text{km}}}{\cancel{\text{s}}}\left(\frac{10^3\text{ m}}{1\ \cancel{\text{km}}}\right)\left(\frac{60\ \cancel{\text{s}}}{1\text{ min}}\right)$$

There are 1000, or 10^3, meters in every kilometer and 60 seconds in every minute.

$$(3 \cdot 60) \times (10^5 \cdot 10^3)\frac{\text{m}}{\text{min}}$$

$$180 \times 10^8 \frac{\text{m}}{\text{min}} = 1.8 \times 10^{10} \frac{\text{m}}{\text{min}}$$

Use the relationship between time, distance, and speed to find the number of minutes it takes light to travel from the Sun to Jupiter.

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{7.8 \times 10^{11}\text{ m}}{1.8 \times 10^{10}\,\frac{\text{m}}{\text{min}}}$$

$$\frac{m}{\left(\frac{m}{min}\right)} = \cancel{m}\left(\frac{min}{\cancel{m}}\right) = min$$

$$= 4.\overline{3} \times 10\text{ min} \approx 43.33\text{ min}$$

It takes light approximately 43.33 minutes to travel from the Sun to Jupiter.

4 Look Back

Light traveling at 3×10^5 km/s for 43.33(60) ≈ 2600 seconds travels a distance of 780,000,000 = 7.8×10^8 km, or 7.8×10^{11}m. The answer is reasonable.

CHECK IT OUT!

5. How many minutes does it take light to travel from the Sun to Earth?

THINK AND DISCUSS

1. Tell which properties of exponents apply only to expressions with the same base.
2. List the steps for writing a number in scientific notation.
3. **GET ORGANIZED** Copy and complete the graphic organizer by providing a numerical and algebraic example of each property.

Know it! Note

Property	Numerical Example	Algebraic Example
Product of Powers		
Quotient of Powers		
Power of a Power		
Power of a Product		
Power of a Quotient		

1-5 Exercises

California Standards
Review of 1A2.0 and 7AF2.1

go.hrw.com
Homework Help Online
KEYWORD: MB7 1-5
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Describe the requirements for a number to be expressed in *scientific notation.*

SEE EXAMPLE 1
p. 34

Write each expression in expanded form.

2. $4(a-b)^2$
3. $(12xy)^4$
4. $-s^3(-2t)^5$
5. $\left(-\frac{1}{2}d\right)^3$

SEE EXAMPLE 2
p. 35

Simplify each expression.

6. $\left(-\frac{3}{5}\right)^{-2}$
7. 5^0
8. $\left(\frac{2}{3}\right)^{-3}$
9. 10^{-1}

SEE EXAMPLE 3
p. 36

Simplify each expression. Assume all variables are nonzero.

10. $(-3a^2b^3)^2$
11. $c^3d^2(c^{-2}d^4)$
12. $\frac{5uv^6}{u^2v^2}$
13. $10\left(\frac{y^5}{x^2}\right)^2$
14. $-2s^{-3}t(7s^{-8}t^5)$
15. $-4m(mn^2)^3$
16. $\frac{(4b)^2}{2b}$
17. $\frac{x^{-1}y^{-2}}{x^3y^{-5}}$

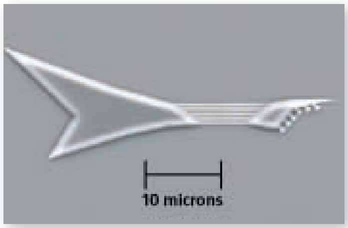
SEE EXAMPLE 4
p. 36

Simplify each expression. Write the answer in scientific notation.

18. $(2.2 \times 10^5)(4.5 \times 10^{11})$
19. $\frac{7.8 \times 10^8}{2.6 \times 10^{-3}}$
20. $\frac{16 \times 10^{-3}}{4.0 \times 10^4}$

SEE EXAMPLE 5
p. 37

21. **Technology** Nanotechnology is a branch of engineering that works with devices that are smaller than 100 nanometers. The width of one string on the playable nanoguitar created by scientists at Cornell University in 2003 is 2.0×10^{-7} meters. If the width of a human hair is about 80 microns, how many nanoguitar strings would have the same width as a human hair? (*Hint:* 1 micron $= 10^{-6}$ meters)



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
22–25	1
26–29	2
30–33	3
34–36	4
37	5

Extra Practice

Skills Practice p. S4

Application Practice p. S32

Write each expression in expanded form.

22. $(m + 2n)^3$ **23.** $5x^3$ **24.** $(-9fg)^3h^4$ **25.** $2a(-b^2 - a)^2$

Simplify each expression.

26. $(-4)^{-2}$ **27.** $\left(-\frac{3}{4}\right)^{-1}$ **28.** $\left(-\frac{5}{2}\right)^{-3}$ **29.** -6^0

Simplify each expression. Assume all variables are nonzero.

30. $\frac{-100s^3t^{-5}}{25s^{-2}t^6}$ **31.** $(-x^4y^2)^5$ **32.** $(16u^4v^6)^{-2}$ **33.** $8a^2b^5(-2a^3b^2)$

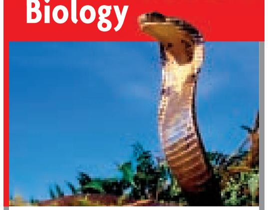
Simplify each expression. Write the answer in scientific notation.

34. $(3.2 \times 10^6)(1.7 \times 10^{-4})$ **35.** $\frac{5.1 \times 10^4}{3.4 \times 10^{-5}}$ **36.** $(6.8 \times 10^3)(9.5 \times 10^5)$

37. **Computer Science** A computer with a 5.4 GHz microprocessor can make 5.4×10^9 calculations in one second. If a total of 5.02×10^{11} calculations are required to convert a given MP3 file to audio, how many minutes will the computer take to convert the file? Round your answer to the nearest hundredth.

LINK

Biology



King cobras are native to Asia. They may grow more than 12 feet in length and feed primarily on other snakes.

38. **Biology** A king cobra bite is fatal to a mouse if the mouse receives at least 0.00173 gram of venom per kilogram of its body mass. What is the smallest amount, in grams, of king cobra venom that will be fatal to a mouse with a mass of 0.02 kilogram? Express your answer in scientific notation.

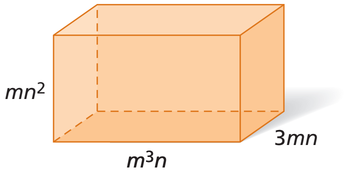
Order each list from least to greatest by first rewriting each number with a base of 2.

39. $8^2, 4^1, 2^5, 16^{-2}$ **40.** $2^{-1}, -4^3, 4^2, 8^{-2}$ **41.** $-8^2, 4^0, 16^1, 2^{-2}$

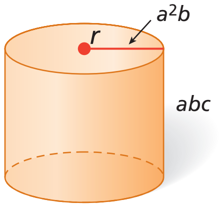
42. **Multi-Step** There are approximately 1.3×10^{15} gallons of water in Lake Michigan. If a faucet is leaking at a rate of 1.5 ounces per minute, how many years would it take for the amount of water that has leaked to be equivalent to the volume of Lake Michigan? (*Hint:* 1 gallon = 128 ounces)

Geometry **Write and simplify an expression for the volume of each figure.**

43.



44.



Simplify each expression. Assume all variables are nonzero.

45. $\frac{27x^3y}{18x^2y^4}$ **46.** $\left(\frac{3a^3b}{2a^{-1}b^2}\right)^2$ **47.** $12a^0b^5(-2a^3b^2)$

48. $\frac{72a^2b^3}{-24a^2b^5}$ **49.** $\left(\frac{5mn}{-3m^2}\right)^{-2}$ **50.** $6x^5y^3(-3x^2y^{-1})$

Measurement **Calculate each of the following.**

51. number of square inches in a square yard

52. number of square centimeters in a square meter

53. number of cubic inches in a cubic foot

54. number of cubic meters in a cubic kilometer

CONCEPT CONNECTION

55. This problem will prepare you for the Concept Connection on page 42.

The *Apollo 11* took approximately 102 hours and 45 minutes to get to the Moon, which is located about 384,500 km from Earth.

a. What was the average speed of the *Apollo 11* to the nearest kilometer per hour?

b. In theory, spaceships of the future might be able to travel at the speed of light, 3×10^5 km/s. How many times as fast is this than the average speed of the *Apollo 11*?

c. How long would it take future space travelers traveling at the speed of light to get to the Moon?

Simplify each expression. Assume all variables are nonzero.

56. $-9a^2b^6(-7ab^{-4})$ **57.** $\dfrac{14x^{-2}y^3}{-8x^{-5}y^5}$ **58.** $-\left(\dfrac{20x^6}{2x^2}\right)^3$

59. $(10x^{-2}y^0z^{-3})^2$ **60.** $(-3a^2b^{-1})^{-3}$ **61.** $(8m^4n^{-2})(-3m^{-2}n)^0$

Geography **Use the map for Exercises 62–66. Identify which country fits the description, and then find its population density, or population per square mile, to the nearest tenth.**

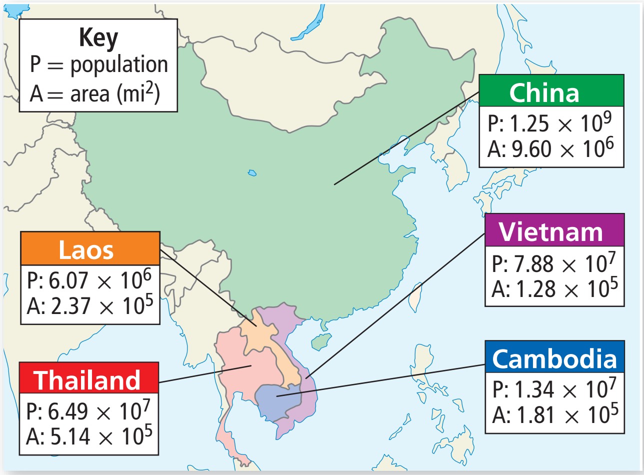
62. greatest population

63. median area

64. median population

65. least area

66. second smallest population



Estimation **Use scientific notation to express each answer.**

67. What is the average number of times a human heart beats in an average lifetime? Use an average rate of 1.2 heartbeats per second and an average lifespan of 75 years.

68. What is the average number of breaths a person takes in a lifetime? Use an average rate of 16 breaths per minute and an average lifespan of 75 years.

69. What is the average number of hairs on a human head? Use an average of 254 hairs per square centimeter and an average scalp size of 500 square centimeters.

Identify the property of exponents illustrated in each equation.

70. $(x^5)^3 = x^{15}$ **71.** $(m^2n^5)^4 = m^8n^{20}$ **72.** $\dfrac{3a^3}{a^{-2}} = 3a^5$ **73.** $\left(\dfrac{st^5}{s^3}\right)^4 = \dfrac{s^4t^{20}}{s^{12}}$

74. **Language** Statements such as "The population of the country is 3.8 million" are commonly used to describe large numbers. Express this value in scientific notation and explain the relationship between the mathematical representation of the number and the words used to describe it.

75. **Critical Thinking** Use the Quotient of Powers Property to show why 0^0 is undefined.

Graphing Calculator The key sequence **2nd** EE **,** on a calculator is used for scientific notation. To enter the number 2.8×10^5 into your calculator, you would enter `2.8` **2nd** EE **,** `5`. The calculator screen will display `2.8E5`. Use your calculator to find the value of each expression.

76. $(3.7 \times 10^{-3})(8.1 \times 10^{-5})$

77. $\dfrac{2.08 \times 10^{-8}}{3.2 \times 10^{6}}$

78. $(4.75 \times 10^{2})(4.2 \times 10^{-7})$

79. $\dfrac{8.4 \times 10^{9}}{2.4 \times 10^{-5}}$

80. $\dfrac{17.068 \times 10^{-4}}{6.8 \times 10^{3}}$

81. $(1.83 \times 10^{13})(6.2 \times 10^{10})$

82. **Write About It** How can you tell which of two numbers written in scientific notation is greater? Use examples to explain your answer.

STANDARDIZED TEST PREP

83. Which number is greatest?

Ⓐ 0.000025 Ⓑ 2.5×10^{-6} Ⓒ 2.5×10^{-4} Ⓓ 2.5×10^{-5}

84. Which number is expressed correctly in scientific notation?

Ⓕ 11×10^{5} Ⓖ 58.5×10^{4} Ⓗ 0.245×10^{-7} Ⓙ 7.25×10^{0}

85. Which expression is equivalent to $(-5)(-5)(-5)(-5)(-5)(-5)$?

Ⓐ 5^{-6} Ⓑ $(-5)^{-6}$ Ⓒ $(-5)^{6}$ Ⓓ -5^{6}

86. If a and c are nonzero, which expression is equivalent to $\dfrac{a^4b^{-3}}{a^2c^0}$?

Ⓕ $\dfrac{a^2}{b^3c}$ Ⓖ $\dfrac{a^2c}{b^3}$ Ⓗ $\dfrac{a^{-2}}{b^{-3}c}$ Ⓙ $\dfrac{a^2}{b^3}$

CHALLENGE AND EXTEND

Simplify each expression. Write your answer in scientific notation.

87. $\left(\dfrac{7.82 \times 10^{6}}{5.48 \times 10^{8}}\right)^2$

88. $[(6.18 \times 10^{7})(2.05 \times 10^{8})]^2$

89. Give examples of numbers that are greater than 1 when raised to the exponent -2. Make a generalization about the types of numbers that are greater than 1 when raised to a negative exponent.

90. Notice that $2^4 = 4^2$. For whole numbers a and b such that $a < b$, give three examples of values of a and b such that $a^b > b^a$ and three examples such that $a^b < b^a$.

