CHAPTER 6

The Standard Deviation as a Ruler and the Normal Model

The women's heptathlon in the Olympics consists of seven track and field events: the 200-m and 800-m runs, 100-m high hurdles, shot put, javelin, high jump, and long jump. To determine who should get the gold medal, somehow the performances in all seven events have to be combined into one score. How can performances in such different events be compared? They don't even have the same units; the races are recorded in minutes and seconds and the throwing and jumping events in meters. In the 2004 Olympics, Austra Skujyté of Lithuania put the shot 16.4 meters, about 3 meters farther than the average of all contestants. Carolina Klüft won the long jump with a 6.78-m jump, about a meter better than the average. Which performance deserves more points? Even though both events are measured in meters, it's not clear how to compare them. The solution to the problem of how to compare scores turns out to be a useful method for comparing all sorts of values whether they have the same units or not.

The Standard Deviation as a Ruler

Grading on a Curve

If you score 79% on an exam, what grade should you get? One teaching philosophy looks only at the raw percentage, 79, and bases the grade on that alone. Another looks at your *relative* performance and bases the grade on how you did compared with the rest of the class. Teachers and students still debate which method is better.

The trick in comparing very different-looking values is to use standard deviations. The standard deviation tells us how the whole collection of values varies, so it's a natural ruler for comparing an individual value to the group. Over and over during this course, we will ask questions such as "How far is this value from the mean?" or "How different are these two statistics?" The answer in every case will be to measure the distance or difference in standard deviations.

The concept of the standard deviation as a ruler is not special to this course. You'll find statistical distances measured in standard deviations throughout Statistics, up to the most advanced levels.[1] This approach is one of the basic tools of statistical thinking.

[1] Other measures of spread could be used as well, but the standard deviation is the most common measure, and it is almost always used as the ruler.

In order to compare the two events, let's start with a picture. This time we'll use stem-and-leaf displays so we can see the individual distances.

Long Jump Stem	Long Jump Leaf	Shot Put Stem	Shot Put Leaf
67	8	16	4
66		15	
65	1	15	
64	2	14	56778
63	0566	14	24
62	11235	13	5789
61	0569	13	012234
60	2223	12	55
59	0278	12	0144
58	4	11	59
57	0	11	23

FIGURE 6.1

Stem-and-leaf displays for both the long jump and the shot put in the 2004 Olympic Heptathlon. Carolina Klüft (green scores) won the long jump, and Austra Skujyté (red scores) won the shot put. Which heptathlete did better for both events combined?

The two winning performances on the top of each stem-and-leaf display appear to be about the same distance from the center of the pack. But look again carefully. What do we mean by the *same distance*? The two displays have different scales. Each line in the stem-and-leaf for the shot put represents half a meter, but for the long jump each line is only a tenth of a meter. It's only because our eyes naturally adjust the scales and use the standard deviation as the ruler that we see each as being about the same distance from the center of the data. How can we make this hunch more precise? Let's see how many standard deviations each performance is from the mean.

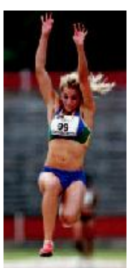
	Event: Long Jump	Event: Shot Put
Mean (all contestants)	6.16 m	13.29 m
SD	0.23 m	1.24 m
n	26	28
Klüft	6.78 m	14.77 m
Skujyté	6.30 m	16.40 m

Klüft's 6.78-m long jump is 0.62 meters longer than the mean jump of 6.16 m. How many *standard deviations* better than the mean is that? The standard deviation for this event was 0.23 m, so her jump was $(6.78 - 6.16)/0.23 = 0.62/0.23 = 2.70$ *standard deviations better* than the mean. Skujyté's winning shot put was $16.40 - 13.29 = 3.11$ meters longer than the mean shot put distance, and that's $3.11/1.24 = 2.51$ standard deviations better than the mean. That's a great performance but not quite as impressive as Klüft's long jump, which was farther above the mean, as measured in *standard deviations*.

Standardizing with *z*-Scores

NOTATION ALERT:

There goes another letter. We always use the letter *z* to denote values that have been standardized with the mean and standard deviation.



To compare these athletes' performances, we determined how many standard deviations from the event's mean each was.

Expressing the distance in standard deviations ***standardizes*** the performances. To standardize a value, we simply subtract the mean performance in that event and then divide this difference by the standard deviation. We can write the calculation as

$$z = \frac{y - \bar{y}}{s}.$$

These values are called **standardized values,** and are commonly denoted with the letter *z*. Usually, we just call them ***z*-scores.**

Standardized values have *no units*. *z*-scores measure the distance of each data value from the mean in standard deviations. A *z*-score of 2 tells us that a data value is 2 standard deviations above the mean. It doesn't matter whether the original variable was measured in inches, dollars, or seconds. Data values below the mean have negative *z*-scores, so a *z*-score of −1.6 means that the data value was 1.6 standard deviations below the mean. Of course, regardless of the direction, the farther a data value is from the mean, the more unusual it is, so a *z*-score of −1.3

is more extraordinary than a z-score of 1.2. Looking at the z-scores, we can see that even though both were winning scores, Klüft's long jump with a z-score of 2.70 is slightly more impressive than Skujyté's shot put with a z-score of 2.51.

FOR EXAMPLE Standardizing skiing times

The men's combined skiing event in the winter Olympics consists of two races: a downhill and a slalom. Times for the two events are added together, and the skier with the lowest total time wins. In the 2006 Winter Olympics, the mean slalom time was 94.2714 seconds with a standard deviation of 5.2844 seconds. The mean downhill time was 101.807 seconds with a standard deviation of 1.8356 seconds. Ted Ligety of the United States, who won the gold medal with a combined time of 189.35 seconds, skied the slalom in 87.93 seconds and the downhill in 101.42 seconds.

Question: On which race did he do better compared with the competition?

For the slalom, Ligety's z-score is found by subtracting the mean time from his time and then dividing by the standard deviation:

$$z_{Slalom} = \frac{87.93 - 94.2714}{5.2844} = -1.2$$

Similarly, his z-score for the downhill is:

$$z_{Downhill} = \frac{101.42 - 101.807}{1.8356} = -0.21$$

The z-scores show that Ligety's time in the slalom is farther below the mean than his time in the downhill. His performance in the slalom was more remarkable.

By using the standard deviation as a ruler to measure statistical distance from the mean, we can compare values that are measured on different variables, with different scales, with different units, or for different individuals. To determine the winner of the heptathlon, the judges must combine performances on seven very different events. Because they want the score to be absolute, and *not* dependent on the particular athletes in each Olympics, they use predetermined tables, but they could combine scores by standardizing each, and then adding the z-scores together to reach a total score. The only trick is that they'd have to switch the sign of the z-score for running events, because unlike throwing and jumping, it's better to have a running time below the mean (with a negative z-score).

To combine the scores Skujyté and Klüft earned in the long jump and the shot put, we standardize both events as shown in the table. That gives Klüft her 2.70 z-score in the long jump and a 1.19 in the shot put, for a total of 3.89. Skujyté's shot put gave her a 2.51, but her long jump was only 0.61 SDs above the mean, so her total is 3.12.

		Event: Long Jump	Event: Shot Put
	Mean	6.16 m	13.29 m
	SD	0.23 m	1.24 m
Klüft	Performance	6.78 m	14.77 m
	z-score	$\frac{6.78 - 6.16}{0.23} = 2.70$	$\frac{14.77 - 13.29}{1.24} = 1.19$
	Total z-score	$2.70 + 1.19 = $ **3.89**	
Skujyté	Performance	6.30 m	16.40 m
	z-score	$\frac{6.30 - 6.16}{0.23} = 0.61$	$\frac{16.40 - 13.29}{1.24} = 2.51$
	Total z-score	$0.61 + 2.51 = $ **3.12**	

Is this the result we wanted? Yes. Each won one event, but Klüft's shot put was second best, while Skujyté's long jump was seventh. The z-scores measure how far each result is from the event mean in standard deviation units. And because they are both in standard deviation units, we can combine them. Not coincidentally, Klüft went on to win the gold medal for the entire seven-event heptathlon, while Skujyté got the silver.

FOR EXAMPLE Combining z-scores

In the 2006 winter Olympics men's combined event, Ivica Kostelić of Croatia skied the slalom in 89.44 seconds and the downhill in 100.44 seconds. He thus beat Ted Ligety in the downhill, but not in the slalom. Maybe he should have won the gold medal.

Question: Considered in terms of standardized scores, which skier did better?

Kostelić's z-scores are:

$$z_{Slalom} = \frac{89.44 - 94.2714}{5.2844} = -0.91 \quad \text{and} \quad z_{Downhill} = \frac{100.44 - 101.807}{1.8356} = -0.74$$

The sum of his z-scores is approximately –1.65. Ligety's z-score sum is only about –1.41. Because the standard deviation of the downhill times is so much smaller, Kostelić's better performance there means that he would have won the event if standardized scores were used.