SPIRAL REVIEW

91. When two people play the game rock, paper, scissors, each person's hand simultaneously shows the player's choice of rock, paper, or scissors. What is the probability that both players will make the same choice? *(Previous course)*

Complete each statement. *(Lesson 1-2)*

92. $\frac{1}{3} \cdot \blacksquare = 1$

93. $4(-3 + \blacksquare) = -12 + 32$

94. $0 = \sqrt{7} + \blacksquare$

Evaluate each expression for the given values of the variables. *(Lesson 1-4)*

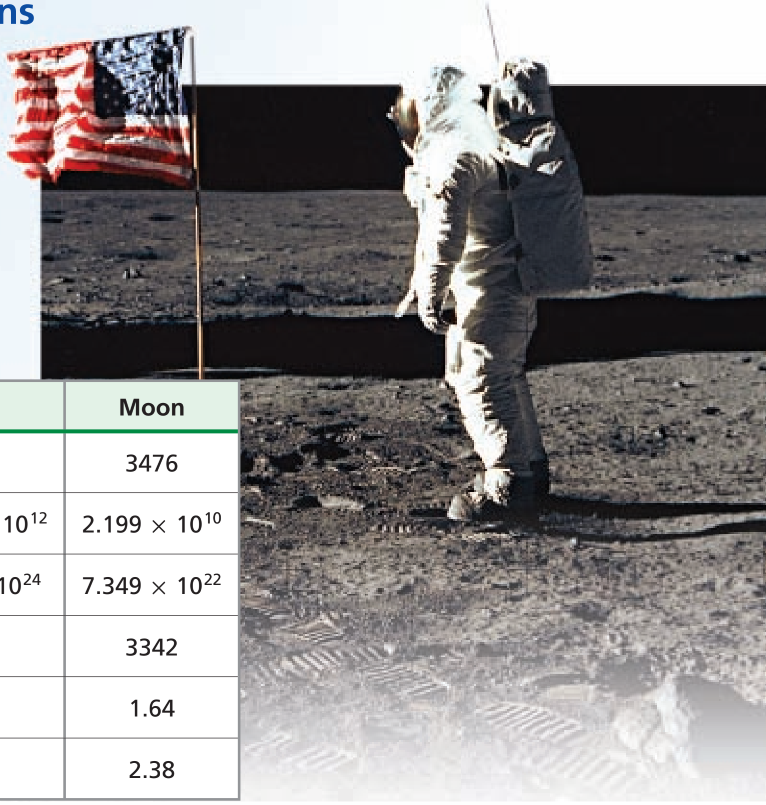
95. $\dfrac{2mn}{n^2 - 2n + 5m}$ for $n = -1$ and $m = 3$

96. $2x(9y - x^2)$ for $x = -3$ and $y = 10$

CONCEPT CONNECTION

Properties and Operations

Man on the Moon On July 20, 1969, the U.S. *Apollo 11* lunar module landed on the Moon. A few hours later, Neil Armstrong was the first human to set foot on the Moon's surface. The *Apollo 11* mission led to many scientific discoveries about Earth's nearest neighbor.



	Earth	Moon
Mean Diameter (km)	12,742	3476
Volume (km^3)	1.08321×10^{12}	2.199×10^{10}
Mass (kg)	5.9736×10^{24}	7.349×10^{22}
Mean Density (kg/m^3)	5515	3342
Surface Gravity (m/s^2)	9.78	1.64
Escape Velocity (km/s)	11.2	2.38

1. *Apollo 11* was launched at 9:32 A.M. eastern daylight time (EDT) on July 16, 1969. The mission lasted 195 h 18 min. What was the date and time when the mission ended?

2. The gravity on the Moon is about $\frac{1}{6}$ of Earth's gravity. Based on the data in the table, is the actual surface gravity on the Moon less than or greater than $\frac{1}{6}$ of Earth's surface gravity? Explain.

3. Classify the numbers in the indicated row of the table by the sets of the real numbers to which they belong.

a. mean diameter **b.** surface gravity **c.** escape velocity

4. The expression $\sqrt{\frac{h}{0.82}}$ can be used to approximate the time in seconds it takes for an object to reach the surface of the Moon when dropped from a height of h meters. The *Apollo 11* lunar module was about 7.2 meters tall. Suppose Neil Armstrong jumped from the top of the lunar module. How long would it have taken him to land on the surface of the Moon?

5. The expression $\sqrt{\frac{h}{4.89}}$ can be used to model the time it takes to reach Earth's surface from a height of h meters. To the nearest tenth of a second, how much longer would it take for an object to fall from a height of 125 m to the surface on the Moon than it would take on Earth?

6. Approximately how many Moons would it take to equal the volume of Earth?

READY TO GO ON?

Quiz for Lessons 1-1 Through 1-5

1-1 Sets of Numbers

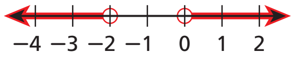
Order the given numbers from least to greatest. Then classify each number by the subsets of the real numbers to which it belongs.

1. $2.5, -3\frac{1}{3}, \sqrt{5}, -\frac{4}{5}, 0.\overline{75}$

2. $\sqrt{3}, -\frac{\pi}{2}, \frac{5}{6}, -1.\overline{15}, -2$

Rewrite each set in the indicated notation.

3. $\{x \mid -4 \le x < 2\}$; interval notation

4. 

set-builder notation

1-2 Properties of Real Numbers

Identify the property demonstrated by each equation.

5. $3(2a + b) = 3(2a) + 3b$

6. $21 + 0 = 21$

7. $(2\pi)r = 2(\pi r)$

8. Use mental math to find the amount of a 12% shipping fee for an item that costs \$250. Explain your steps.

1-3 Square Roots

9. A rental company rents portable dance floors in three different sizes: 75 square feet, 125 square feet, and 150 square feet. Estimate the dimensions of each square dance floor to the nearest tenth of a foot. Then identify which of the three sizes is the largest dance floor that would fit in a room 11 feet wide and 13 feet long.

Simplify each expression.

10. $-\sqrt{72}$

11. $5\sqrt{12} + 9\sqrt{3}$

12. $\frac{-4\sqrt{10}}{\sqrt{2}}$

13. $\sqrt{32} \cdot \sqrt{6}$

1-4 Simplifying Algebraic Expressions

Evaluate each expression for the given values of the variables.

14. $\frac{a^2}{3} + \frac{ab}{4}$ for $a = 3$ and $b = -4$

15. $\frac{d^2}{2cd}$ for $c = -1$ and $d = 2$

Simplify each expression.

16. $2x^2 - 3y + 5x^2 - x^2$

17. $3(x + 2y) - 5x + y$

1-5 Properties of Exponents

Simplify each expression. Assume all variables are nonzero.

18. $(x^{11}y^{-2})^4$

19. $\frac{-3s^3t^2}{s^{-2}t^8}$

20. $4(a^2b^6)^{-3}$

21. $\left(\frac{m^4}{-5m^{-2}n^3}\right)^2$

22. The atomic mass of an element from the periodic table is the mass, in grams, of one *mole*, or 6.02×10^{23} atoms. Suppose a sample of oxygen contains 4.515×10^{26} atoms. How many moles of oxygen atoms are in the sample?

1-6

Relations and Functions

Objectives

Identify the domain and range of relations and functions.

Determine whether a relation is a function.

Vocabulary

- relation
- domain
- range
- function

California Standards

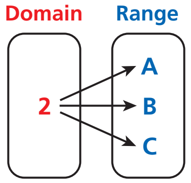
Preparation for 24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

Why learn this?

The relationship between the numbers and the letters on the keys of a cell phone can be described using relations.

When you create a text message on a cell phone, you enter letters by pressing the numbered keys that they appear on. For instance, you would press the 2 key to enter an *A*, *B*, or *C*. This relationship can be represented by a mapping diagram or a set of ordered pairs.

Mapping Diagram



Set of Ordered Pairs

{(2, A), (2, B), (2, C)}

(x, y) → (input, output) → (domain, range)

A **relation** is a pairing of input values with output values. It can be shown as a set of ordered pairs (x, y), where x is an input and y is an output.

The set of input values for a relation is called the **domain**, and the set of output values is called the **range**.

EXAMPLE 1 Identifying Domain and Range

Give the domain and range for the relation shown.

First-Class Stamp Rates

Year	1900	1920	1940	1960	1980	2000
Rate (¢)	2	2	3	4	15	33

Helpful Hint

Notice that the rate 2¢ appears twice in the table but is listed only once in the set of range values. When the domain or range of a relation is listed, each value is listed only once.

List the set of ordered pairs:

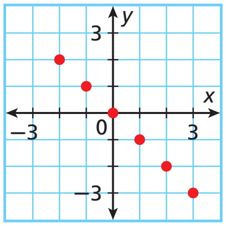
$\{(1900, 2), (1920, 2), (1940, 3), (1960, 4), (1980, 15), (2000, 33)\}$

Domain: $\{1900, 1920, 1940, 1960, 1980, 2000\}$ *The set of x-coordinates*

Range: $\{2, 3, 4, 15, 33\}$ *The set of y-coordinates*

CHECK IT OUT!

1. Give the domain and range for the relation shown in the graph.



Suppose you are told that a person entered a word into a text message using the numbers 6, 2, 8, and 4 on a cell phone. It would be difficult to determine the word without seeing it because each number can be used to enter three different letters.

Number		{Number, Letter}
	→	$\{(6, \mathrm{M}), (6, \mathrm{N}), (6, \mathrm{O})\}$
	→	$\{(2, \mathrm{A}), (2, \mathrm{B}), (2, \mathrm{C})\}$
	→	$\{(8, \mathrm{T}), (8, \mathrm{U}), (8, \mathrm{V})\}$
	→	$\{(4, \mathrm{G}), (4, \mathrm{H}), (4, \mathrm{I})\}$

The numbers 6, 2, 8, and 4 each appear as the first coordinate of three different ordered pairs.

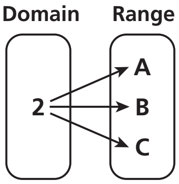
However, if you are told to enter the word *MATH* into a text message, you can easily determine that you must use the numbers 6, 2, 8, and 4, because each letter appears on only one numbered key.

$\{(\mathrm{M}, 6), (\mathrm{A}, 2), (\mathrm{T}, 8), (\mathrm{H}, 4)\}$

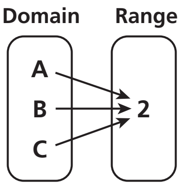
The first coordinate is different in each ordered pair.

A relation in which the first coordinate is never repeated is called a *function*. In a **function**, there is only one output for each input, so each element of the domain is mapped to exactly one element in the range.

Although a single input in a function cannot be mapped to more than one output, two or more different inputs can be mapped to the same output.



Not a function: The relationship from number to letter is *not* a function because the domain value 2 is mapped to the range values A, B, and C.



Function: The relationship from letter to number is a function because each letter in the domain is mapped to only one number in the range.

EXAMPLE 2 Determining Whether a Relation Is a Function

Determine whether each relation is a function.

A

Instant Rice Cooking Times				
Servings	2	4	6	8
Cooking Time (min)	5	8	10	11

There is only one cooking time for each number of servings. The relation from number of servings to cooking time is a function.

B **from last name to Social Security number**

A last name, such as Smith, from the domain would be associated with many different Social Security numbers. The relation from last name to Social Security number is not a function.

CHECK IT OUT! **Determine whether each relation is a function.**

2a.

Shoe Prices			
Size	7	8	9
Price ($)	35	35	35

2b. from the number of items in a grocery cart to the total cost of the items in the cart

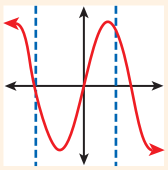
Every point on a vertical line has the same x-coordinate, so a vertical line cannot represent a function. If a vertical line passes through more than one point on the graph of a relation, the relation must have more than one point with the same x-coordinate. Therefore the relation is not a function.

Vertical-Line Test

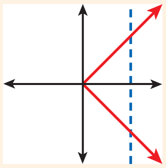
WORDS

If any vertical line passes through more than one point on the graph of a relation, the relation is not a function.

EXAMPLES



Function

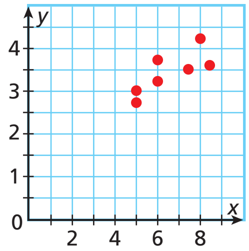


Not a Function

EXAMPLE 3 Using the Vertical-Line Test

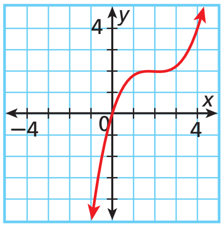
Use the vertical-line test to determine whether each relation is a function. If not, identify two points a vertical line would pass through.

A



This is *not* a function. A vertical line at $x = 6$ would pass through $(6, 3.25)$ and $(6, 3.75)$.

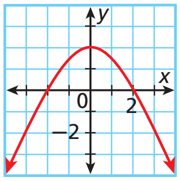
B



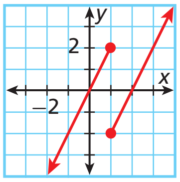
This *is* a function. Any vertical line would pass through only one point on the graph.

CHECK IT OUT! **Use the vertical-line test to determine whether each relation is a function. If not, identify two points a vertical line would pass through.**

3a.

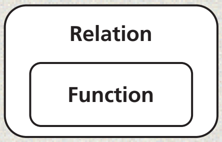


3b.



THINK AND DISCUSS

1. Name four different ways to represent a relation or function.
2. Explain why the vertical-line test works.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give an example of a table, a graph, and a set of ordered pairs.



1-6 Exercises

California Standards
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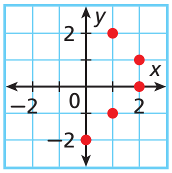
GUIDED PRACTICE

1. **Vocabulary** The set of output values of a function is its ___?___. (*domain* or *range*)

SEE EXAMPLE 1
p. 44

Give the domain and range for each relation.

2.



3.

Average Movie Ticket Price	
Year	**Price**
2000	\$5.39
2001	\$5.65
2002	\$5.80
2003	\$6.03

SEE EXAMPLE 2
p. 45

Determine whether each relation is a function.

4.

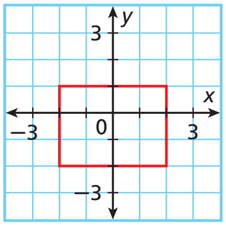
Math Test Scores				
Name	Jan	Helen	Luke	Soren
Score	90	84	88	84

5. from car models to car colors

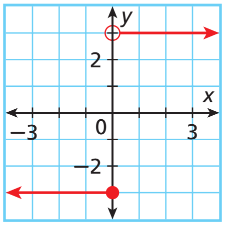
SEE EXAMPLE 3
p. 46

Use the vertical-line test to determine whether each relation is a function. If not, identify two points a vertical line would pass through.

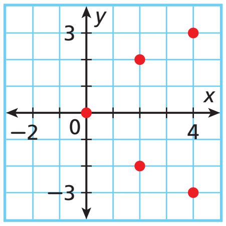
6.



7.



8.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–10	1
11–12	2
13–15	3

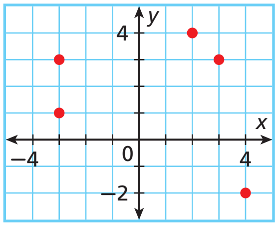
Extra Practice
Skills Practice p. S5
Application Practice p. S32

Give the domain and range for each relation.

9.

Basketball Points Scored				
Player	Irene	Anna	Lea	Kate
Points	22	12	16	12

10.

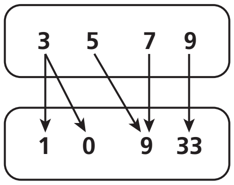


Determine whether each relation is a function.

11.

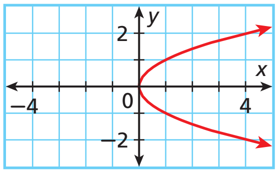
Women's Glove Sizes			
Size	S	M	L
Maximum Hand Length (in.)	6.5	7.5	8.5

12.

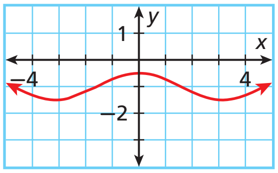


Use the vertical-line test to determine whether each relation is a function. If not, identify two points a vertical line would pass through.

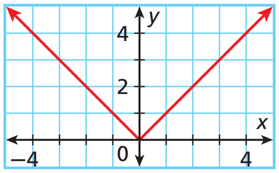
13.



14.



15.

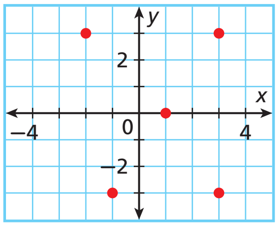


Give the domain and range of each relation and make a mapping diagram.

16. $\{(-5, 0), (0, -5), (5, 0), (0, 5)\}$

17. $\{(-2, -2), (-1, -2), (0, 0), (1, 2), (2, 2)\}$

18.



19.

Average Egg Weights

Size	Weight (oz)
Jumbo	2.5
Extra large	2.25
Large	2
Medium	1.75

20. from each unique letter in the word *seven* to the number that represents the position of that letter in the alphabet

21. **Money** In 1999 the U.S. Mint began releasing quarters to commemorate each of the 50 states. The release schedule specified that each year for a total of 10 years, new quarters commemorating 5 different states would be released. Explain whether each relation is a function.

- **a.** from each year to the number of states with new quarters released in that year
- **b.** from each state to the year its quarter is released
- **c.** from each year to the states with new quarters released in that year
- **d.** from each year to the total number of states with quarters released by the end of that year
- **e.** from the number of new quarters released each year to the year

LINK

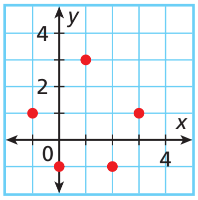
Money

By the end of 2004, there were more than 17.6 billion state quarters in circulation.

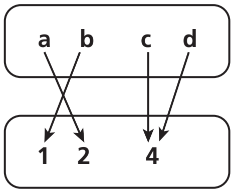
Source: www.usmint.gov

Give the domain and range of each relation. Then explain whether the relation is a function.

22.



23.



24. $\{(7, 1), (7, 2), (7, 3), (7, 4), (7, 6)\}$

25. $\{(9, 3), (7, 3), (5, 3), (3, 3), (1, 3)\}$

26.

x	3	0	0	−1	−3
y	−4	−3	−1	−2	0

27.

x	7	6	5	4	3
y	−1	2	−1	2	3

28. From the months of the year to the number of days in that month in a non-leap year

29. From day of the week to the number of hours in that day

CONCEPT CONNECTION

30. This problem will prepare you for the Concept Connection on page 74.

a. The relation $(0, 20)$, $(-20, 0)$, $(0, -20)$, and $(20, 0)$ can be plotted to produce the vertices of a shape very common in Native American art. What shape is this?

b. Does this relation represent a function? Why or why not?

c. What is the domain of this relation?

d. What is the range of this relation?

Explain whether the relation from A to B is a function, the relation from B to A is a function, or both are functions.

	A	B
31.	Date of birth	Person
32.	Thumbprint	Person
33.	Area code	State
34.	Amount of sales tax	Purchase total
35.	Sales tax percentage	Purchase total
36.	Jersey number	NFL football player
37.	Jersey number	current Cleveland Browns player

38. **ERROR ANALYSIS** Identify which statement is incorrect. Explain the error.

A

The relation
$\{(-1, 9), (0, 8), (0, 7), (1, 6)\}$
is a function.

B

The relation
$\{(4, 5), (5, 5), (6, 5), (7, 5)\}$
is a function.

Carpentry **Use the table for Exercises 39–41.**

Common Wire Nail Data

Size	Length (in.)	Gauge	Number (per lb)
2d	1	15	876
3d	$1\frac{1}{4}$	14	568
4d	$1\frac{1}{2}$	$12\frac{1}{2}$	316
5d	$1\frac{3}{4}$	$12\frac{1}{2}$	271
6d	2	$11\frac{1}{2}$	181

39. If you know the gauge of a nail, can you determine its size? What does this indicate about the relation from gauge to size?

40. Identify the pattern in the nail lengths as size increases. Does the pattern indicate that the relation from length to size is a function?

41. Consider the relation from nail size to the number of nails per pound.

a. Does the relation represent a function?

b. Explain the relationship between a nail's size and its average weight.

c. Confirm your answer to part **b** by finding the average weight for each nail size. (*Hint:* 1 pound = 16 ounces)

42. **Critical Thinking** If you switch the domain and range of any function, will the resulting relation always be a function? Explain by using examples.

43. **Write About It** Explain how you would determine whether each of the following represents a function: a set of ordered pairs, a mapping diagram, and a graph.

STANDARDIZED TEST PREP

44. Which relation is NOT a function?

Ⓐ $\{(0, 1), (1, 0), (2, 0), (3, 1)\}$

Ⓑ $\{(-1, 5), (-2, 4), (-2, 3), (-3, 2)\}$

Ⓒ $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Ⓓ $\left\{\left(2, \frac{1}{2}\right), \left(4, \frac{1}{4}\right), \left(8, \frac{1}{8}\right), \left(16, \frac{1}{16}\right)\right\}$

45. Which set represents the domain of $\{(99, -2), (99, -3), (96, -4), (96, -5)\}$?

Ⓕ $\{96, 99\}$

Ⓖ All negative integers

Ⓗ $\{-2, -3, -4, -5\}$

Ⓙ $\{-2, -3, -4, -5, 96, 99\}$

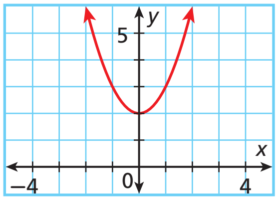
46. Which is an element of the range of the graphed function?

Ⓐ −2

Ⓑ 0

Ⓒ 1

Ⓓ 4

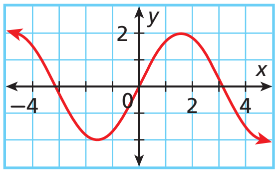


CHALLENGE AND EXTEND

47. Find the conditions for a and b that make $\{(a, b), (-a, b), (2a, b), (a^2, b)\}$ a function.

A *one-to-one function* is a function in which each output corresponds to only one input. Explain whether each function is one to one.

48.

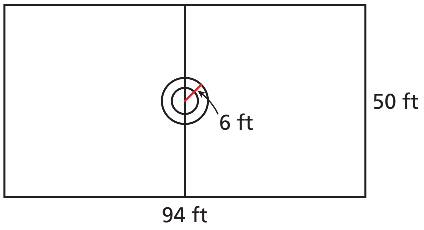


49. from length in inches to length in feet

50. Find the conditions for a and b that make $\left\{(2, b), (3, ab), \left(4, \frac{ab}{2}\right)\right\}$ a one-to-one function.

SPIRAL REVIEW

Use the diagram of the basketball court for Exercises 51–53. *(Previous course)*



51. What is the perimeter of the basketball court?

52. What is the area of the basketball court?

53. To the nearest tenth, what is the area of the outermost circle at center court?

Estimate to the nearest tenth. *(Lesson 1-3)*

54. $\sqrt{42}$

55. $\sqrt{22}$

56. $-\sqrt{8}$

57. $\sqrt{90}$

Simplify each expression. Assume all variables are nonzero. *(Lesson 1-5)*

58. $(-3y^4)^3$

59. $\frac{(10w^2)^2}{5w^5}$

60. $(4c^6d^2)^2$

61. $\left(\frac{x^3}{z}\right)^7$

1-7

Function Notation

Objectives

Write functions using function notation.

Evaluate and graph functions.

Vocabulary

function notation
dependent variable
independent variable

California Standards

Preparation for 24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions.

Why learn this?

Function notation can be used to indicate the distance traveled by a Japanese bullet train. (See Example 3.)

Some sets of ordered pairs can be described by using an equation. When the set of ordered pairs described by an equation satisfies the definition of a function, the equation can be written in **function notation**.

$$\overbrace{f(x)}^{\text{Output value}} = 5\overset{\text{Input value}}{x} + 3 \qquad \overbrace{f(1)}^{\text{Output value}} = 5(\overset{\text{Input value}}{1}) + 3$$

f of *x* equals 5 times *x* plus 3. *f* of 1 equals 5 times 1 plus 3.

The function described by $f(x) = 5x + 3$ is the same as the function described by $y = 5x + 3$. And both of these functions are the same as the set of ordered pairs $(x, 5x + 3)$.

$$y = 5x + 3 \rightarrow (x, y) \rightarrow (x, 5x + 3)$$
$$f(x) = 5x + 3 \rightarrow (x, f(x)) \rightarrow (x, 5x + 3)$$

Notice that $y = f(x)$ for each x.

The graph of a function is a picture of the function's ordered pairs.

EXAMPLE 1

Evaluating Functions

For each function, evaluate $f(0)$, $f\left(\frac{1}{2}\right)$, and $f(-2)$.

A $f(x) = 7 - 2x$

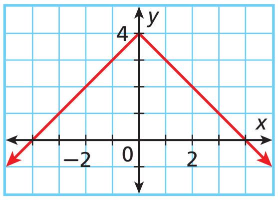
Substitute each value for x and evaluate.

$f(0) = 7 - 2(0) = 7$

$f\left(\frac{1}{2}\right) = 7 - 2\left(\frac{1}{2}\right) = 6$

$f(-2) = 7 - 2(-2) = 11$

B



Use the graph to find the corresponding y-value for each x-value.

$f(0) = 4 \quad f\left(\frac{1}{2}\right) = 3\frac{1}{2} \quad f(-2) = 2$

Caution!

$f(x)$ is *not* "f times x" or "f multiplied by x." $f(x)$ means "the value of f at x." So $f(1)$ represents the value of f at $x = 1$.

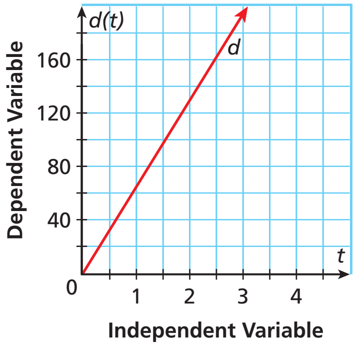
CHECK IT OUT! For each function, evaluate $f(0)$, $f\left(\frac{1}{2}\right)$, and $f(-2)$.

1a. $f(x) = x^2 - 4x$

1b. $f(x) = -2x + 1$

In the notation $f(x)$, f is the *name* of the function. The output $f(x)$ of a function is called the **dependent variable** because it *depends* on the input value of the function. The input x is called the **independent variable**. When a function is graphed, the independent variable is graphed on the horizontal axis and the dependent variable is graphed on the vertical axis.

$$\underbrace{d(t)}_{\text{Dependent variable}} = 65\underset{\uparrow\ \text{Independent variable}}{t}$$



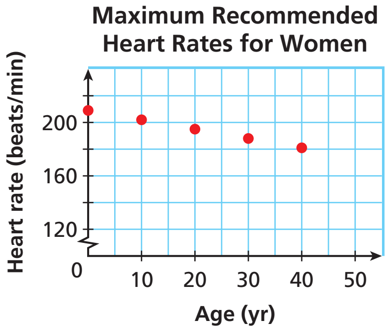
EXAMPLE 2 Graphing Functions

Graph each function.

A **The diagram shows the maximum recommended heart rate for women by age.**

Age (yr)	Heart Rate (beats/min)
0	209
10	202
20	195
30	188
40	181

Graph the points.



Do not connect the points, because the values between the given points have not been defined.

Reading Math

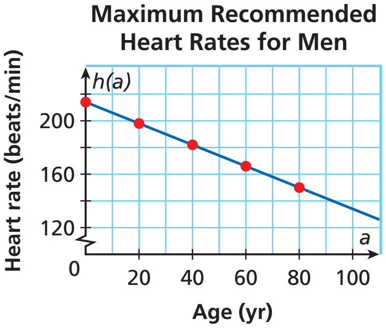
A function whose graph is made up of unconnected points is called a *discrete* function.

B **The maximum recommended heart rate h for men is a function of age a and can be calculated with $h(a) = 214 - 0.8a$.**

Make a table.

a	$214 - 0.8a$	$h(a)$
0	$214 - 0.8(0)$	214
20	$214 - 0.8(20)$	198
40	$214 - 0.8(40)$	182
60	$214 - 0.8(60)$	166
80	$214 - 0.8(80)$	150

Graph the points.



Connect the points with a line because the function is defined for $0 \le a \le 100$.

CHECK IT OUT! **Graph each function.**

2a.

3	2
5	6
7	10
9	10

2b. $f(x) = 2x + 1$

The algebraic expression used to define a function is called the function rule. The function described by $f(x) = 5x + 3$ is defined by the function rule $5x + 3$. To write a function rule, first identify the independent and dependent variables.

EXAMPLE 3 Transportation Application

The Japanese bullet train that travels from Tokyo to Kyoto averages about 156 km/h. The distance from Tokyo to Kyoto is 380 km.

a. Write a function to represent the distance remaining on the trip after a certain amount of time.

Time traveled is the independent variable, and distance remaining is the dependent variable.

Let t be the time in hours and let d be the distance in kilometers remaining on the trip.

Write a word equation to represent the problem situation. Then replace the words with expressions.

distance remaining	=	total distance	−	distance traveled
$d(t)$	=	380	−	$156t$

b. What is the value of the function for an input of 1.5, and what does it represent?