When we standardize data to get a z-score, we do two things. First, we shift the data by subtracting the mean. Then, we rescale the values by dividing by their standard deviation. We often shift and rescale data. What happens to a grade distribution if *everyone* gets a five-point bonus? Everyone's grade goes up, but does the shape change? (*Hint:* Has anyone's distance from the mean changed?) If we switch from feet to meters, what happens to the distribution of heights of students in your class? Even though your intuition probably tells you the answers to these questions, we need to look at exactly how shifting and rescaling work.

JUST CHECKING

1. Your Statistics teacher has announced that the lower of your two tests will be dropped. You got a 90 on test 1 and an 80 on test 2. You're all set to drop the 80 until she announces that she grades "on a curve." She standardized the scores in order to decide which is the lower one. If the mean on the first test was 88 with a standard deviation of 4 and the mean on the second was 75 with a standard deviation of 5,

a) Which one will be dropped?
b) Does this seem "fair"?

Shifting Data

Since the 1960s, the Centers for Disease Control's National Center for Health Statistics has been collecting health and nutritional information on people of all ages and backgrounds. A recent survey, the National Health and Nutrition Examination Survey (NHANES) 2001–2002,[2] measured a wide variety of variables, including body measurements, cardiovascular fitness, blood chemistry, and demographic information on more than 11,000 individuals.

[2] www.cdc.gov/nchs/nhanes.htm

WHO	80 male participants of the NHANES survey between the ages of 19 and 24 who measured between 68 and 70 inches tall
WHAT	Their weights
UNIT	Kilograms
WHEN	2001–2002
WHERE	United States
WHY	To study nutrition, and health issues and trends
HOW	National survey

Included in this group were 80 men between 19 and 24 years old of average height (between 5′8″ and 5′10″ tall). Here are a histogram and boxplot of their weights:

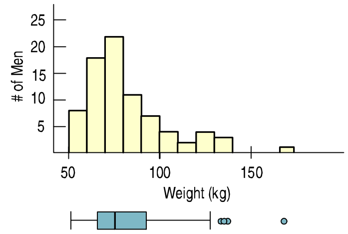


FIGURE 6.2

Histogram and boxplot for the men's weights. The shape is skewed to the right with several high outliers.

Their mean weight is 82.36 kg. For this age and height group, the National Institutes of Health recommends a maximum healthy weight of 74 kg, but we can see that some of the men are heavier than the recommended weight. To compare their weights to the recommended maximum, we could subtract 74 kg from each of their weights. What would that do to the center, shape, and spread of the histogram? Here's the picture:

A S ***Activity:* Changing the Baseline.** What happens when we shift data? Do measures of center and spread change?

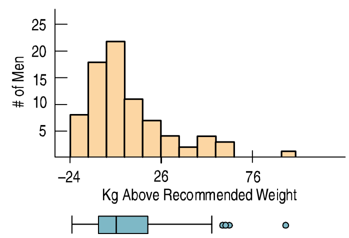


FIGURE 6.3

Subtracting 74 kilograms shifts the entire histogram down but leaves the spread and the shape exactly the same.

Doctors' height and weight charts sometimes give ideal weights for various heights that include 2-inch heels. If the mean height of adult women is 66 inches including 2-inch heels, what is the mean height of women without shoes? Each woman is shorter by 2 inches when barefoot, so the mean is decreased by 2 inches, to 64 inches.

On average, they weigh 82.36 kg, so on average they're 8.36 kg overweight. And, after subtracting 74 from each weight, the mean of the new distribution is $82.36 - 74 = 8.36$ kg. In fact, when we **shift** the data by adding (or subtracting) a constant to each value, all measures of position (center, percentiles, min, max) will increase (or decrease) by the same constant.

What about the spread? What does adding or subtracting a constant value do to the spread of the distribution? Look at the two histograms again. Adding or subtracting a constant changes each data value equally, so the entire distribution just shifts. Its shape doesn't change and neither does the spread. None of the measures of spread we've discussed—not the range, not the IQR, not the standard deviation—changes.

Adding (or subtracting) a constant to every data value adds (or subtracts) the same constant to measures of position, but leaves measures of spread unchanged.

Rescaling Data

Not everyone thinks naturally in metric units. Suppose we want to look at the weights in pounds instead. We'd have to **rescale** the data. Because there are about 2.2 pounds in every kilogram, we'd convert the weights by multiplying each value by 2.2. Multiplying or dividing each value by a constant changes the measurement units. Here are histograms of the two weight distributions, plotted on the same scale, so you can see the effect of multiplying:

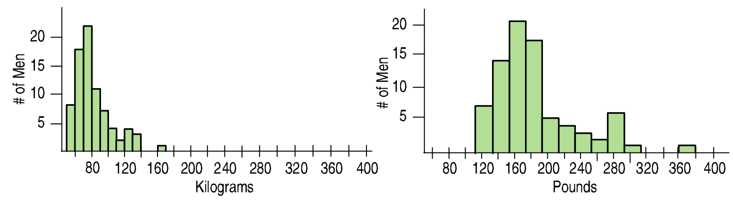


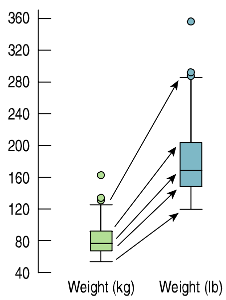
FIGURE 6.4
Men's weights in both kilograms and pounds. How do the distributions and numerical summaries change?

A S ***Simulation:* Changing the Units.** Change the center and spread values for a distribution and watch the summaries change (or not, as the case may be).

What happens to the shape of the distribution? Although the histograms don't look exactly alike, we see that the shape really hasn't changed: Both are unimodal and skewed to the right.

What happens to the mean? Not too surprisingly, it gets multiplied by 2.2 as well. The men weigh 82.36 kg on average, which is 181.19 pounds. As the boxplots and 5-number summaries show, all measures of position act the same way. They all get multiplied by this same constant.

What happens to the spread? Take a look at the boxplots. The spread in pounds (on the right) is larger. How much larger? If you guessed 2.2 times, you've figured out how measures of spread get rescaled.



	Weight (kg)	Weight (lb)
Min	54.3	119.46
Q1	67.3	148.06
Median	76.85	169.07
Q3	92.3	203.06
Max	161.5	355.30
IQR	25	55
SD	22.27	48.99

FIGURE 6.5
The boxplots (drawn on the same scale) show the weights measured in kilograms (on the left) and pounds (on the right). Because 1 kg is 2.2 lb, all the points in the right box are 2.2 times larger than the corresponding points in the left box. So each measure of position and spread is 2.2 times as large when measured in pounds rather than kilograms.

When we multiply (or divide) all the data values by any constant, all measures of position (such as the mean, median, and percentiles) and measures of spread (such as the range, the IQR, and the standard deviation) are multiplied (or divided) by that same constant.

FOR EXAMPLE **Rescaling the slalom**

Recap: The times in the men's combined event at the winter Olympics are reported in minutes and seconds. Previously, we converted these to seconds and found the mean and standard deviation of the slalom times to be 94.2714 seconds and 5.2844 seconds, respectively.

Question: Suppose instead that we had reported the times in minutes—that is, that each individual time was divided by 60. What would the resulting mean and standard deviation be?

Dividing all the times by 60 would divide both the mean and the standard deviation by 60:

$$\text{Mean} = 94.2714/60 = 1.5712 \text{ minutes}; \quad SD = 5.2844/60 = 0.0881 \text{ minutes}.$$

JUST CHECKING

2. In 1995 the Educational Testing Service (ETS) adjusted the scores of SAT tests. Before ETS recentered the SAT Verbal test, the mean of all test scores was 450.

a) How would adding 50 points to each score affect the mean?

b) The standard deviation was 100 points. What would the standard deviation be after adding 50 points?

c) Suppose we drew boxplots of test takers' scores a year before and a year after the recentering. How would the boxplots of the two years differ?

3. A company manufactures wheels for in-line skates. The diameters of the wheels have a mean of 3 inches and a standard deviation of 0.1 inches. Because so many of their customers use the metric system, the company decided to report their production statistics in millimeters (1 inch = 25.4 mm). They report that the standard deviation is now 2.54 mm. A corporate executive is worried about this increase in variation. Should he be concerned? Explain.

Back to *z*-scores

A S ***Activity:*** **Standardizing.** What if we both shift and rescale? The result is so nice that we give it a name.

Standardizing data into *z*-scores is just shifting them by the mean and rescaling them by the standard deviation. Now we can see how standardizing affects the distribution. When we subtract the mean of the data from every data value, we shift the mean to zero. As we have seen, such a shift doesn't change the standard deviation.

When we *divide* each of these shifted values by *s*, however, the standard deviation should be divided by *s* as well. Since the standard deviation was *s* to start with, the new standard deviation becomes 1.

How, then, does standardizing affect the distribution of a variable? Let's consider the three aspects of a distribution: the shape, center, and spread.