$d(1.5) = 380 - 156(1.5)$ *Substitute 1.5 for t and simplify.*

$d(1.5) = 146$

The value of the function for an input of 1.5 is 146. This means that there are 146 kilometers remaining in the trip after 1.5 hours.

CHECK IT OUT! **A local photo shop will develop and print the photos from a disposable camera for $0.27 per print.**

3a. Write a function to represent the cost of photo processing.

3b. What is the value of the function for an input of 24, and what does it represent?

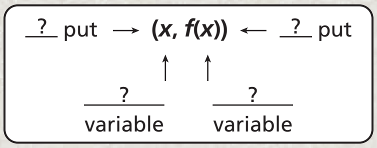
THINK AND DISCUSS

1. Identify a reasonable domain for the function in Example 3. Explain your answer.

2. Explain three things you can determine about a function from the notation $g(t)$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each blank, fill in the missing portion of the label.



1-7 Exercises

California Standards

Preparation for 24.0

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GUIDED PRACTICE

1. **Vocabulary** In function notation, the variable x is generally used to represent the __?__ variable. (*dependent* or *independent*)

SEE EXAMPLE 1
p. 51

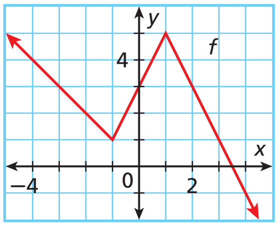
For each function, evaluate $f(0)$, $f(1.5)$, and $f(-4)$.

2. $f(x) = 3x - 4$

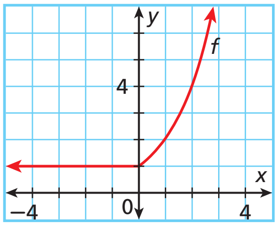
3. $f(x) = x^2 + 9$

4. $f(x) = 3x^2 - x + 2$

5.



6.



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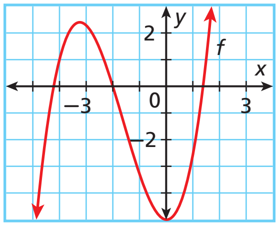
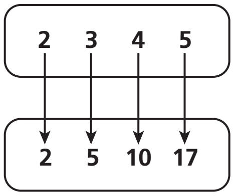


SEE EXAMPLE 2
p. 52

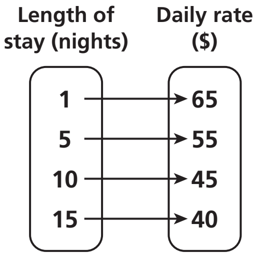
Graph each function.

8.



9. $g(x) = -3x + 12$

10.



SEE EXAMPLE 3
p. 53

11. **Business** A furniture company misprinted a sales ad for a living room set but honors the advertised price. For each customer who purchases the living room set, the company suffers a loss of \$125. Write a function to represent the company's total loss. What is the value of the function for an input of 50, and what does it represent?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–17	1
18–20	2
21	3

Extra Practice
Skills Practice p. S5
Application Practice p. S32

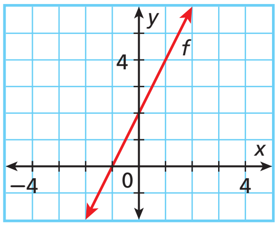
For each function, evaluate $f(0)$, $f\left(\frac{3}{2}\right)$, and $f(-1)$.

12. $f(x) = 7x - 4$

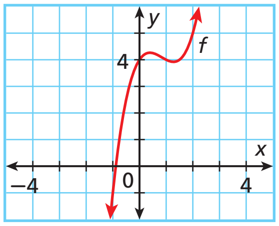
13. $f(x) = -x^2 + x$

14. $f(x) = -2x^2 + 1$

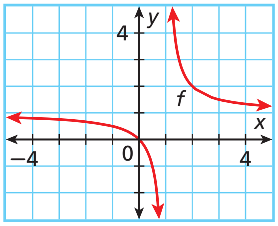
15.



16.



17.



Graph each function.

18.

2003 Federal Income Tax Rates					
Income (\$)	25,000	50,000	75,000	100,000	150,000
Tax Rate (%)	15	25	28	28	33

19. $f(x) = \sqrt{x}$ for $x \geq 0$

20. $f(x) = \frac{1}{2}x + 1$ for $-6 < x < 6$

Recreation

At a depth of 33 feet, a scuba diver is exposed to approximately twice the pressure he or she would experience at the surface.

21. **Safety** In a certain county, the fines for speeding in a school zone are \$160 plus an additional \$4 for every mile per hour over the speed limit. Write a function to represent the speeding fines. What is the value of the function for an input of 8, and what does it represent?

22. **Recreation** In order to scuba dive safely, divers must be aware that the water pressure in the ocean is a function of depth. The water pressure increases by 0.445 pounds per square inch (psi) for each foot of depth. The pressure at the surface is 14.7 psi. Write a function to represent water pressure. What is the value of the function for an input of 50, and what does it represent?

A set of input values is sometimes referred to as the *replacement set* for the independent variable. Evaluate each function for the given replacement set.

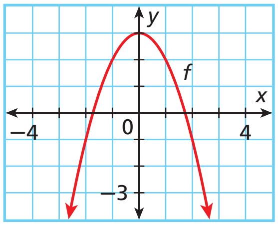
23. $f(x) = 3x - 6; \left\{-3.5, -1, \frac{1}{4}, 2, 11\right\}$

24. $f(x) = x(1 - 2x); \left\{-8, \frac{2}{3}, 1, 9, 4\right\}$

25. $f(x) = \frac{2x - 1}{3}; \left\{-4, 0, \frac{1}{2}, 5\right\}$

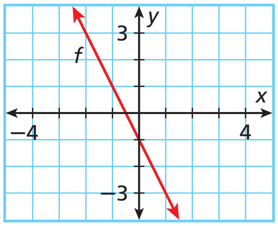
26. $f(x) = (x - 1)^2 + 4; \left\{-6, -\frac{3}{2}, 1, 4\right\}$

27.



$\left\{-2, -1, 1, 2\right\}$

28.



$\left\{-\frac{3}{2}, -1, 0, \frac{1}{2}\right\}$

Explain what a reasonable domain and range would be for each situation. Then explain why the situation represents a function.

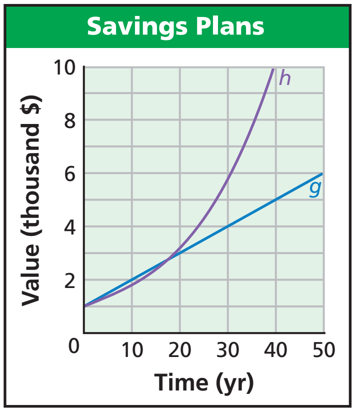
29. the number of boxes of kitchen tile that must be purchased to cover a floor with an area of *A* square feet

30. the number of horseshoes needed to shoe *h* horses

31. the vertical position of a diver in relation to the surface of the pool *t* seconds after diving from a 10-meter platform into a 16-foot-deep pool (*Hint:* 1 meter ≈ 3.28 feet)

32. the temperature in degrees Fahrenheit at an Antarctic research station *h* hours after 12:00 A.M.

Banking **The graph at right shows the functions that represent two different savings plans. Use the graph for Exercises 33–37.**



33. If *t* represents time in years, for what value of *t* will *h* have a value of \$7500? What does this value of *t* represent?

34. Use function notation to represent the value of each savings plan at 25 years. Estimate these values.

35. For what value of *t* is $g(t)$ approximately equal to $\frac{1}{2}h(t)$? Explain what this value of *t* represents.

36. Approximately how many years will it take for each savings plan to double from its original value?

37. What is the value of $h(40) - g(40)$, and what is its real-world meaning?

CONCEPT CONNECTION

38. This problem will prepare you for the Concept Connection on page 74.

a. The function $c(p) = 175 + 3.5p$ can be used to define the cost of producing up to 200 ceramic pots. If the materials are \$175 and the additional cost to produce each pot is \$3.50, how much will it cost to produce 125 pots?

b. How many pots can be produced if the budget is limited to \$450?

c. What would the graph of this function look like?

Critical Thinking For Exercises 39–41, explain why $-5 < x < 5$ is not a reasonable domain for each function.

39. $f(x) = \dfrac{1}{x-3}$ **40.** $g(x) = \sqrt{x-1}$

41. $f(x)$ is the distance traveled in x hours at a rate of 55 mi/h.

42. If $f(2) = 8$ and $f(3) = 11$, name two points that lie on the graph of f.

Identify the independent and dependent variable for each situation. Then state a reasonable domain.

43. As long as a minimum of 15 shirts are ordered, the cost for an order of T-shirts is \$4.25 per shirt.

44. Belinda's medical insurance states that she must pay the first \$500 for a hospital stay plus 15% of the remaining charges.

Write a function to represent each situation. Graph your function.

45. The price for a tank of gasoline is \$2.37 per gallon.

46. Raul earns \$7.50 per hour for baby-sitting.

47. The sale price is 20% off of the original price.

48. Leona's weekly salary is \$250 plus 5% of her total sales for the week.

49. **Write About It** Explain what is meant by reasonable domain and range. Give examples.

STANDARDIZED TEST PREP

50. If $f(x) = 5 - 3x$ and $g(x) = 12x + 2$, which statement is NOT true?

Ⓐ $f(0) > g(0)$ Ⓑ $g(5) > f(5)$ Ⓒ $f(1) > g(1)$ Ⓓ $f(-1) > g(-1)$

51. The function $h(t) = 20t - 5t^2$ gives the height of an object t seconds after it has been thrown into the air. Which statement is true?

Ⓕ The height at 4 seconds is the same as the height at 2 seconds.

Ⓖ The height at 2 seconds is less than the height at 3 seconds.

Ⓗ The height at 3 seconds is the same as the height at 1 second.

Ⓙ The height at 4 seconds is greater than the height at 1 second.

52. A function is described by the equation $f(x) = -3x^2 + 12$. If the replacement set for the independent variable is $\{1, 3, 4, 9, 10\}$, which is an element of the corresponding set for the dependent variable?

Ⓐ 1 Ⓑ 3 Ⓒ 4 Ⓓ 9

53. **Gridded Response** Given $f(x) = 3(x - 2)^2 + 4$, find $f(-1)$.

CHALLENGE AND EXTEND

Determine each value for the given function. Simplify your answer.

54. $f(2c)$ for $f(x) = \sqrt{x^3}$

55. $g\left(-\frac{h}{4}\right)$ for $g(x) = \frac{6x + h}{2x}$, where $h \neq 0$

56. $h(t^2 + 3t)$ for $h(x) = 4x + 7t$

57. $r(t^4)$ for $r(x) = \sqrt{x^2 + \left(\frac{2}{x}\right)^2}$

58. **Geometry** The area of a triangle is $\frac{1}{2}$ the product of its base length b and its height h.

- **a.** If $b = 4$, explain whether the equation for the area of a triangle represents a function.
- **b.** Explain whether the equation that represents the area of a triangle is a function for the domain $\{(b, h) \mid b > 0 \text{ and } h > 0\}$.

SPIRAL REVIEW

Simplify each expression. Assume all variables are nonzero. *(Lesson 1-4)*

59. $4(x + 2) - x(y - 8)$

60. $(2a)^2 + 6a^2$

61. $\frac{3c - 10 + 2c}{5c}$

62. $s(s + 7) - 4s$

Name the conditions for b that would make each set of ordered pairs a function. *(Lesson 1-6)*

63. $\{(1, 2), (6, 0), (0, 1), (-8, b)\}$

64. $\{(b, 2), (0, 3), (5, 4), (-3, 5)\}$

Determine whether each relation is a function. *(Lesson 1-6)*

65. $\left\{(-1, -5), (-2, 0.5), (-4, 5), \left(-5, \frac{1}{2}\right)\right\}$

66. $\{(-1, 3), (-1, 4), (-1, 5), (-1, 6)\}$

Career Path

go.hrw.com
Career Resources Online
KEYWORD: MB7 Career

Adam Leung
Radio announcer

Q: What math classes did you take in high school?

A: I took Algebra 1, Geometry, and Algebra 2.

Q: What are some of your duties as an announcer?

A: During the day, I read the traffic reports and the news in addition to playing music. On weekends, I have more freedom to play music and take calls from listeners.

Q: How is math used in your job?

A: I've got to be sure all the scheduled music, ads, news, and traffic reports are covered in my shift. I use math to calculate how much time I need. I also use math to help create contests.

Q: What are your plans for the future?

A: I'll probably continue to work as an announcer for a while. After that, I'd like to become a station manager or a radio engineer. Engineers are responsible for making sure all the equipment at the station works properly.

1-8

Exploring Transformations

Objectives

Apply transformations to points and sets of points.

Interpret transformations of real-world data.

Vocabulary

transformation
translation
reflection
stretch
compression

Why learn this?

Changes in recording studio fees can be modeled by transformations. (See Example 4.)

A **transformation** is a change in the position, size, or shape of a figure. A **translation**, or slide, is a transformation that moves each point in a figure the same distance in the same direction.

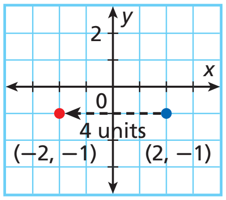
California Standards

Preparation for 9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation $y = a(x - b)^2 + c$.

EXAMPLE 1 Translating Points

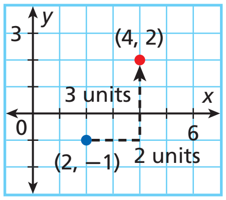
Perform the given translation on the point $(2, -1)$. Give the coordinates of the translated point.

A 4 units left



Translating $(2, -1)$ 4 units left results in the point $(-2, -1)$.

B 2 units right and 3 units up



Translating $(2, -1)$ 2 units right and 3 units up results in the point $(4, 2)$.

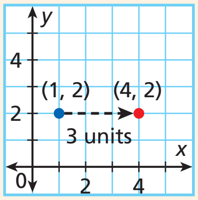
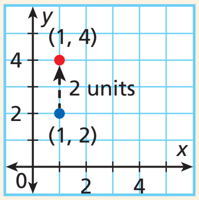
CHECK IT OUT! **Perform the given translation on the point $(-1, 3)$. Give the coordinates of the translated point.**

1a. 4 units right

1b. 1 unit left and 2 units down

Notice that when you translate **left or right,** the ***x*-coordinate** changes, and when you translate **up or down,** the ***y*-coordinate** changes.