***z*-scores have mean 0 and standard deviation 1.**

- *Standardizing into z-scores does not change the* ***shape*** *of the distribution of a variable.*
- *Standardizing into z-scores changes the* ***center*** *by making the mean 0.*
- *Standardizing into z-scores changes the* ***spread*** *by making the standard deviation 1.*

STEP-BY-STEP EXAMPLE Working with Standardized Variables

Many colleges and universities require applicants to submit scores on standardized tests such as the SAT Writing, Math, and Critical Reading (Verbal) tests. The college your little sister wants to apply to says that while there is no minimum score required, the middle 50% of their students have combined SAT scores between 1530 and 1850. You'd feel confident if you knew her score was in their top 25%, but unfortunately she took the ACT test, an alternative standardized test.

Question: How high does her ACT need to be to make it into the top quarter of equivalent SAT scores?

To answer that question you'll have to standardize all the scores, so you'll need to know the mean and standard deviations of scores for some group on both tests. The college doesn't report the mean or standard deviation for their applicants on either test, so we'll use the group of all test takers nationally. For college-bound seniors, the average combined SAT score is about 1500 and the standard deviation is about 250 points. For the same group, the ACT average is 20.8 with a standard deviation of 4.8.

Plan State what you want to find out. **Variables** Identify the variables and report the W's (if known). Check the appropriate conditions.	I want to know what ACT score corresponds to the upper-quartile SAT score. I know the mean and standard deviation for both the SAT and ACT scores based on all test takers, but I have no individual data values. ✔ **Quantitative Data Condition:** Scores for both tests are quantitative but have no meaningful units other than points.
Mechanics Standardize the variables. The y-value we seek is z standard deviations above the mean.	The middle 50% of SAT scores at this college fall between 1530 and 1850 points. To be in the top quarter, my sister would have to have a score of at least 1850. That's a z-score of $z = \frac{(1850 - 1500)}{250} = 1.40$ So an SAT score of 1850 is 1.40 standard deviations above the mean of all test takers. For the ACT, 1.40 standard deviations above the mean is $20.8 + 1.40(4.8) = 27.52$.
Conclusion Interpret your results in context.	To be in the top quarter of applicants in terms of combined SAT score, she'd need to have an ACT score of at least 27.52.

When Is a *z*-score BIG?

A *z*-score gives us an indication of how unusual a value is because it tells us how far it is from the mean. If the data value sits right at the mean, it's not very far at all and its *z*-score is 0. A *z*-score of 1 tells us that the data value is 1 standard deviation above the mean, while a *z*-score of −1 tells us that the value is 1 standard deviation below the mean. How far from 0 does a *z*-score have to be to be interesting or unusual? There is no universal standard, but the larger the score is (negative or positive), the more unusual it is. We know that 50% of the data lie between the quartiles. For symmetric data, the standard deviation is usually a bit smaller than the IQR, and it's not uncommon for at least half of the data to have *z*-scores between −1 and 1. But no matter what the shape of the distribution, a *z*-score of 3 (plus or minus) or more is rare, and a *z*-score of 6 or 7 shouts out for attention.

To say more about how big we expect a *z*-score to be, we need to *model* the data's distribution. A model will let us say much more precisely how often we'd be likely to see *z*-scores of different sizes. Of course, like all models of the real world, the model will be wrong—wrong in the sense that it can't match

Is Normal Normal?

Don't be misled. The name "Normal" doesn't mean that these are the *usual* shapes for histograms. The name follows a tradition of positive thinking in Mathematics and Statistics in which functions, equations, and relationships that are easy to work with or have other nice properties are called "normal", "common", "regular", "natural", or similar terms. It's as if by calling them ordinary, we could make them actually occur more often and simplify our lives.

"All models are wrong—but some are useful."

—George Box, famous statistician

reality exactly. But it can still be useful. Like a physical model, it's something we can look at and manipulate in order to learn more about the real world.

Models help our understanding in many ways. Just as a model of an airplane in a wind tunnel can give insights even though it doesn't show every rivet,[3] models of data give us summaries that we can learn from and use, even though they don't fit each data value exactly. It's important to remember that they're only *models* of reality and not reality itself. But without models, what we can learn about the world at large is limited to only what we can say about the data we have at hand.

There is no universal standard for z-scores, but there is a model that shows up over and over in Statistics. You may have heard of "bell-shaped curves." Statisticians call them Normal models. **Normal models** are appropriate for distributions whose shapes are unimodal and roughly symmetric. For these distributions, they provide a measure of how extreme a z-score is. Fortunately, there is a Normal model for every possible combination of mean and standard deviation. We write $N(\mu, \sigma)$ to represent a Normal model with a mean of μ and a standard deviation of σ. Why the Greek? Well, *this* mean and standard deviation are not numerical summaries of data. They are part of the model. They don't come from the data. Rather, they are numbers that we choose to help specify the model. Such numbers are called **parameters** of the model.

NOTATION ALERT:

$N(\mu, \sigma)$ always denotes a Normal model. The μ, pronounced "mew," is the Greek letter for "m" and always represents the mean in a model. The σ, sigma, is the lowercase Greek letter for "s" and always represents the standard deviation in a model.

We don't want to confuse the parameters with summaries of the data such as $\bar{y}$ and s, so we use special symbols. In Statistics, we almost always use Greek letters for parameters. By contrast, summaries of data are called **statistics** and are usually written with Latin letters.

If we model data with a Normal model and standardize them using the corresponding μ and σ, we still call the standardized value a ***z*-score,** and we write

$$z = \frac{y - \mu}{\sigma}.$$

Usually it's easier to standardize data first (using its mean and standard deviation). Then we need only the model $N(0,1)$. The Normal model with mean 0 and standard deviation 1 is called the **standard Normal model** (or the **standard Normal distribution**).

Is the Standard Normal a standard?

Yes. We call it the "Standard Normal" because it models standardized values. It is also a "standard" because this is the particular Normal model that we almost always use.

But be careful. You shouldn't use a Normal model for just any data set. Remember that standardizing won't change the shape of the distribution. If the distribution is not unimodal and symmetric to begin with, standardizing won't make it Normal.

When we use the Normal model, we assume that the distribution of the data is, well, Normal. Practically speaking, there's no way to check whether this **Normality Assumption** is true. In fact, it almost certainly is not true. Real data don't behave like mathematical models. Models are idealized; real data are real. The good news, however, is that to use a Normal model, it's sufficient to check the following condition:

A S *Activity:* **Working with Normal Models.** Learn more about the Normal model and see what data drawn at random from a Normal model might look like.

> **Nearly Normal Condition.** The shape of the data's distribution is unimodal and symmetric. Check this by making a histogram (or a Normal probability plot, which we'll explain later).

Don't model data with a Normal model without checking whether the condition is satisfied.

All models make **assumptions.** Whenever we model—and we'll do that often—we'll be careful to point out the assumptions that we're making. And, what's even more important, we'll check the associated **conditions** in the data to make sure that those assumptions are reasonable.

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[3] In fact, the model is useful *because* it doesn't have every rivet. It is because models offer a simpler view of reality that they are so useful as we try to understand reality.

The 68–95–99.7 Rule

One in a Million

These magic 68, 95, 99.7 values come from the Normal model. As a model, it can give us corresponding values for any z-score. For example, it tells us that fewer than 1 out of a million values have z-scores smaller than −5.0 or larger than +5.0. So if someone tells you you're "one in a million," they must really admire your z-score.

TI-*nspire*

The 68–95–99.7 Rule. See it work for yourself.

Normal models give us an idea of how extreme a value is by telling us how likely it is to find one that far from the mean. We'll soon show how to find these numbers precisely—but one simple rule is usually all we need.

It turns out that in a Normal model, about 68% of the values fall within 1 standard deviation of the mean, about 95% of the values fall within 2 standard deviations of the mean, and about 99.7%—almost all—of the values fall within 3 standard deviations of the mean. These facts are summarized in a rule that we call (let's see . . .) the **68–95–99.7 Rule.**[4]

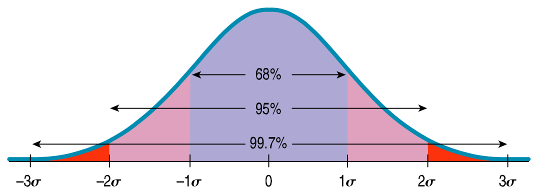


FIGURE 6.6

Reaching out one, two, and three standard deviations on a Normal model gives the 68–95–99.7 Rule, seen as proportions of the area under the curve.

FOR EXAMPLE Using the 68–95–99.7 Rule

Question: In the 2006 Winter Olympics men's combined event, Jean-Baptiste Grange of France skied the slalom in 88.46 seconds—about 1 standard deviation faster than the mean. If a Normal model is useful in describing slalom times, about how many of the 35 skiers finishing the event would you expect skied the slalom *faster* than Jean-Baptiste?

From the 68–95–99.7 Rule, we expect 68% of the skiers to be within one standard deviation of the mean. Of the remaining 32%, we expect half on the high end and half on the low end. 16% of 35 is 5.6, so, conservatively, we'd expect about 5 skiers to do better than Jean-Baptiste.

JUST CHECKING

4. As a group, the Dutch are among the tallest people in the world. The average Dutch man is 184 cm tall—just over 6 feet (and the average Dutch woman is 170.8 cm tall—just over 5′7″). If a Normal model is appropriate and the standard deviation for men is about 8 cm, what percentage of all Dutch men will be over 2 meters (6′6″) tall?

5. Suppose it takes you 20 minutes, on average, to drive to school, with a standard deviation of 2 minutes. Suppose a Normal model is appropriate for the distributions of driving times.

a) How often will you arrive at school in less than 22 minutes?

b) How often will it take you more than 24 minutes?

c) Do you think the distribution of your driving times is unimodal and symmetric?

d) What does this say about the accuracy of your predictions? Explain.

[4] This rule is also called the "Empirical Rule" because it originally came from observation. The rule was first published by Abraham de Moivre in 1733, 75 years before the Normal model was discovered. Maybe it should be called "de Moivre's Rule," but that wouldn't help us remember the important numbers, 68, 95, and 99.7.

The First Three Rules for Working with Normal Models

A S ***Activity:* Working with Normal Models.** Well, actually playing with them. This interactive tool lets you do what this chapter's figures can't do, move them!

1. Make a picture.
2. Make a picture.
3. Make a picture.

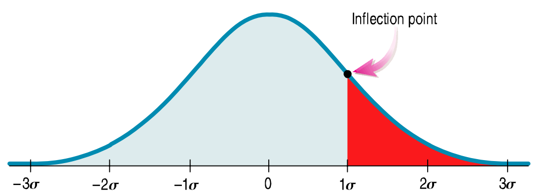
Although we're thinking about models, not histograms of data, the three rules don't change. To help you think clearly, a simple hand-drawn sketch is all you need. Even experienced statisticians sketch pictures to help them think about Normal models. You should too.

Of course, when we have data, we'll also need to make a histogram to check the **Nearly Normal Condition** to be sure we can use the Normal model to model the data's distribution. Other times, we may be told that a Normal model is appropriate based on prior knowledge of the situation or on theoretical considerations.

A S ***Activity:* Normal Models.** Normal models have several interesting properties—see them here.

How to Sketch a Normal Curve That Looks Normal To sketch a good Normal curve, you need to remember only three things:

- The Normal curve is bell-shaped and symmetric around its mean. Start at the middle, and sketch to the right and left from there.
- Even though the Normal model extends forever on either side, you need to draw it only for 3 standard deviations. After that, there's so little left that it isn't worth sketching.
- The place where the bell shape changes from curving downward to curving back up—the *inflection point*—is exactly one standard deviation away from the mean.



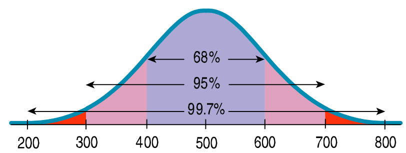
STEP-BY-STEP EXAMPLE

Working with the 68–95–99.7 Rule

The SAT Reasoning Test has three parts: Writing, Math, and Critical Reading (Verbal). Each part has a distribution that is roughly unimodal and symmetric and is designed to have an overall mean of about 500 and a standard deviation of 100 for all test takers. In any one year, the mean and standard deviation may differ from these target values by a small amount, but they are a good overall approximation.

Question: Suppose you earned a 600 on one part of your SAT. Where do you stand among all students who took that test?

You could calculate your z-score and find out that it's $z = (600 - 500)/100 = 1.0$, but what does that tell you about your percentile? You'll need the Normal model and the 68–95–99.7 Rule to answer that question.

THINK **Plan** State what you want to know.	I want to see how my SAT score compares with the scores of all other students. To do that, I'll need to model the distribution.
Variables Identify the variable and report the W's.	Let y = my SAT score. Scores are quantitative but have no meaningful units other than points.
Be sure to check the appropriate conditions.	✔ **Nearly Normal Condition:** If I had data, I would check the histogram. I have no data, but I am told that the SAT scores are roughly unimodal and symmetric.
Specify the parameters of your model.	I will model SAT score with a N(500, 100) model.
SHOW **Mechanics** Make a picture of this Normal model. (A simple sketch is all you need.)	
Locate your score.	My score of 600 is 1 standard deviation above the mean. That corresponds to one of the points of the 68–95–99.7 Rule.
TELL **Conclusion** Interpret your result in context.	About 68% of those who took the test had scores that fell no more than 1 standard deviation from the mean, so 100% − 68% = 32% of all students had scores more than 1 standard deviation away. Only half of those were on the high side, so about 16% (half of 32%) of the test scores were better than mine. My score of 600 is higher than about 84% of all scores on this test.

The bounds of SAT scoring at 200 and 800 can also be explained by the 68–95–99.7 Rule. Since 200 and 800 are three standard deviations from 500, it hardly pays to extend the scoring any farther on either side. We'd get more information only on 100 − 99.7 = 0.3% of students.

The Worst-Case Scenario* Suppose we encounter an observation that's 5 standard deviations above the mean. Should we be surprised? We've just seen that when a Normal model is appropriate, such a value is exceptionally rare. After all, 99.7% of all the values should be within 3 standard deviations of the mean, so anything farther away would be unusual indeed.

But our handy 68–95–99.7 Rule applies only to Normal models, and the Normal is such a *nice* shape. What if we're dealing with a distribution that's strongly skewed (like the CEO salaries), or one that is uniform or bimodal or something really strange? A Normal model has 68% of its observations within one standard deviation of the mean, but a bimodal distribution could even be entirely empty in the middle. In that case could we still say anything at all about an observation 5 standard deviations above the mean?

Remarkably, even with really weird distributions, the worst case can't get all that bad. A Russian mathematician named Pafnuty Tchebycheff[5] answered the question by proving this theorem:

In any distribution, at least $1 - \frac{1}{k^2}$ *of the values must lie within* $\pm k$ *standard deviations of the mean.*

What does that mean?

- For $k = 1$, $1 - \frac{1}{1^2} = 0$; if the distribution is far from Normal, it's possible that none of the values are within 1 standard deviation of the mean. We should be really cautious about saying anything about 68% unless we think a Normal model is justified. (Tchebycheff's theorem really is about the worst case; it tells us nothing about the middle; only about the extremes.)
- For $k = 2$, $1 - \frac{1}{2^2} = \frac{3}{4}$; no matter how strange the shape of the distribution, at least 75% of the values must be within 2 standard deviations of the mean. Normal models may expect 95% in that 2-standard-deviation interval, but even in a worst-case scenario it can never go lower than 75%.
- For $k = 3$, $1 - \frac{1}{3^2} = \frac{8}{9}$; in any distribution, at least 89% of the values lie within 3 standard deviations of the mean.

What we see is that values beyond 3 standard deviations from the mean are uncommon, Normal model or not. Tchebycheff tells us that at least 96% of all values must be within 5 standard deviations of the mean. While we can't always apply the 68–95–99.7 Rule, we can be sure that the observation we encountered 5 standard deviations above the mean is unusual.

Finding Normal Percentiles

A S ***Activity:*** **Your Pulse *z*-Score.** Is your pulse rate high or low? Find its *z*-score with the Normal Model Tool.

An SAT score of 600 is easy to assess, because we can think of it as one standard deviation above the mean. If your score was 680, though, where do you stand among the rest of the people tested? Your *z*-score is 1.80, so you're somewhere between 1 and 2 standard deviations above the mean. We figured out that no more than 16% of people score better than 600. By the same logic, no more than 2.5% of people score better than 700. Can we be more specific than "between 16% and 2.5%"?

When the value doesn't fall exactly 1, 2, or 3 standard deviations from the mean, we can look it up in a table of **Normal percentiles** or use technology.[6] Either way, we first convert our data to *z*-scores before using the table. Your SAT score of 680 has a *z*-score of $(680 - 500)/100 = 1.80$.

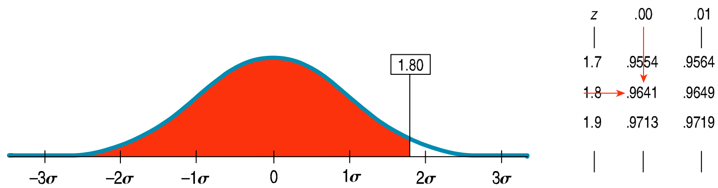
A S ***Activity:*** **The Normal Table.** Table Z just sits there, but this version of the Normal table changes so it always Makes a Picture that fits.

[5] He may have made the worst case for deviations clear, but the English spelling of his name is not. You'll find his first name spelled Pavnutii or Pavnuty and his last name spelled Chebsheff, Cebysev, and other creative versions.

[6] See Table Z in Appendix G, if you're curious. But your calculator (and any statistics computer package) does this, too—and more easily!

FIGURE 6.7

A table of Normal percentiles (Table Z in Appendix G) lets us find the percentage of individuals in a Standard Normal distribution falling below any specified z-score value.