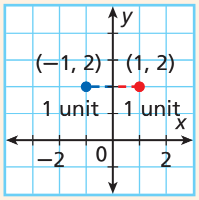
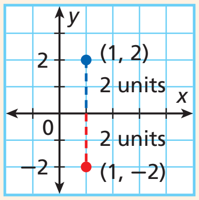
Know it! Note

Translations	
Horizontal Translation	**Vertical Translation**
Each point shifts *right* or *left* by a number of units.	Each point shifts *up* or *down* by a number of units.
 y 4 (1, 2) (4, 2) 2 3 units x 0 2 4	 y (1, 4) 4 2 units 2 (1, 2) x 0 2 4
The ***x***-coordinate changes. $(1, 2) \to (1 + 3, 2)$ $(x, y) \to (x + h, y)$	The ***y***-coordinate changes. $(1, 2) \to (1, 2 + 2)$ $(x, y) \to (x, y + k)$
left if $h < 0$ right if $h > 0$	down if $k < 0$ up if $k > 0$

A **reflection** is a transformation that flips a figure across a line called the line of reflection. Each reflected point is the same distance from the line of reflection, but on the opposite side of the line.

Know it! Note

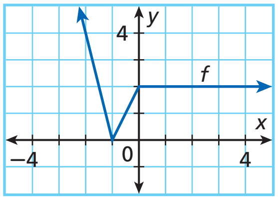
Reflections

Reflection Across y-axis	Reflection Across x-axis
Each point flips across the y-axis.	Each point flips across the x-axis.
 The x-coordinate changes. $(1, 2) \rightarrow (-1, 2)$ $(x, y) \rightarrow (-x, y)$	 The y-coordinate changes. $(1, 2) \rightarrow (1, -2)$ $(x, y) \rightarrow (x, -y)$

You can transform a function by transforming its ordered pairs. When a function is translated or reflected, the original graph and the graph of the transformation are *congruent* because the size and shape of the graphs are the same.

EXAMPLE 2 Translating and Reflecting Functions

Use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.

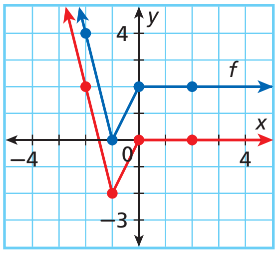


A translation 2 units down

Identify important points from the graph and make a table.

x	y	$y - 2$
−2	4	$4 - 2 = 2$
−1	0	$0 - 2 = -2$
0	2	$2 - 2 = 0$
2	2	$2 - 2 = 0$

The entire graph shifts 2 units down. Subtract 2 from each y-coordinate.



Helpful Hint

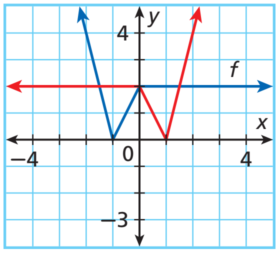
Transform x by adding a table column on the left side; transform y by adding a column on the right side.

B reflection across y-axis

Identify important points from the graph and make a table.

$-x$	x	y
$-1(-2) = 2$	−2	4
$-1(-1) = 1$	−1	0
$-1(0) = 0$	0	2
$-1(2) = -2$	2	2

Multiply each x-coordinate by −1. The entire graph flips across the y-axis.



CHECK IT OUT!

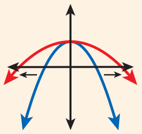
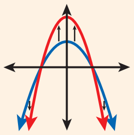
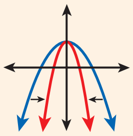
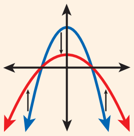
For the function from Example 2, use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.

2a. translation 3 units right **2b.** reflection across x-axis

Imagine grasping two points on the graph of a function that lie on opposite sides of the y-axis. If you pull the points away from the y-axis, you would create a horizontal **stretch** of the graph. If you push the points towards the y-axis, you would create a horizontal **compression**.

Stretches and compressions are not congruent to the original graph.

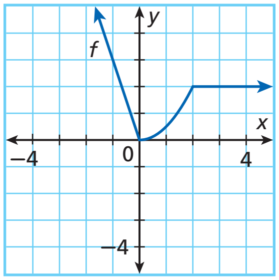
Stretches and Compressions

	Horizontal	Vertical
Stretch	Each point is *pulled away* from the y-axis.  The x-coordinate changes. $(4, 0) \to (2(4), 0)$ $(x, y) \to (bx, y)$ $\lvert b \rvert > 1$	Each point is *pulled away* from the x-axis.  The y-coordinate changes. $(0, 4) \to (0, 2(4))$ $(x, y) \to (x, ay)$ $\lvert a \rvert > 1$
Compression	Each point is *pushed toward* the y-axis.  The x-coordinate changes. $(4, 0) \to \left(\frac{1}{2}(4), 0\right)$ $(x, y) \to (bx, y)$ $0 < \lvert b \rvert < 1$	Each point is *pushed toward* the x-axis.  The y-coordinate changes. $(0, 4) \to \left(0, \frac{1}{2}(4)\right)$ $(x, y) \to (x, ay)$ $0 < \lvert a \rvert < 1$

EXAMPLE 3

Stretching and Compressing Functions

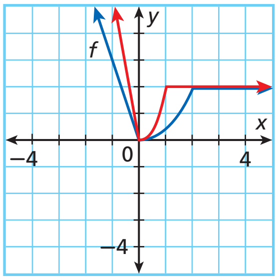
Use a table to perform a horizontal compression of $y = f(x)$ by a factor of $\frac{1}{2}$. Use the same coordinate plane as the original function.



Identify important points from the graph and make a table.

$\frac{1}{2}x$	x	y
$\frac{1}{2}(-1) = -\frac{1}{2}$	-1	3
$\frac{1}{2}(0) = 0$	0	0
$\frac{1}{2}(2) = 1$	2	2
$\frac{1}{2}(4) = 2$	4	2

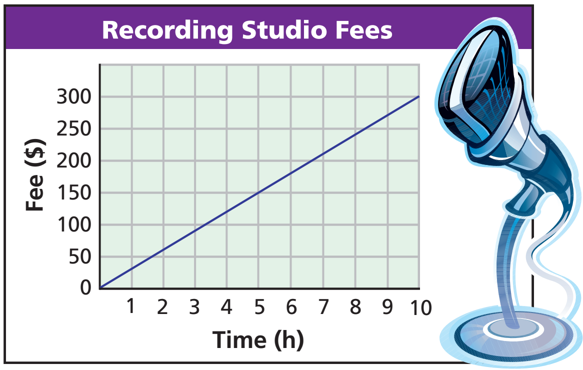
Multiply each x-coordinate by $\frac{1}{2}$.



3. For the function from Example 3, use a table to perform a vertical stretch of $y = f(x)$ by a factor of 2. Graph the transformed function on the same coordinate plane as the original function.

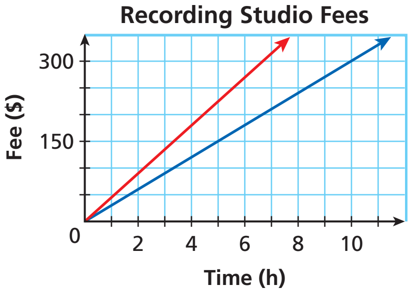
EXAMPLE 4 *Business Application*

Recording studio fees are usually based on an hourly rate, but the rate can be modified due to various options. The graph shows a basic hourly studio rate. Sketch a graph to represent each situation below and identify the transformation of the original graph that it represents.



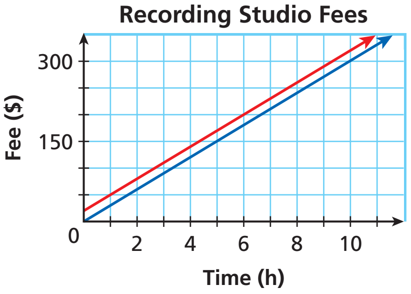
A **The engineer's time is needed, so the hourly rate is 1.5 times the original rate.**

If the fees are 1.5 times the basic hourly rate, the value of each *y*-coordinate would be multiplied by 1.5. This represents a vertical stretch by a factor of 1.5.



B **A $20 setup fee is added to the basic hourly rate.**

If the prices are $20 more than the original estimate, the value of each *y*-coordinate would increase by 20. This represents a vertical translation up 20 units.



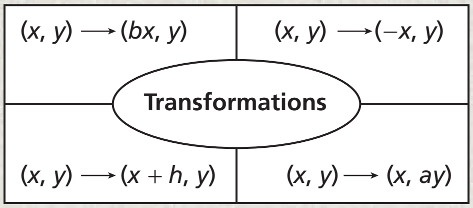
CHECK IT OUT! **4. What if...?** Suppose that a discounted rate is $\frac{3}{4}$ of the original rate. Sketch a graph to represent the situation and identify the transformation of the original graph that it represents.

THINK AND DISCUSS

1. Describe two ways to transform (4, 2) to (2, 2).

2. Compare a vertical stretch with a horizontal compression.

Know it! Note **3. GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the transformations indicated by the given rule.



1-8 Exercises

California Standards
Preparation for 🔑 9.0 and 24.0

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KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** A transformation that pushes a graph toward the *x*-axis is a __?__. (*reflection* or *compression*)

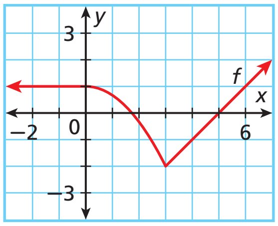
SEE EXAMPLE 1
p. 59

Perform the given translation on the point $(4, 2)$ and give the coordinates of the translated point.

2. 5 units left
3. 3 units down
4. 1 unit right, 6 units up

SEE EXAMPLE 2
p. 60

Use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.



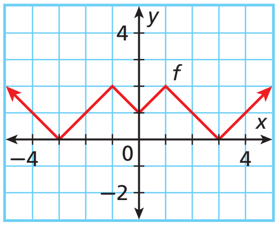
5. translation 2 units up

6. reflection across the *y*-axis

7. reflection across the *x*-axis

SEE EXAMPLE 3
p. 61

Use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.



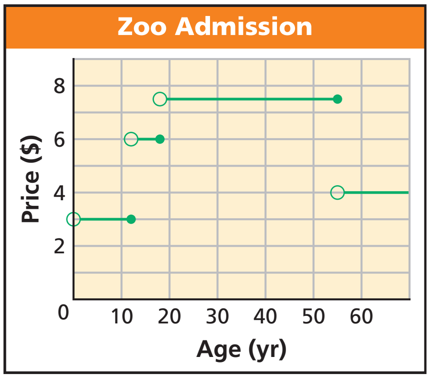
8. horizontal stretch by a factor of 3

9. vertical stretch by a factor of 3

10. vertical compression by a factor of $\frac{1}{3}$

SEE EXAMPLE 4
p. 62

Recreation **The graph shows the price for admission by age at a local zoo. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.**



11. Admission is half price on Wednesdays.

12. To raise funds for endangered species, the zoo charges $1.50 extra per ticket.

13. The maximum age for each ticket price is increased by 5 years.

PRACTICE AND PROBLEM SOLVING

Independent Practice

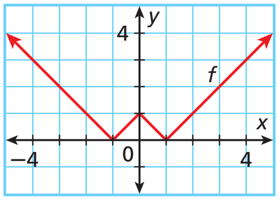
For Exercises	See Example
14–16	1
17–20	2
21–24	3
25–27	4

Extra Practice
Skills Practice p. S5
Application Practice p. S32

Perform the given translation on $(3, 1)$. Give the coordinates of the translated point.

14. 2 units right
15. 4 units up
16. 5 units left, 4 units down

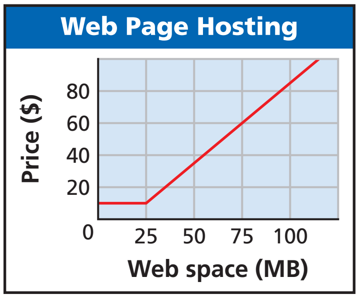
Use a table to perform each transformation of $y = f(x)$. Use the same coordinate plane as the original function.



17. translation 2 units down
18. reflection across the *x*-axis
19. translation 3 units right
20. reflection across the *y*-axis

21. vertical compression by a factor of $\frac{2}{3}$
22. horizontal compression by a factor of $\frac{1}{2}$
23. horizontal stretch by a factor of $\frac{3}{2}$
24. vertical stretch by a factor of 2

Technology **The graph shows the cost of Web page hosting depending on the Web space used. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.**



25. The prices are reduced by \$5.

26. The prices are discounted by 25%.

27. A special is offered for double the amount of Web space for the same price.

Estimation **The table gives the coordinates for the vertices of a triangle. Estimate the area of each transformed triangle by graphing it and counting the number of squares it covers on the coordinate plane. How does the area of each transformed triangle compare with the area of the original triangle?**

x	y
−2	2
2	−4
4	−2

28. reflection across the y-axis

29. 5 units left, 3 units up

30. horizontal stretch by a factor of 2

31. horizontal compression by a factor of $\frac{2}{3}$

32. vertical compression by a factor of $\frac{2}{3}$

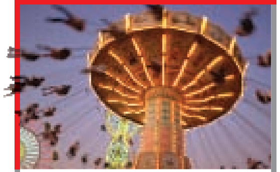
33. reflection across the x-axis

34. 1 unit left, 6 units down

35. vertical stretch by a factor of 3

LINK

Entertainment



The amusement park industry in the United States includes about 700 parks and accounted for over \$8.5 billion in revenues in 2001.

Source: Statistical Abstract of the United States

36. Entertainment The revenue from an amusement park ride is given by the admission price of \$3 times the number of riders. As part of a promotion, the first 10 riders ride for free.

a. What kind of transformation describes the change in the revenue based on the promotion?

b. Write a function rule for this transformation.

37. Business An automotive mechanic charges \$50 to diagnose the problem in a vehicle and \$65 per hour for labor to fix it.

a. If the mechanic increases his diagnostic fee to \$60, what kind of transformation is this to the graph of the total repair bill?

b. If the mechanic increases his labor rate to \$75 per hour, what kind of transformation is this to the graph of the total repair bill?

c. If it took 3 hours to repair your car, which of the two rate increases would have a greater effect on your total bill?

CONCEPT CONNECTION

38. This problem will prepare you for the Concept Connection on page 74.

The student council wants to buy vases for the flowers for the school prom. A florist charges a \$20 delivery fee plus \$1.25 per vase. A home-decorating store charges a \$10 delivery fee plus \$1.25 per vase.

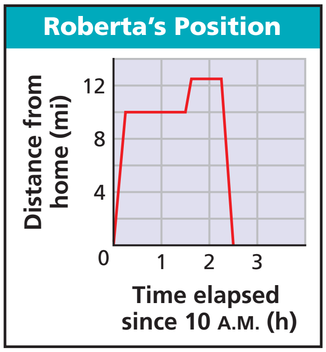
a. The function $f(x) = 20 + 1.25x$ models the cost of ordering x vases from the florist, and the function $g(x) = 10 + 1.25x$ models the cost of ordering x vases from the home-decorating store. What do the graphs of these functions look like?

b. How are the graphs related to each other?

c. How could you modify these functions so that their graphs are identical?

d. If the florist decided to waive the \$20 delivery fee as long as the number of vases ordered was more than 150, how would the graph of f change? How would it compare with the graph of the other function?

Transportation **Use the graph and the following information for Exercises 39–43.**



Roberta left her house at 10:00 A.M. and drove to the library. She was at the library studying until 11:30 A.M. Then she drove to the grocery store. At 12:15 P.M. Roberta left the grocery store and drove home. The graph shows Roberta's position with respect to time.

Sketch a graph to reflect each change to the original story. Assume the time Roberta spends inside each building remains the same.

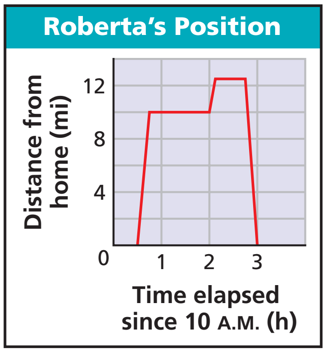
39. Roberta drove at half the speed from her house to the library.

40. The grocery store she went to is twice as far from the library.

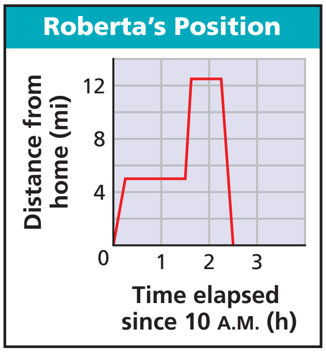
41. The grocery store is 2.5 miles closer to the house than the library is.

Change the original story about Roberta to match each graph.

42.



43.



44. **Critical Thinking** Suppose two transformations are performed on a single point: a translation and a reflection. Does the order in which the transformations are performed make a difference? Does the type of translation or reflection matter? Explain your reasoning.

45. **Write About It** Describe how transformations might make graphing easier.

STANDARDIZED TEST PREP

46. The function $c(p) = 0.99p$ represents the cost in dollars of p pounds of peaches. If the cost per pound increases by 10%, how will the graph of the function change?

Ⓐ Translation 0.1 unit up

Ⓑ Translation 0.1 unit right

Ⓒ Horizontal stretch by a factor of 1.1

Ⓓ Vertical stretch by a factor of 1.1

47. Which transformation would change the point $(5, 3)$ into $(-5, 3)$?

Ⓕ Reflection across the x-axis

Ⓖ Translation 5 units down

Ⓗ Reflection across the y-axis

Ⓙ Translation 5 units left

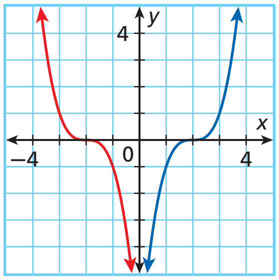
48. The graph of the function f is a line that intersects the y-axis at the point $(0, 3)$ and the x-axis at the point $(3, 0)$. Which transformation of f does NOT intersect the y-axis at the point $(0, 6)$?

Ⓐ Translation 3 units up

Ⓑ Translation 3 units right

Ⓒ Vertical stretch by a factor of 2

Ⓓ Horizontal compression by a factor of $\frac{1}{2}$



49. Which transformation is displayed in the graph?

- Ⓕ Reflection across the x-axis
- Ⓖ Translation 5 units down
- Ⓗ Reflection across the y-axis
- Ⓙ Translation 5 units left

50. Which represents a translation 4 units right and 2 units down?

- Ⓐ From $(4, 2)$ to $(0, 0)$
- Ⓑ From $(4, -2)$ to $(0, 0)$
- Ⓒ From $(-4, -2)$ to $(0, 0)$
- Ⓓ From $(-4, 2)$ to $(0, 0)$

51. **Short Response** Graph the points $(-1, 3)$ and $(-1, -3)$. Describe two different transformations that would transform $(-1, 3)$ to $(-1, -3)$.

CHALLENGE AND EXTEND

52. Suppose the rule $(x, y) \rightarrow (2x, y - 3)$ is used to translate a point. If the coordinates of the translated point are $(22, 7)$, what was the original point?

53. **History** From 1999 to 2001 the cost for mailing n first class letters through the United States Postal Service was $c(n) = 0.33n$. In 2001 the rate was increased by \$0.01 per letter. In 2002 the rate was increased an additional \$0.03 per letter.

- **a.** Write an equation that represents the cost of mailing n first class letters in 2002.
- **b.** What transformation describes the total change in price?
- **c.** Graph both functions and estimate the maximum number of first class letters you could mail for \$5.00 in both 1999 and 2002.
- **d.** Explain the effect of the reasonable domain and range for these functions on your answer for part **c**.

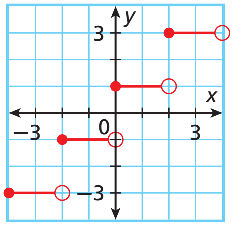
54. Name a point that when reflected across the x-axis has the same coordinates as if it were reflected across the y-axis. How many points are there that satisfy this condition?

SPIRAL REVIEW

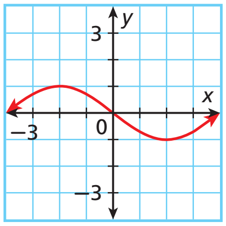
55. **Sports** Katrina's mean bowling score for three games was 144. If the score of her first game was 172 and the score of her second game was 150, what was the score of her third game? *(Previous course)*

Use the vertical-line test to determine whether each relation is a function. *(Lesson 1-6)*

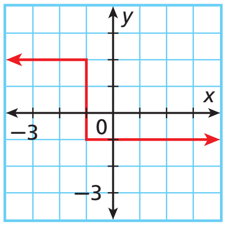
56.



57.



58.



For each function evaluate $f(1)$, $f(-3)$, and $f\left(\frac{1}{4}\right)$. *(Lesson 1-7)*

59. $f(x) = \dfrac{4x - 5}{2}$

60. $f(x) = 2x^3$

61. $f(x) = (1 - x^2)^2$

1-9

Introduction to Parent Functions

Objectives

Identify parent functions from graphs and equations.

Use parent functions to model real-world data and make estimates for unknown values.

Vocabulary

parent function

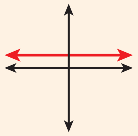
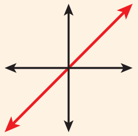
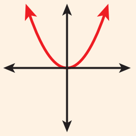
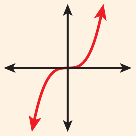
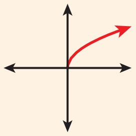
Who uses this?

Oceanographers use transformations of parent functions to approximate data sets such as wave height versus wind speed. (See Example 3.)

Similar to the way that numbers are classified into sets based on common characteristics, functions can be classified into *families of functions*. The **parent function** is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent function.

Know it! Note

Parent Functions

Family	Constant	Linear	Quadratic	Cubic	Square root
Rule	$f(x) = c$	$f(x) = x$	$f(x) = x^2$	$f(x) = x^3$	$f(x) = \sqrt{x}$
Graph					
Domain	ℝ	ℝ	ℝ	ℝ	$x \geq 0$
Range	$y = c$	ℝ	$y \geq 0$	ℝ	$y \geq 0$
Intersects *y*-axis	$(0, c)$	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$

California Standards

Preparation for 10.0 Students graph quadratic functions and determine the maxima, minima, and zeros of the function.

Also covered: **Preparation for 9.0 and 24.0**

EXAMPLE 1

Identifying Transformations of Parent Functions

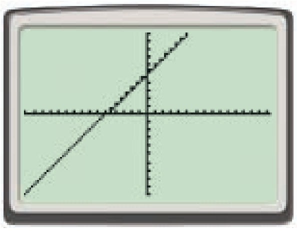
Identify the parent function for *g* from its function rule. Then graph *g* on your calculator and describe what transformation of the parent function it represents.

A $g(x) = x + 5$

$g(x) = x + 5$ is linear. *x has a power of 1.*

The linear parent function $f(x) = x$ intersects the y-axis at the point $(0, 0)$.

Graph $\mathbf{Y_1 = X + 5}$ on a graphing calculator. The function $g(x) = x + 5$ intersects the y-axis at the point $(0, 5)$.



So $g(x) = x + 5$ represents a vertical translation of the linear parent function 5 units up.

Helpful Hint

To make graphs appear accurate on a graphing calculator, use the standard square window. Press ZOOM, choose **6:ZStandard**, press ZOOM again, and choose **5:ZSquare.**

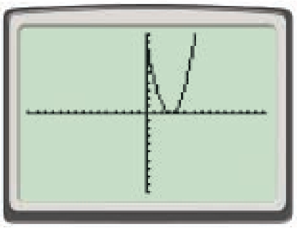
Identify the parent function for g from its function rule. Then graph g on your calculator and describe what transformation of the parent function it represents.

B $g(x) = (x - 3)^2$

$g(x) = (x - 3)^2$ is quadratic. *$x - 3$ has a power of 2.*

The quadratic parent function $f(x) = x^2$ intersects the x-axis at the point $(0, 0)$.

Graph $\mathbf{Y_1 = (X - 3)^2}$ on a graphing calculator. The function $g(x) = (x - 3)^2$ intersects the x-axis at the point $(3, 0)$.



So $g(x) = (x - 3)^2$ represents a horizontal translation of the quadratic parent function 3 units right.

CHECK IT OUT! **Identify the parent function for g from its function rule. Then graph g on your calculator and describe what transformation of the parent function it represents.**

1a. $g(x) = x^3 + 2$

1b. $g(x) = (-x)^2$

It is often necessary to work with a set of data points like the ones represented by the table at right.

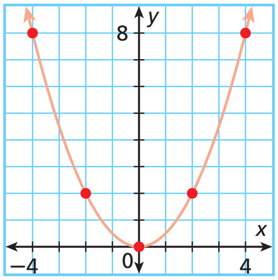
x	−4	−2	0	2	4
y	8	2	0	2	8

With only the information in the table, it is impossible to know the exact behavior of the data between and beyond the given points. However, a working knowledge of the parent functions can allow you to sketch a curve to approximate those values not found in the table.

EXAMPLE 2 Identifying Parent Functions to Model Data Sets

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

x	−4	−2	0	2	4
y	8	2	0	2	8



The graph of the data points resembles the shape of the quadratic parent function $f(x) = x^2$.

The quadratic parent function passes through the points $(2, 4)$ and $(4, 16)$. The data set contains the points

$$(2, 2) = \left(2, \frac{1}{2}(4)\right) \text{ and } (4, 8) = \left(2, \frac{1}{2}(16)\right).$$

The data set seems to represent a vertical compression of the quadratic parent function by a factor of $\frac{1}{2}$.

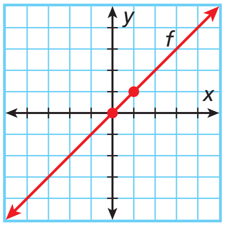
CHECK IT OUT! **2. Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.**

x	−4	−2	0	2	4
y	−12	−6	0	6	12

Consider the two data points $(0, 0)$ and $(1, 1)$. If you plot them on a coordinate plane you might very well think that they are part of a linear function. In fact they belong to each of the parent functions below.

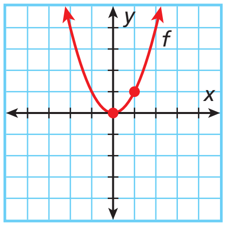
Linear

$f(x) = x$



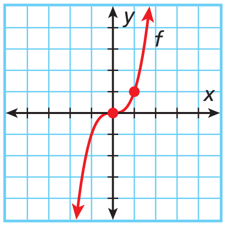
Quadratic

$f(x) = x^2$



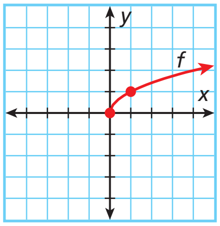
Cubic

$f(x) = x^3$



Square Root

$f(x) = \sqrt{x}$



Remember that any parent function you use to approximate a set of data should never be considered exact. However, these function approximations are often useful for estimating unknown values.

Helpful Hint

A greater number of data points increases your chances of correctly identifying the parent function that best describes the data.

EXAMPLE 3 *Oceanography Application*

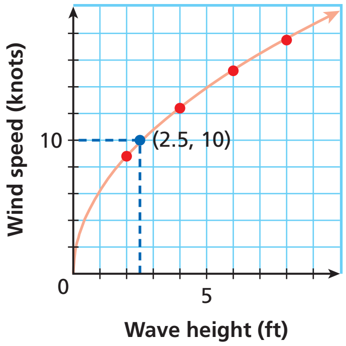
An oceanographer wants to determine a model that can be used to estimate wind speed based upon wave height. Graph the relationship from wave height to wind speed and identify which parent function best describes it. Then use the graph to estimate the wave height when the wind speed is 10 knots.

Ocean Waves

Wave Height (ft)	Wind Speed (knots)
2	8.8
4	12.4
6	15.2
8	17.5
10	19.6

Step 1 Graph the relation.

Graph the points given in the table. Draw a smooth curve through them to help you see the shape.



Step 2 Identify the parent function.

The graph of the data set resembles the shape of the square-root parent function $f(x) = \sqrt{x}$.

Step 3 Estimate the wave height when the wind speed is 10 knots.

The curve indicates that a wind speed of 10 knots would create a wave that is approximately 2.5 feet high.

CHECK IT OUT!

3. The cost of playing an online video game depends on the number of months for which the online service is used. Graph the relationship from number of months to cost, and identify which parent function best describes the data. Then use the graph to estimate the cost for 5 months of online service.