TI-*nspire*

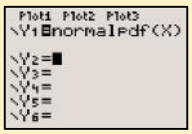
Normal percentiles. Explore the relationship between *z*-scores and areas in a Normal model.

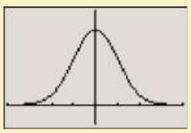
In the piece of the table shown, we find your *z*-score by looking down the left column for the first two digits, 1.8, and across the top row for the third digit, 0. The table gives the percentile as 0.9641. That means that 96.4% of the *z*-scores are less than 1.80. Only 3.6% of people, then, scored better than 680 on the SAT.

Most of the time, though, you'll do this with your calculator.

TI Tips

Finding Normal percentages



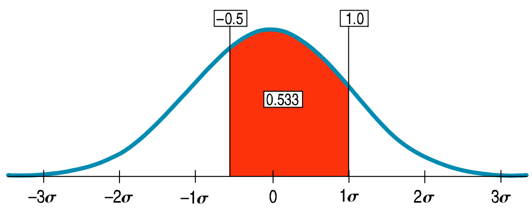


Your calculator knows the Normal model. Have a look under 2nd DISTR. There you will see three "norm" functions, normalpdf(, normalcdf(, and invNorm(. Let's play with the first two.

- normalpdf(calculates *y*-values for graphing a Normal curve. You probably won't use this very often, if at all. If you want to try it, graph Y1=normalpdf(X) in a graphing WINDOW with Xmin=-4, Xmax=4, Ymin=-0.1, and Ymax=0.5.
- normalcdf(finds the proportion of area under the curve between two *z*-score cut points, by specifying normalcdf(zLeft,zRight). Do make friends with this function; you will use it often!

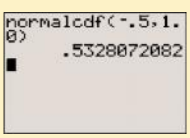
Example 1

The Normal model shown shades the region between $z = -0.5$ and $z = 1.0$.



To find the shaded area:

Under 2nd DISTR select normalcdf(; hit ENTER.
Specify the cut points: normalcdf(-.5,1.0) and hit ENTER again.



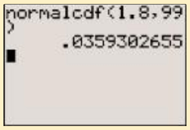
There's the area. Approximately 53% of a Normal model lies between half a standard deviation below and one standard deviation above the mean.

Example 2

In the example in the text we used Table Z to determine the fraction of SAT scores above your score of 680. Now let's do it again, this time using your TI.

First we need *z*-scores for the cut points:

- Since 680 is 1.8 standard deviations above the mean, your *z*-score is 1.8; that's the left cut point.



- Theoretically the standard Normal model extends rightward forever, but you can't tell the calculator to use infinity as the right cut point. Recall that for a Normal model almost all the area lies within ±3 standard deviations of the mean, so any upper cut point beyond, say, $z = 5$ does not cut off anything very important. We suggest you always use 99 (or −99) when you really want infinity as your cut point—it's easy to remember and way beyond any meaningful area.

Now you're ready. Use the command `normalcdf(1.8,99)`.

There you are! The Normal model estimates that approximately 3.6% of SAT scores are higher than 680.

STEP-BY-STEP EXAMPLE — Working with Normal Models Part I

The Normal model is our first model for data. It's the first in a series of modeling situations where we step away from the data at hand to make more general statements about the world. We'll become more practiced in thinking about and learning the details of models as we progress through the book. To give you some practice in thinking about the Normal model, here are several problems that ask you to find percentiles in detail.

Question: What proportion of SAT scores fall between 450 and 600?

THINK

Plan State the problem.

I want to know the proportion of SAT scores between 450 and 600.

Variables Name the variable.

Let y = SAT score.

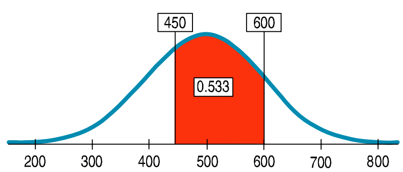
Check the appropriate conditions and specify which Normal model to use.

✓ **Nearly Normal Condition:** We are told that SAT scores are nearly Normal.

I'll model SAT scores with a $N(500, 100)$ model, using the mean and standard deviation specified for them.

SHOW

Mechanics Make a picture of this Normal model. Locate the desired values and shade the region of interest.



Find z-scores for the cut points 450 and 600. Use technology to find the desired proportions, represented by the area under the curve. (This was Example 1 in the TI Tips—take another look.)

Standardizing the two scores, I find that

$$z = \frac{(y - \mu)}{\sigma} = \frac{(600 - 500)}{100} = 1.00$$

and

$$z = \frac{(450 - 500)}{100} = -0.50$$

(If you use a table, then you need to subtract the two areas to find the area *between* the cut points.)

So,

Area $(450 < y < 600) = \text{Area}(-0.5 < z < 1.0)$
$= 0.5328$

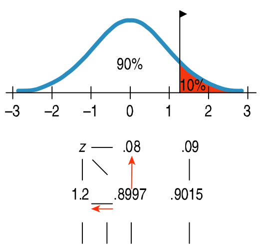
(**OR**: From Table Z, the *area* $(z < 1.0) = 0.8413$ and *area* $(z < -0.5) = 0.3085$, so the proportion of z-scores *between* them is $0.8413 - 0.3085 = 0.5328$, or 53.28%.)

TELL

Conclusion Interpret your result in context.

The Normal model estimates that about 53.3% of SAT scores fall between 450 and 600.

From Percentiles to Scores: *z* in Reverse



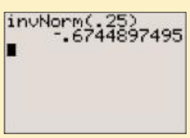
Finding areas from *z*-scores is the simplest way to work with the Normal model. But sometimes we start with areas and are asked to work backward to find the corresponding *z*-score or even the original data value. For instance, what *z*-score cuts off the top 10% in a Normal model?

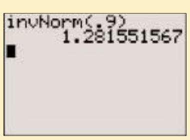
Make a picture like the one shown, shading the rightmost 10% of the area. Notice that this is the 90th percentile. Look in Table Z for an area of 0.900. The exact area is not there, but 0.8997 is pretty close. That shows up in the table with 1.2 in the left margin and .08 in the top margin. The *z*-score for the 90th percentile, then, is approximately $z = 1.28$.

Computers and calculators will determine the cut point more precisely (and more easily).

TI Tips

Finding Normal cutpoints





To find the *z*-score at the 25th percentile, go to `2nd DISTR` again. This time we'll use the third of the "norm" functions, `invNorm(`.

Just specify the desired percentile with the command `invNorm(.25)` and hit `ENTER`. The calculator says that the cut point for the leftmost 25% of a Normal model is approximately $z = -0.674$.

One more example: What *z*-score cuts off the highest 10% of a Normal model? That's easily done—just remember to specify the *percentile*. Since we want the cut point for the *highest* 10%, we know that the other 90% must be *below* that *z*-score. The cut point, then, must stand at the 90th percentile, so specify `invNorm(.90)`.

Only 10% of the area in a Normal model is more than about 1.28 standard deviations above the mean.

STEP-BY-STEP EXAMPLE — Working with Normal Models Part II

Question: Suppose a college says it admits only people with SAT Verbal test scores among the top 10%. How high a score does it take to be eligible?

THINK

Plan State the problem.

How high an SAT Verbal score do I need to be in the top 10% of all test takers?

Variable Define the variable.

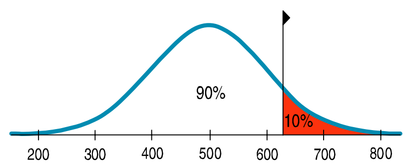
Let y = my SAT score.

Check to see if a Normal model is appropriate, and specify which Normal model to use.

✓ **Nearly Normal Condition:** I am told that SAT scores are nearly Normal. I'll model them with $N(500, 100)$.

SHOW

Mechanics Make a picture of this Normal model. Locate the desired percentile approximately by shading the rightmost 10% of the area.



The college takes the top 10%, so its cutoff score is the 90th percentile. Find the corresponding z-score using your calculator as shown in the TI Tips. (**OR**: Use Table Z as shown on p. 119.)

The cut point is $z = 1.28$.

Convert the z-score back to the original units.

A z-score of 1.28 is 1.28 standard deviations above the mean. Since the SD is 100, that's 128 SAT points. The cutoff is 128 points above the mean of 500, or 628.

TELL

Conclusion Interpret your results in the proper context.

Because the school wants SAT Verbal scores in the top 10%, the cutoff is 628. (Actually, since SAT scores are reported only in multiples of 10, I'd have to score at least a 630.)

TI-nspire

Normal models. Watch the Normal model react as you change the mean and standard deviation.

STEP-BY-STEP EXAMPLE More Working with Normal Models

Working with Normal percentiles can be a little tricky, depending on how the problem is stated. Here are a few more worked examples of the kind you're likely to see.

A cereal manufacturer has a machine that fills the boxes. Boxes are labeled "16 ounces," so the company wants to have that much cereal in each box, but since no packaging process is perfect, there will be minor variations. If the machine is set at exactly 16 ounces and the Normal model applies (or at least the distribution is roughly symmetric), then about half of the boxes will be underweight, making consumers unhappy and exposing the company to bad publicity and possible lawsuits. To prevent underweight boxes, the manufacturer has to set the mean a little higher than 16.0 ounces.