Cost of Online Video Game

Time (mo)	1	3	6	9	12
Cost ($)	40	56	80	104	128

THINK AND DISCUSS

1. Explain how to determine the parent function for a given equation.
2. Explain why recognizing parent functions is useful for graphing.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give the appropriate information for a translation of the parent function 3 units up.

Know it! Note

Transformed Parent Functions

Family	Linear	Quadratic	Square root
Rule			
Graph			
Domain			
Range			
Intersects *y*-axis			

1-9 Exercises

California Standards

Preparation for 🔑 9.0, 🔑 10.0, 🔑 12.0, and 24.0

go.hrw.com

Homework Help Online

KEYWORD: MB7 1-9

Parent Resources Online

KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain how transformations, families of functions, and *parent functions* are related.

SEE EXAMPLE 1 p. 67

Identify the parent function for *g* from its function rule. Then graph *g* on your calculator and describe what transformation of the parent function it represents.

2. $g(x) = (x - 1)^3$
3. $g(x) = (x + 1)^2$
4. $g(x) = -x$
5. $g(x) = \sqrt{x + 3}$
6. $g(x) = x^2 + 4$
7. $g(x) = x - \sqrt{2}$

SEE EXAMPLE 2 p. 68

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

8.

x	−3	−1	0	1	3
y	−15	−5	0	5	15

9.

x	−3	−1	0	1	3
y	−1	$-\frac{1}{27}$	0	$\frac{1}{27}$	1

SEE EXAMPLE 3 p. 69

10. **Physics** The time it takes a pendulum to make one complete swing back and forth depends on its string length.
    a. Graph the relationship from string length to time.
    b. Identify which parent function best describes the data.
    c. Use your graph to estimate the string length of a pendulum that takes 4.5 seconds to make one complete swing.
    d. Use your graph to estimate the time it takes to make a complete swing for a string of length 14 meters.

Pendulum Swing

String Length (m)	Time (s)
2	2.8
4	4.0
6	4.9
8	5.7
10	6.3

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
11–13	1
14–15	2
16	3

Extra Practice

Skills Practice p. S5

Application Practice p. S32

Identify the parent function for g from its function rule. Then graph g on your calculator and describe what transformation of the parent function it represents.

11. $g(x) = x^2 - 1$ **12.** $g(x) = \sqrt{x - 2}$ **13.** $g(x) = x^3 + 3$

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

14.

x	-3	-1	0	1	3
y	3	$\frac{1}{3}$	0	$\frac{1}{3}$	3

15.

x	0	1	4	9	16
y	0	2	4	6	8

16. Geometry The number of segments required to connect a given number of points is shown in the table.

a. Graph the relationship from the number of points to the number of segments.

b. Identify which parent function best describes the data.

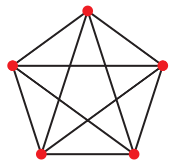
c. Use your graph to estimate the number of points if there are 45 segments.

d. Use your graph to estimate the number of segments if there are 7 points.

Connecting Points

Number of Points	2	5	8	11
Number of Segments	1	10	28	55

2 points
1 segment



5 points
10 segments

Graphing Calculator **Graph each function with a graphing calculator. Identify the domain and range of the function, and describe the transformation from its parent function.**

17. $g(x) = 3\sqrt{x}$ **18.** $g(x) = \frac{2}{3}x$ **19.** $g(x) = -\sqrt{x}$

20. $g(x) = -(x - 2)^2$ **21.** $g(x) = -x^2 + 1$ **22.** $g(x) = -\frac{1}{2}x^3$

23. Sports Based on the information in the table, what is the total cost of 15 tickets to the hockey game? Explain how you determined your answer.

Hockey Tickets

Number of Tickets	1	5	8	12
Total Cost ($)	13	65	104	156

Graph each function. Identify the parent function that best describes the set of points, and describe the transformation from the parent function.

24. $\{(-2, 8), (-1, 1), (0, 0), (1, -1), (2, -8)\}$ **25.** $\{(5, 4), (7, 0), (9, 4), (10, 9), (11, 16)\}$

26. $\{(0, 0), (-1, 1), (-4, 2), (-9, 3), (-16, 4)\}$ **27.** $\{(-4, 3), (-2, 1), (0, -1), (2, -3), (4, -5)\}$

CONCEPT CONNECTION

28. This problem will prepare you for the Concept Connection on page 74.

a. One function used in the Multi-Step Test Prep in Lesson 1-8 was $f(x) = 20 + 1.25x$. What is its parent function?

b. The graph for a given function has a U shape. What could be the parent function?

c. Plot the data set $\{(0, 0), (1, 2), (4, 4), (9, 6), (16, 8), (25, 10)\}$. Which parent function best models the data set?

Photography **When resizing a digital photo, it is often important to preserve its *aspect ratio*, the ratio of its width to its height. Use the table for Exercises 29–31.**

Digital Photos with Aspect Ratio 3:2

Width (pixels)	Height (pixels)	File Size (KB)
640	427	220
800	533	254
1024	683	413
1280	853	750

29. Graph the relationship from width to height and identify which parent function best describes the data. Use the graph to estimate the width of a photo with a height of 1000 pixels.

30. Graph the relationship from height to width and identify which parent function best describes the data. Use the graph to estimate the height of a photo with a width of 500 pixels.

31. Resizing a photo changes the file size. Graph the relationship from width to file size and identify which parent function best describes the data. Use the graph to estimate the width of a photo with a file size of 1000 KB.

Sketch a graph for each situation and identify the related parent function. Then explain what the reasonable domain and range for the function is and compare it with the domain and range of the parent function.

32. distance traveled after h hours at a speed of 55 mi/h

33. volume of a cube with side length ℓ

34. area of a room with width w and a length of 15 feet

35. cost to wash n loads of laundry at $1.00 per load

36. cost of an item with original price p after a 15% discount

37. side length of a square with area A

LINK

Chemistry

Aerogel has been called the world's lowest density solid. It is 99.8% air and is an excellent heat insulator. As shown above, a layer of aerogel can prevent a flame from melting crayons.

38. **Chemistry** The table shows properties of aerogel. Graph the relationship from mass to volume, and then estimate the volume of 1 gram of aerogel.

Aerogel Properties

Mass (mg)	30	90	300	450
Volume (cm^3)	10	30	100	150

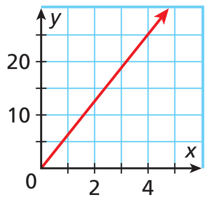
39. **What if...?** Use the set of points $\{(-1, -1), (0, 0), (1, 1)\}$ to answer each question.

- **a.** What parent function best describes the set of points?
- **b.** If the points $(-2, 8)$ and $(2, 8)$ were added, what parent function would best describe the set?
- **c.** If the point $(1, 1)$ were replaced with $(1, -1)$, what parent function would best describe the set?
- **d.** If the point $(-1, -1)$ were replaced with $(4, 2)$, what parent function would best describe the set?
- **e.** **Multi-Step** If the x-coordinate of each point were doubled and 3 were added to each y-coordinate, what parent function would best describe the set? What transformation of the parent function would the set represent?

40. **Critical Thinking** Explain any relationship you have noticed between the quadratic parent function and a function rule that represents a horizontal translation, a vertical translation, or a reflection across the x-axis.

41. **Write About It** Order the parent functions covered in this lesson from least to greatest by the rate at which $f(x)$ increases as x increases for $x > 1$. Explain your answer.

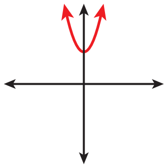
STANDARDIZED TEST PREP



42. Which situation could be represented by the graph?

Ⓐ The area of a circle based on its radius
Ⓑ The volume of a sphere based on its radius
Ⓒ The surface area of a sphere based on its radius
Ⓓ The circumference of a circle based on its radius

43. Which graph best represents the function $f(x) = 2x^2 - 2$?

Ⓕ


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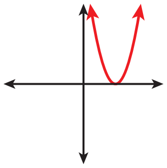

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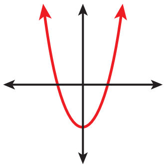

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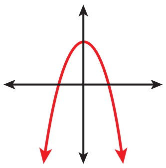


44. Which equation describes a relationship in which every nonzero real number x corresponds to a negative real number y?

Ⓐ $y = -x^3$
Ⓑ $y = -x^2$
Ⓒ $y = (-x)^2$
Ⓓ $y = -x$

45. For which function is -1 NOT an element of the range?

Ⓕ $y = -1$
Ⓖ $y = (-x)^2$
Ⓗ $y = -x$
Ⓙ $y = x^3$

46. What type of function can be used to determine the side length of a square if the independent variable is the square's area?

Ⓐ Cubic
Ⓑ Linear
Ⓒ Quadratic
Ⓓ Square root

CHALLENGE AND EXTEND

Identify the parent function for each function.

47. $g(x) = 3(x-1)^2 - 6$

48. $h(x) = \left(4x^3\right)^0 + 2$

49. $g(x) = 5(3x-2) - 11x$

50. Another parent function is an exponential function of the form $f(x) = a^x$.

- **a.** Graph $f(x) = 2^x$.
- **b.** Find the domain and range of the function.
- **c.** Identify the point where the function crosses the y-axis.
- **d.** Predict where $f(x) = 3^x$ crosses the y-axis and explain your answer.

SPIRAL REVIEW

Simplify each expression. Write each answer in scientific notation. *(Lesson 1-5)*

51. $\left(1.5 \times 10^{-4}\right)\left(5.0 \times 10^{13}\right)$

52. $\left(8.1 \times 10^{3}\right)^2$

53. $\dfrac{1.9 \times 10^{-6}}{9.5 \times 10^{18}}$

Evaluate each function for the given set of input values. *(Lesson 1-7)*

54. $f(x) = \frac{1}{2}x + 3; \left\{-3, 0, \frac{1}{3}, 6\right\}$

55. $f(x) = x(x+2); \left\{-5, -\frac{2}{3}, 1.6, 4\right\}$

Perform each transformation on the point $(3, -5)$. Give the coordinates of the translated point. *(Lesson 1-8)*

56. left 2 , up 6

57. right 1, down 5

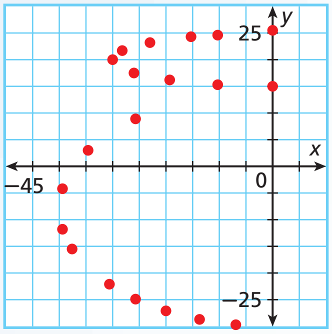
58. reflected across the y-axis

CONCEPT CONNECTION

Introduction to Functions

Native American Art Much of Native American art, in particular Navajo and Cherokee, displays symmetrical designs.

To reproduce these designs, artists can determine the points that make up the designs and transform them. The set of ordered pairs in the table defines the outline of the left side of a handmade Navajo vase. The graph shows the plotted points.



Vase Outline

x	y
0	25.5
−10.3	24.6
−15.3	24.3
−23.0	23.2
−28.2	21.7
−30.0	20.0
−26.0	17.5
−19.3	16.2
−10.3	15.3
0	15.0
−25.7	8.9
−34.6	3.0
−39.4	−4.2
−39.4	−11.8
−37.6	−15.5
−30.6	−22.1
−25.7	−24.9
−20.0	−27.1
−13.7	−28.7
−6.9	−29.7

1. What is the domain of this relation?
2. What is the range of this relation?
3. Is this relation also a function? Explain why or why not.
4. If the coordinates were plotted such that the vase appears to be on its side (that is, the *x*- and *y*-coordinates switched places), would the relation be a function? Explain why or why not.
5. If the vase appears to be on its side, which parent function would best represent the bottom of the vase?
6. What transformation would create the right side of the upright vase?
7. What kind of transformation could be done on the relation in the table to make the vase shorter?
8. What kind of transformation could be done on the relation in the table to make the vase narrower?

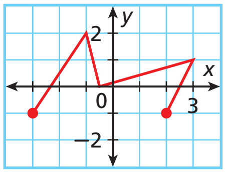
READY TO GO ON?

Quiz for Lessons 1-6 Through 1-9

1-6 Relations and Functions

Give the domain and range for each relation. Then tell whether the relation is a function.

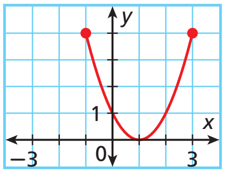
1.



2.

x	0	2	4	6	2
y	5	8	10	20	12

3.



1-7 Function Notation

For each function, evaluate $f(0)$, $f(1)$, and $f(-2)$.

4. $f(x) = 12 - 3x$

5. $f(x) = 3x^3 + 1$

6. $f(x) = 4 - x^2$

7. In a certain city, taxi fares are regulated at \$1.75 per ride plus \$0.25 for each $\frac{1}{4}$ mile.

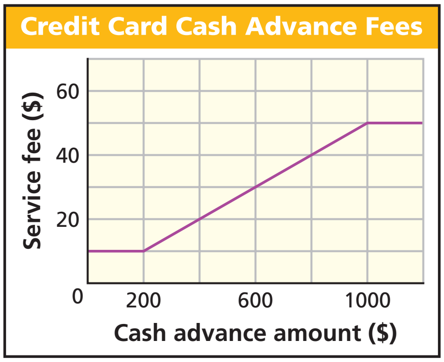
- **a.** Write a function to represent the taxi fare per mile.
- **b.** Graph your function.
- **c.** What is the value of the function for an input of 5.5, and what does it represent?

1-8 Exploring Transformations

The graph shows some credit card fees for cash advances. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.

8. Each fee is increased by \$15.

9. Each fee is decreased by 40%.


Credit Card Cash Advance Fees

1-9 Introduction to Parent Functions

Identify the parent function for *g* from its equation. Then graph *g* on your calculator and describe what transformation of the parent function it represents.

10. $g(x) = -x^2$

11. $g(x) = \sqrt{x - 3}$

12. $g(x) = 1.5x$

13. The table lists the maximum load of a three-strand nylon rope based on its diameter. Graph the relationship from diameter to maximum load and identify which parent function best describes the data. Then use your graph to estimate the diameter of a three-strand nylon rope that has a maximum load of 7920 kilograms.