Based on their experience with the packaging machine, the company believes that the amount of cereal in the boxes fits a Normal model with a standard deviation of 0.2 ounces. The manufacturer decides to set the machine to put an average of 16.3 ounces in each box. Let's use that model to answer a series of questions about these cereal boxes.

Question 1: What fraction of the boxes will be underweight?

THINK

Plan State the problem.

What proportion of boxes weigh less than 16 ounces?

Variable Name the variable.

Let y = weight of cereal in a box.

Check to see if a Normal model is appropriate.

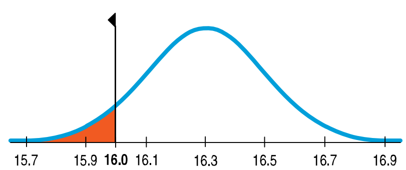
✓ **Nearly Normal Condition:** I have no data, so I cannot make a histogram, but I am told that the company believes the distribution of weights from the machine is Normal.

Specify which Normal model to use.

I'll use a N(16.3, 0.2) model.

SHOW

Mechanics Make a picture of this Normal model. Locate the value you're interested in on the picture, label it, and shade the appropriate region.



REALITY CHECK

Estimate from the picture the percentage of boxes that are underweight. (This will be useful later to check that your answer makes sense.) It looks like a low percentage. Less than 20% for sure.

I want to know what fraction of the boxes will weigh less than 16 ounces.

Convert your cutoff value into a z-score.

$$z = \frac{y - \mu}{\sigma} = \frac{16 - 16.3}{0.2} = -1.50$$

Find the area with your calculator (or use the Normal table).

$$\text{Area}\,(y < 16) = \text{Area}(z < -1.50) = 0.0668$$

TELL

Conclusion State your conclusion, and check that it's consistent with your earlier guess. It's below 20%—seems okay.

I estimate that approximately 6.7% of the boxes will contain less than 16 ounces of cereal.

Question 2: The company's lawyers say that 6.7% is too high. They insist that no more than 4% of the boxes can be underweight. So the company needs to set the machine to put a little more cereal in each box. What mean setting do they need?

THINK

Plan State the problem.

What mean weight will reduce the proportion of underweight boxes to 4%?

Variable Name the variable.

Let y = weight of cereal in a box.

Check to see if a Normal model is appropriate.

✔ **Nearly Normal Condition:** I am told that a Normal model applies.

Specify which Normal model to use. This time you are not given a value for the mean!

I don't know μ, the mean amount of cereal. The standard deviation for this machine is 0.2 ounces. The model is $N(\mu, 0.2)$.

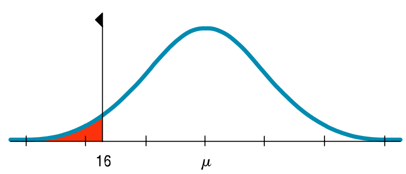
REALITY CHECK

We found out earlier that setting the machine to $\mu = 16.3$ ounces made 6.7% of the boxes too light. We'll need to raise the mean a bit to reduce this fraction.

No more than 4% of the boxes can be below 16 ounces.

SHOW

Mechanics Make a picture of this Normal model. Center it at μ (since you don't know the mean), and shade the region below 16 ounces.



Using your calculator (or the Normal table), find the z-score that cuts off the lowest 4%.

The z-score that has 0.04 area to the left of it is $z = -1.75$.

Use this information to find μ. It's located 1.75 standard deviations to the right of 16. Since σ is 0.2, that's 1.75×0.2, or 0.35 ounces more than 16.

For 16 to be 1.75 standard deviations below the mean, the mean must be

$$16 + 1.75(0.2) = 16.35 \text{ ounces.}$$

TELL

Conclusion Interpret your result in context.
(This makes sense; we knew it would have to be just a bit higher than 16.3.)

The company must set the machine to average 16.35 ounces of cereal per box.

Question 3: The company president vetoes that plan, saying the company should give away less free cereal, not more. Her goal is to set the machine no higher than 16.2 ounces and still have only 4% underweight boxes. The only way to accomplish this is to reduce the standard deviation. What standard deviation must the company achieve, and what does that mean about the machine?

THINK

Plan State the problem.

What standard deviation will allow the mean to be 16.2 ounces and still have only 4% of boxes underweight?

Variable Name the variable.

Let y = weight of cereal in a box.

Check conditions to be sure that a Normal model is appropriate.

✔ **Nearly Normal Condition:** The company believes that the weights are described by a Normal model.

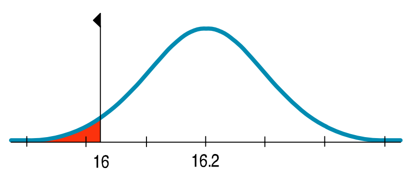
Specify which Normal model to use. This time you don't know σ.

I know the mean, but not the standard deviation, so my model is $N(16.2, \sigma)$.

REALITY CHECK We know the new standard deviation must be less than 0.2 ounces.

SHOW

Mechanics Make a picture of this Normal model. Center it at 16.2, and shade the area you're interested in. We want 4% of the area to the left of 16 ounces.



Find the z-score that cuts off the lowest 4%.

I know that the z-score with 4% below it is $z = -1.75$.

Solve for σ. (We need 16 to be 1.75 σ's below 16.2, so 1.75 σ must be 0.2 ounces. You could just start with that equation.)

$$z = \frac{y - \mu}{\sigma}$$

$$-1.75 = \frac{16 - 16.2}{\sigma}$$

$$1.75\,\sigma = 0.2$$

$$\sigma = 0.114$$

TELL

Conclusion Interpret your result in context.

As we expected, the standard deviation is lower than before—actually, quite a bit lower.

The company must get the machine to box cereal with a standard deviation of only 0.114 ounces. This means the machine must be more consistent (by nearly a factor of 2) in filling the boxes.

Are You Normal? Find Out with a Normal Probability Plot

In the examples we've worked through, we've assumed that the underlying data distribution was roughly unimodal and symmetric, so that using a Normal model makes sense. When you have data, you must *check* to see whether a Normal model is reasonable. How? Make a picture, of course! Drawing a histogram of the data and looking at the shape is one good way to see if a Normal model might work.

TI-*nspire*

Normal probability plots and histograms. See how a normal probability plot responds as you change the shape of a distribution.

There's a more specialized graphical display that can help you to decide whether the Normal model is appropriate: the **Normal probability plot.** If the distribution of the data is roughly Normal, the plot is roughly a diagonal straight line. Deviations from a straight line indicate that the distribution is not Normal. This plot is usually able to show deviations from Normality more clearly than the corresponding histogram, but it's usually easier to understand *how* a distribution fails to be Normal by looking at its histogram.

Some data on a car's fuel efficiency provide an example of data that are nearly Normal. The overall pattern of the Normal probability plot is straight. The two trailing low values correspond to the values in the histogram that trail off the low end. They're not quite in line with the rest of the data set. The Normal probability plot shows us that they're a bit lower than we'd expect of the lowest two values in a Normal model.

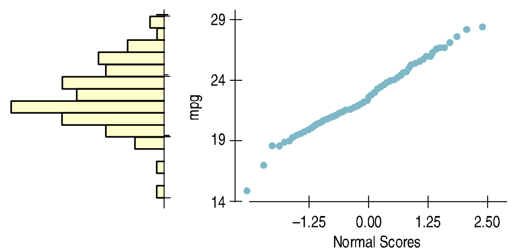


FIGURE 6.9

Histogram and Normal probability plot for gas mileage (mpg) recorded by one of the authors over the 8 years he owned a 1989 Nissan Maxima. The vertical axes are the same, so each dot on the probability plot would fall into the bar on the histogram immediately to its left.

By contrast, the Normal probability plot of the men's *Weight*s from the NHANES Study is far from straight. The weights are skewed to the high end, and the plot is curved. We'd conclude from these pictures that approximations using the 68–95–99.7 Rule for these data would not be very accurate.

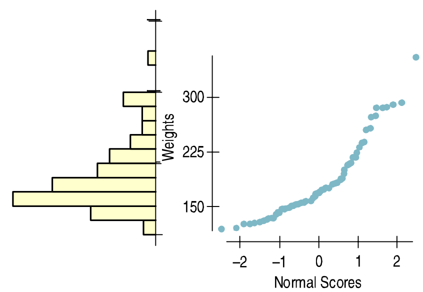
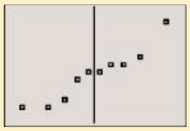


FIGURE 6.10

Histogram and Normal probability plot for men's weights. Note how a skewed distribution corresponds to a bent probability plot.

TI Tips

Creating a Normal probability plot



Let's make a Normal probability plot with the calculator. Here are the boys' agility test scores we looked at in Chapter 5; enter them in L1:

22, 17, 18, 29, 22, 23, 24, 23, 17, 21

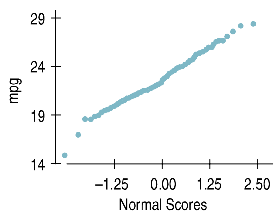
Now you can create the plot:

- Turn a STATPLOT On.
- Tell it to make a Normal probability plot by choosing the last of the icons.
- Specify your datalist and which axis you want the data on. (We'll use Y so the plot looks like the others we showed you.)
- Specify the Mark you want the plot to use.
- Now ZoomStat does the rest.

The plot doesn't look very straight. Normality is certainly questionable here.

(Not that it matters in making this decision, but that vertical line is the y-axis. Points to the left have negative z-scores and points to the right have positive z-scores.)

How Does a Normal Probability Plot Work?



A S ***Activity:*** **Assessing Normality.** This activity guides you through the process of checking the Nearly Normal condition using your statistics package.

Why does the Normal probability plot work like that? We looked at 100 fuel efficiency measures for the author's Nissan car. The smallest of these has a z-score of -3.16. The Normal model can tell us what value to expect for the smallest z-score in a batch of 100 if a Normal model were appropriate. That turns out to be -2.58. So our first data value is smaller than we would expect from the Normal.

We can continue this and ask a similar question for each value. For example, the 14th-smallest fuel efficiency has a z-score of almost exactly -1, and that's just what we should expect (well, -1.1 to be exact). A Normal probability plot takes each data value and plots it against the z-score you'd expect that point to have if the distribution were perfectly Normal.[7]

When the values match up well, the line is straight. If one or two points are surprising from the Normal's point of view, they don't line up. When the entire distribution is skewed or different from the Normal in some other way, the values don't match up very well at all and the plot bends.

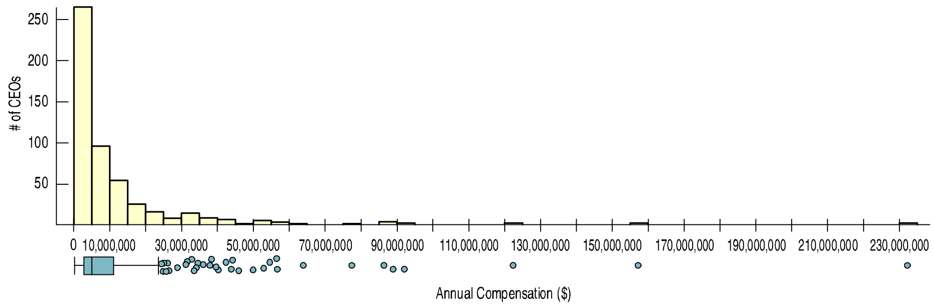
It turns out to be tricky to find the values we expect. They're called *Normal scores*, but you can't easily look them up in the tables. That's why probability plots are best made with technology and not by hand.

The best advice on using Normal probability plots is to see whether they are straight. If so, then your data look like data from a Normal model. If not, make a histogram to understand how they differ from the model.

[7] Sometimes the Normal probability plot switches the two axes, putting the data on the x-axis and the z-scores on the y-axis.

WHAT CAN GO WRONG?

- **Don't use a Normal model when the distribution is not unimodal and symmetric.** Normal models are so easy and useful that it is tempting to use them even when they don't describe the data very well. That can lead to wrong conclusions. Don't use a Normal model without first checking the **Nearly Normal Condition.** Look at a picture of the data to check that it is unimodal and symmetric. A histogram, or a Normal probability plot, can help you tell whether a Normal model is appropriate.



The CEOs (p. 90) had a mean total compensation of \$10,307,311.87 with a standard deviation of \$17,964,615.16. Using the Normal model rule, we should expect about 68% of the CEOs to have compensations between −\$7,657,303.29 and \$28,271,927.03. In fact, more than 90% of the CEOs have annual compensations in this range. What went wrong? The distribution is skewed, not symmetric. Using the 68–95–99.7 Rule for data like these will lead to silly results.

- **Don't use the mean and standard deviation when outliers are present.** Both means and standard deviations can be distorted by outliers, and no model based on distorted values will do a good job. A z-score calculated from a distribution with outliers may be misleading. It's always a good idea to check for outliers. How? Make a picture.
- **Don't round your results in the middle of a calculation.** We *reported* the mean of the heptathletes' long jump as 6.16 meters. More precisely, it was 6.16153846153846 meters.

  You should use all the precision available in the data for all the intermediate steps of a calculation. Using the more precise value for the mean (and also carrying 15 digits for the SD), the z-score calculation for Klüft's long jump comes out to

$$z = \frac{6.78 - 6.16153846153846}{0.2297597407326585} = 2.691775053755667700$$

  We'd report that as 2.692, as opposed to the rounded-off value of 2.70 we got earlier from the table.
- **Don't worry about minor differences in results.** Because various calculators and programs may carry different precision in calculations, your answers may differ slightly from those we show in the text and in the Step-By-Steps, or even from the values given in the answers in the back of the book. Those differences aren't anything to worry about. They're not the main story Statistics tries to tell.

CONNECTIONS

Changing the center and spread of a variable is equivalent to changing its *units*. Indeed, the only part of the data's context changed by standardizing is the units. All other aspects of the context do not depend on the choice or modification of measurement units. This fact points out an important distinction between the numbers the data provide for calculation and the meaning of the variables and the relationships among them. Standardizing can make the numbers easier to work with, but it does not alter the meaning.

Another way to look at this is to note that standardizing may change the center and spread values, but it does not affect the *shape* of a distribution. A histogram or boxplot of standardized values looks just the same as the histogram or boxplot of the original values except, perhaps, for the numbers on the axes.

When we summarized *shape, center,* and *spread* for histograms, we compared them to unimodal, symmetric shapes. You couldn't ask for a nicer example than the Normal model. And if the shape *is* like a Normal, we'll use the the mean and standard deviation to standardize the values.

WHAT HAVE WE LEARNED?

We've learned that the story data can tell may be easier to understand after shifting or rescaling the data.

- Shifting data by adding or subtracting the same amount from each value affects measures of center and position but not measures of spread.
- Rescaling data by multiplying or dividing every value by a constant, changes all the summary statistics—center, position, and spread.

We've learned the power of standardizing data.

- Standardizing uses the standard deviation as a ruler to measure distance from the mean, creating *z*-scores.
- Using these *z*-scores, we can compare apples and oranges—values from different distributions or values based on different units.
- And a *z*-score can identify unusual or surprising values among data.

We've learned that the 68–95–99.7 Rule can be a useful rule of thumb for understanding distributions.

- For data that are unimodal and symmetric, about 68% fall within 1 SD of the mean, 95% fall within 2 SDs of the mean, and 99.7% fall within 3 SDs of the mean (see p. 130).

Again we've seen the importance of *Thinking* about whether a method will work.

- **Normality Assumption:** We sometimes work with Normal tables (Table Z). Those tables are based on the Normal model.
- Data can't be exactly Normal, so we check the **Nearly Normal Condition** by making a histogram (is it unimodal, symmetric, and free of outliers?) or a Normal probability plot (is it straight enough?). (See p. 125.)

Terms

Standardizing 105. We standardize to eliminate units. Standardized values can be compared and combined even if the original variables had different units and magnitudes.

Standardized value 105. A value found by subtracting the mean and dividing by the standard deviation.

Shifting 107. Adding a constant to each data value adds the same constant to the mean, the median, and the quartiles, but does not change the standard deviation or IQR.

Rescaling 108. Multiplying each data value by a constant multiplies both the measures of position (mean, median, and quartiles) and the measures of spread (standard deviation and IQR) by that constant.

Normal model 112. A useful family of models for unimodal, symmetric distributions.

Parameter 112. A numerically valued attribute of a model. For example, the values of μ and σ in a $N(\mu, \sigma)$ model are parameters.

Statistic 112. A value calculated from data to summarize aspects of the data. For example, the mean, $\bar{y}$ and standard deviation, s, are statistics.

***z*-score** 105. A *z*-score tells how many standard deviations a value is from the mean; *z*-scores have a mean of 0 and a standard deviation of 1. When working with data, use the statistics $\bar{y}$ and s:

$$z = \frac{y - \bar{y}}{s}.$$

112. When working with models, use the parameters μ and σ:

$$z = \frac{y - \mu}{\sigma}.$$

Standard Normal model 112. A Normal model, $N(\mu, \sigma)$ with mean $\mu = 0$ and standard deviation $\sigma = 1$. Also called the **standard Normal distribution.**

Nearly Normal Condition 112. A distribution is nearly Normal if it is unimodal and symmetric. We can check by looking at a histogram or a Normal probability plot.

68–95–99.7 Rule 113. In a Normal model, about 68% of values fall within 1 standard deviation of the mean, about 95% fall within 2 standard deviations of the mean, and about 99.7% fall within 3 standard deviations of the mean.

Normal percentile 116. The Normal percentile corresponding to a *z*-score gives the percentage of values in a standard Normal distribution found at that *z*-score or below.