Nylon Rope Maximum Load

Diameter (mm)	8	10	12	14	16
Maximum Load (kg)	1920	2720	3750	5100	6640

CHAPTER 1

Study Guide: Review

Vocabulary

compression	61
dependent variable	52
domain	44
element	6
empty set	6
finite set	7
function	45
function notation	51
independent variable	52
infinite set	7
interval notation	7
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radicand	21
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relation	44
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scientific notation	36
set	6
set-builder notation	8
stretch	61
subset	6
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Complete the sentence below with vocabulary words from the list above.

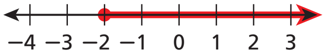
1. For a function, the ___?___ is the set of input values, and the ___?___ is the set of output values.

1-1 Sets of Numbers *(pp. 6–13)*

Prep for 🔑 **1.0** and 🔑 **2.0**

EXAMPLES

Rewrite each set in the indicated notation.

- 

; interval notation

  The interval is the real numbers greater than or equal to -2.

  $[-2, \infty)$ *-2 included, but infinity is not.*

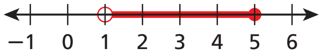
- $(-1, 6)$; set builder notation

  $\{x \mid -1 < x < 6\}$ *Neither endpoint is included.*

EXERCISES

Rewrite each set in the indicated notation.

2. $[-5, \infty)$; set-builder notation

3. 

interval notation

4. $\{x \mid x > 3 \text{ and } x \in \mathbb{N}\}$; roster notation

5. $(-\infty, -2)$ or $(5, \infty)$; set-builder notation

6. $\{x \mid -4 < x \leq 5 \text{ and } x \in \mathbb{Z}\}$; words

7. $5.5 \leq x \leq 5.6$; interval notation

1-2 Properties of Real Numbers *(pp. 14–19)*

Prep for **25.0**

EXAMPLE

- **Identify the property demonstrated by the equation $3(8x) = (3 \cdot 8)x$.**

  In the equation, the factors have been regrouped. The property of multiplication that allows regrouping is the Associative Property.

EXERCISES

Identify the property demonstrated by each equation.

8. $2x\sqrt{3} = \sqrt{3} \cdot (2x)$

9. $9.9x - 2x = (9.9 - 2)x$

Find the additive and multiplicative inverse of each number.

10. 0.55

11. $-\frac{7}{8}$

12. $1.\overline{2}$

1-3 Square Roots *(pp. 21–26)*

Review of 🔑 **1A2.0**

EXAMPLE

- **Simplify the expression $\frac{3\sqrt{2}}{\sqrt{6}}$.**

$\frac{3\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$	*Rationalize the denominator.*
$\frac{3\sqrt{12}}{6}$	*Product Property of Square Roots*
$\frac{3\sqrt{4 \cdot 3}}{6}$	*Product Property of Square Roots*
$\frac{6\sqrt{3}}{6} = \sqrt{3}$	

EXERCISES

Estimate to the nearest tenth.

13. $\sqrt{12}$

14. $\sqrt{55}$

15. $\sqrt{74}$

16. $\sqrt{29}$

Simplify each expression.

17. $\sqrt{32}$

18. $\frac{\sqrt{64}}{\sqrt{4}}$

19. $2\sqrt{2} - \sqrt{72}$

20. $\sqrt{3} \cdot \sqrt{21}$

21. $\frac{7}{\sqrt{2}}$

22. $\frac{2\sqrt{20}}{5\sqrt{8}}$

1-4 Simplifying Algebraic Expressions *(pp. 27–32)*

Review of **7AF1.1** and **7AF1.2**

EXAMPLES

- **Evaluate $6c - 3c^2 + d^3$ for $c = -1$ and $d = 3$.**

$6(-1) - 3(-1)^2 + (3)^3$	*Substitute −1 for c and 3 for d.*
$-6 - 3(1) + 27 = 18$	

- **Simplify the expression $3m + (m - 5n)2$.**

$3m + (2m - 10n)$	*Distribute the 2.*
$3m + 2m - 10n$	*Identify like terms.*
$5m - 10n$	*Combine like terms.*

EXERCISES

Evaluate each expression for the given values of the variables.

23. $x^2y - xy^2$ for $x = 6$ and $y = -2$

24. $-\frac{x^2}{2} + 5xy - 9y$ for $x = 4$ and $y = 2$

25. $\frac{n^2 + mn - 1}{4m^2n}$ for $m = 2$ and $n = -1$

Simplify each expression.

26. $-x - 2y + 9x - y + 3x$

27. $7 - (5a - b) + 11$

28. $-4(2x + 3y) + 5x$

29. $c(a^2 - b) + 3bc$

1-5 Properties of Exponents *(pp. 34–41)*

Review of 🔑 **1A2.0** and **7AF2.1**

EXAMPLE

- **Simplify the expression $\frac{6m^4n^{-3}}{18m^3n}$. Assume all variables are nonzero.**

$\frac{6}{18}\left(m^{4-3}n^{-3-1}\right)$	*Quotient of Powers Property*
$\frac{1}{3}\left(mn^{-4}\right)$	*Simplify.*
$\frac{m}{3n^4}$	*Negative Exponent Property*

EXERCISES

Simplify each expression. Assume all variables are nonzero.

30. $\left(-2x^5y^{-3}\right)^3$

31. $\frac{-24x^4y^{-6}}{14x^{-3}y^3}$

32. $\left(\frac{r^2s}{s^3}\right)^2$

33. $4mn\left(m^5n^{-5}\right)$

Simplify each expression. Write each answer in scientific notation.

34. $\frac{7.7 \times 10^5}{1.1 \times 10^{-2}}$

35. $\left(4.5 \times 10^{-2}\right)\left(1.2 \times 10^3\right)$

1-6 Relations and Functions *(pp. 44–50)*

Prep for **24.0**

EXAMPLE

- **Give the domain and range for the relation. Then determine whether the relation is a function.**

Arcade Game Costs				
Games	1	2	3	4
Cost ($)	0.50	1.00	1.50	2.00

Domain: $\{1, 2, 3, 4\}$ *Independent variable*

Range: $\{0.50, 1.00, 1.50, 2.00\}$ *Dependent variable*

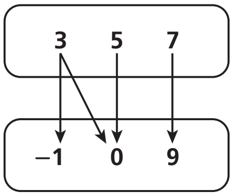
Each number of games has only one cost associated with it.

The relation from number of games to cost is a function.

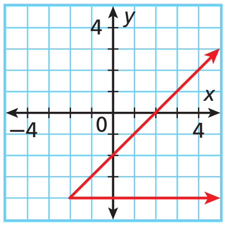
EXERCISES

Give the domain and range for each relation. Then determine whether the relation is a function.

36.



37.



38. $\{(3, 4), (4, 3), (0, 3), (-2, 4)\}$

39.

x	5	10	15	20	25
y	−5	−4	−3	−2	−1

40. from the first three letters of the alphabet to the U.S. states that begin with that letter

1-7 Function Notation *(pp. 51–57)*

Prep for **24.0**

EXAMPLE

- **A cell phone company charges $40 per month for the first 500 minutes plus $0.75 for each additional minute used. Write a function to represent the total monthly cost based on the number of minutes used. What is the value of the function for an input of 30, and what does it represent?**

Let *c* be the total monthly cost and *m* be the number of additional minutes used.

$$\text{cost} = \text{monthly fee} + \text{rate} \cdot \text{additional minutes}$$

$$c(m) = 40 + 0.75 \cdot m$$

$$\begin{aligned} c(30) &= 40 + 0.75(30) \\ &= 40 + 22.5 \\ &= 62.5 \end{aligned}$$

The value of $c(m)$ for an input of 30 is $c(30) = 62.5$. This means that the monthly cost when 30 additional minutes are used is $62.50.

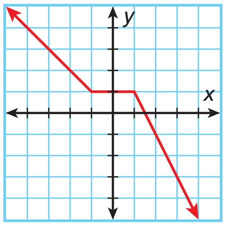
EXERCISES

For each function, find $f(2)$, $f\left(\frac{1}{2}\right)$, and $f(-2)$.

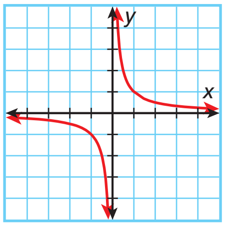
41. $f(x) = -x^2 + 2$

42. $f(x) = -5x - 6$

43.

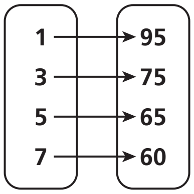


44.



Graph each function.

45.



46. $f(x) = 10 - 2x$

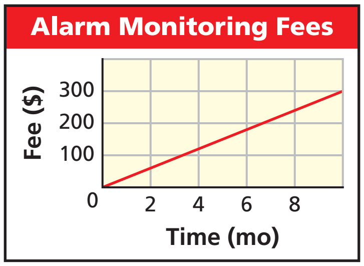
47. **Geometry** The surface area of a cube is 6 times the square of its side length. Write a function to represent the surface area of a cube. What is the value of the function for an input of 10 centimeters, and what does it represent?

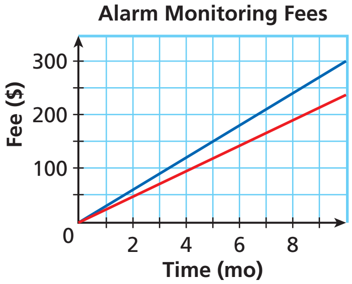
1-8 Exploring Transformations *(pp. 59–66)*

Prep for 🔑 9.0

EXAMPLE

- **The graph shows household alarm monitoring fees. Sketch a graph to represent a $\frac{1}{5}$ fee reduction on long-term contracts. Then identify the transformation of the original graph that the new graph represents.**





Each price is $\frac{4}{5}$ of the original price. This represents a vertical compression of the graph by a factor of $\frac{4}{5}$.

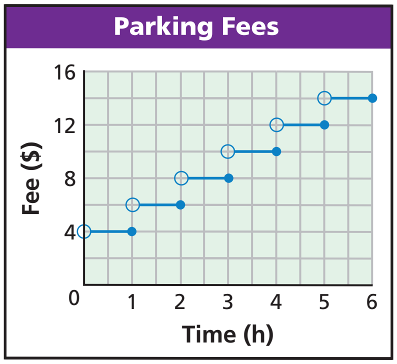
EXERCISES

Perform the given transformation to the point $(5, -1)$. Give the coordinates of the new point.

48. 5 units left, 4 units down

49. reflection across the x-axis

The graph shows parking garage fees. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.



50. The fees are half price on weekends.

51. The fees are increased by 10%.

52. All fees are increased by \$1.00.

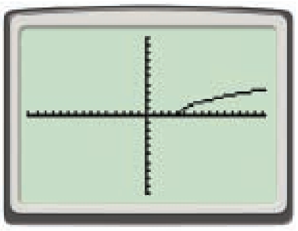
1-9 Introduction to Parent Functions *(pp. 67–73)*

Prep for 🔑 9.0, 🔑 10.0, and 24.0

EXAMPLE

- **Identify the parent function for $g(x) = \sqrt{x-4}$ from its equation. Then graph g on your calculator and describe what transformation of the parent function it represents.**

$g(x) = \sqrt{x-4}$ is a square-root function.



The graph of the square-root parent function intersects the x-axis at the point $(0, 0)$.

The graph of the function $g(x) = \sqrt{x-4}$ intersects the x-axis at the point $(4, 0)$.

So $g(x) = \sqrt{x-4}$ represents a translation of the square-root parent function 4 units right.

EXERCISES

Identify the parent function for g from its equation. Then graph g on your calculator and describe what transformation of the parent function it represents.

53. $g(x) = x^2 - 1$

54. $g(x) = -\sqrt{x}$

55. Graph the data from the table. Describe the parent function that would best approximate the data set. Then use the graph to estimate the tire pressure for a 95-pound rider.

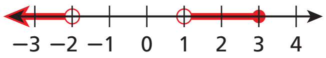
Bicycle Road-Tire Pressures

Weight of Rider (lb)	110	140	170	200	230
Pressure (psi)	95	105	115	125	135

CHAPTER TEST

1. Order $1.\overline{5}$, -2, 0.95, $-\sqrt{3}$, and 1 from least to greatest. Then classify each number by the subsets of the real numbers to which it belongs.

Rewrite each set in the indicated notation.

2. 

interval notation

3. $(-\infty, 12]$; set-builder notation

Identify the property demonstrated by each equation.

4. $x + y = y + x$

5. $9 \cdot 2 + 9 \cdot 7 = 9 \cdot (2 + 7)$

6. $x = (1)x$

7. A company manufactures square windows that come in three sizes: 6 square feet, 8 square feet, and 15 square feet. Estimate the side length of each window to the nearest tenth of a foot. Then identify which window is the largest one that could fit in a wall with a width of 3 feet.

Simplify each expression.

8. $-2\sqrt{3} + \sqrt{75}$

9. $\sqrt{24} - \sqrt{54}$

10. $\sqrt{22} \cdot \sqrt{55}$

11. $2(x + 1) + 9x$

12. $5x - 5y - 7x + y$

13. $12x + 4(x + y) - 6y$

Simplify each expression. Assume all variables are nonzero.

14. $8a^2b^5(-2a^3b^2)$

15. $\dfrac{28u^{-2}v^3}{4u^2v^2}$

16. $(5x^4y^{-3})^{-2}$

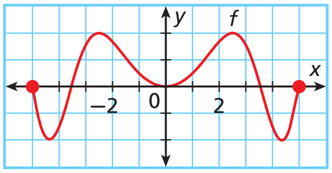
17. $\left(\dfrac{3x^2y}{xy^2}\right)^{-1}$

18. German shepherds are often used as police dogs because they have 2.25×10^8 smell receptors in their nose. Humans average only 5×10^6 smell receptors in their nose. How many times as great is the number of smell receptors in a German shepherd's nose as that in a human's nose?

Give the domain and range for each relation. Then tell whether each relation is a function.

19.

x	10	9	8	9	10
y	2	4	6	8	10

20. 

For each function, evaluate $f(-2)$, $f\left(\frac{1}{2}\right)$, and $f(0)$.

21. $f(x) = -4x$

22. $f(x) = -3x^2 + x$

23. $f(x) = \sqrt{x + 3}$

24. The table shows how the distance from the top of a building to the horizon depends on the building's height. Graph the relationship from building height to horizon distance, and identify which parent function best describes the data. Then use your graph to estimate the distance to the horizon from the top of a building with a height of 80 m.

Horizon Distances

Height of Building (m)	5	10	20	40	100
Distance to Horizon (km)	8.0	11.3	15.9	22.5	35.6