Normal probability plot 124. A display to help assess whether a distribution of data is approximately Normal. If the plot is nearly straight, the data satisfy the **Nearly Normal Condition.**

Skills

THINK

- Understand how adding (subtracting) a constant or multiplying (dividing) by a constant changes the center and/or spread of a variable.
- Recognize when standardization can be used to compare values.
- Understand that standardizing uses the standard deviation as a ruler.
- Recognize when a Normal model is appropriate.

SHOW

- Know how to calculate the *z*-score of an observation.
- Know how to compare values of two different variables using their *z*-scores.
- Be able to use Normal models and the 68–95–99.7 Rule to estimate the percentage of observations falling within 1, 2, or 3 standard deviations of the mean.
- Know how to find the percentage of observations falling below any value in a Normal model using a Normal table or appropriate technology.
- Know how to check whether a variable satisfies the **Nearly Normal Condition** by making a Normal probability plot or a histogram.

TELL

- Know what *z*-scores mean.
- Be able to explain how extraordinary a standardized value may be by using a Normal model.

NORMAL PLOTS ON THE COMPUTER

The best way to tell whether your data can be modeled well by a Normal model is to make a picture or two. We've already talked about making histograms. Normal probability plots are almost never made by hand because the values of the Normal scores are tricky to find. But most statistics software make Normal plots, though various packages call the same plot by different names and array the information differently.

EXERCISES

1. **Shipments.** A company selling clothing on the Internet reports that the packages it ships have a median weight of 68 ounces and an IQR of 40 ounces.
   a) The company plans to include a sales flyer weighing 4 ounces in each package. What will the new median and IQR be?
   b) If the company recorded the shipping weights of these new packages in pounds instead of ounces, what would the median and IQR be? (1 lb. = 16 oz.)

2. **Hotline.** A company's customer service hotline handles many calls relating to orders, refunds, and other issues. The company's records indicate that the median length of calls to the hotline is 4.4 minutes with an IQR of 2.3 minutes.
   a) If the company were to describe the duration of these calls in seconds instead of minutes, what would the median and IQR be?
   b) In an effort to speed up the customer service process, the company decides to streamline the series of push-button menus customers must navigate, cutting the time by 24 seconds. What will the median and IQR of the length of hotline calls become?

3. **Payroll.** Here are the summary statistics for the weekly payroll of a small company: lowest salary = \$300, mean salary = \$700, median = \$500, range = \$1200, IQR = \$600, first quartile = \$350, standard deviation = \$400.
   a) Do you think the distribution of salaries is symmetric, skewed to the left, or skewed to the right? Explain why.
   b) Between what two values are the middle 50% of the salaries found?
   c) Suppose business has been good and the company gives every employee a \$50 raise. Tell the new value of each of the summary statistics.
   d) Instead, suppose the company gives each employee a 10% raise. Tell the new value of each of the summary statistics.

4. **Hams.** A specialty foods company sells "gourmet hams" by mail order. The hams vary in size from 4.15 to 7.45 pounds, with a mean weight of 6 pounds and standard deviation of 0.65 pounds. The quartiles and median weights are 5.6, 6.2, and 6.55 pounds.
   a) Find the range and the IQR of the weights.
   b) Do you think the distribution of the weights is symmetric or skewed? If skewed, which way? Why?
   c) If these weights were expressed in ounces (1 pound = 16 ounces) what would the mean, standard deviation, quartiles, median, IQR, and range be?
   d) When the company ships these hams, the box and packing materials add 30 ounces. What are the mean, standard deviation, quartiles, median, IQR, and range of weights of boxes shipped (in ounces)?
   e) One customer made a special order of a 10-pound ham. Which of the summary statistics of part d might *not* change if that data value were added to the distribution?

5. **SAT or ACT?** Each year thousands of high school students take either the SAT or the ACT, standardized tests used in the college admissions process. Combined SAT Math and Verbal scores go as high as 1600, while the maximum ACT composite score is 36. Since the two exams use very different scales, comparisons of performance are difficult. A convenient rule of thumb is $SAT = 40 \times ACT + 150$; that is, multiply an ACT score by 40 and add 150 points to estimate the equivalent SAT score. An admissions officer reported the following statistics about the ACT scores of 2355 students who applied to her college one year. Find the summaries of equivalent SAT scores.

   Lowest score = 19   Mean = 27   Standard deviation = 3
   Q3 = 30   Median = 28   IQR = 6

6. **Cold U?** A high school senior uses the Internet to get information on February temperatures in the town where he'll be going to college. He finds a Web site with some statistics, but they are given in degrees Celsius. The conversion formula is $°F = 9/5\ °C + 32$. Determine the Fahrenheit equivalents for the summary information below.

   Maximum temperature = 11°C   Range = 33°
   Mean = 1°   Standard deviation = 7°
   Median = 2°   IQR = 16°

7. **Stats test.** Suppose your Statistics professor reports test grades as *z*-scores, and you got a score of 2.20 on an exam. Write a sentence explaining what that means.

8. **Checkup.** One of the authors has an adopted grandson whose birth family members are very short. After examining him at his 2-year checkup, the boy's pediatrician said that the *z*-score for his height relative to American 2-year-olds was −1.88. Write a sentence explaining what that means.

9. Stats test, part II. The mean score on the Stats exam was 75 points with a standard deviation of 5 points, and Gregor's z-score was -2. How many points did he score?

10. Mensa. People with z-scores above 2.5 on an IQ test are sometimes classified as geniuses. If IQ scores have a mean of 100 and a standard deviation of 16 points, what IQ score do you need to be considered a genius?

11. Temperatures. A town's January high temperatures average 36°F with a standard deviation of 10°, while in July the mean high temperature is 74° and the standard deviation is 8°. In which month is it more unusual to have a day with a high temperature of 55°? Explain.

12. Placement exams. An incoming freshman took her college's placement exams in French and mathematics. In French, she scored 82 and in math 86. The overall results on the French exam had a mean of 72 and a standard deviation of 8, while the mean math score was 68, with a standard deviation of 12. On which exam did she do better compared with the other freshmen?

13. Combining test scores. The first Stats exam had a mean of 65 and a standard deviation of 10 points; the second had a mean of 80 and a standard deviation of 5 points. Derrick scored an 80 on both tests. Julie scored a 70 on the first test and a 90 on the second. They both totaled 160 points on the two exams, but Julie claims that her total is better. Explain.

14. Combining scores again. The first Stat exam had a mean of 80 and a standard deviation of 4 points; the second had a mean of 70 and a standard deviation of 15 points. Reginald scored an 80 on the first test and an 85 on the second. Sara scored an 88 on the first but only a 65 on the second. Although Reginald's total score is higher, Sara feels she should get the higher grade. Explain her point of view.

15. Final exams. Anna, a language major, took final exams in both French and Spanish and scored 83 on each. Her roommate Megan, also taking both courses, scored 77 on the French exam and 95 on the Spanish exam. Overall, student scores on the French exam had a mean of 81 and a standard deviation of 5, and the Spanish scores had a mean of 74 and a standard deviation of 15.
a) To qualify for language honors, a major must maintain at least an 85 average for all language courses taken. So far, which student qualifies?
b) Which student's overall performance was better?

16. MP3s. Two companies market new batteries targeted at owners of personal music players. DuraTunes claims a mean battery life of 11 hours, while RockReady advertises 12 hours.
a) Explain why you would also like to know the standard deviations of the battery lifespans before deciding which brand to buy.
b) Suppose those standard deviations are 2 hours for DuraTunes and 1.5 hours for RockReady. You are headed for 8 hours at the beach. Which battery is most likely to last all day? Explain.
c) If your beach trip is all weekend, and you probably will have the music on for 16 hours, which battery is most likely to last? Explain.

17. Cattle. The Virginia Cooperative Extension reports that the mean weight of yearling Angus steers is 1152 pounds. Suppose that weights of all such animals can be described by a Normal model with a standard deviation of 84 pounds.
a) How many standard deviations from the mean would a steer weighing 1000 pounds be?
b) Which would be more unusual, a steer weighing 1000 pounds or one weighing 1250 pounds?

T **18. Car speeds.** John Beale of Stanford, CA, recorded the speeds of cars driving past his house, where the speed limit read 20 mph. The mean of 100 readings was 23.84 mph, with a standard deviation of 3.56 mph. (He actually recorded every car for a two-month period. These are 100 representative readings.)
a) How many standard deviations from the mean would a car going under the speed limit be?
b) Which would be more unusual, a car traveling 34 mph or one going 10 mph?

19. More cattle. Recall that the beef cattle described in Exercise 17 had a mean weight of 1152 pounds, with a standard deviation of 84 pounds.
a) Cattle buyers hope that yearling Angus steers will weigh at least 1000 pounds. To see how much over (or under) that goal the cattle are, we could subtract 1000 pounds from all the weights. What would the new mean and standard deviation be?
b) Suppose such cattle sell at auction for 40 cents a pound. Find the mean and standard deviation of the sale prices for all the steers.

T **20. Car speeds again.** For the car speed data of Exercise 18, recall that the mean speed recorded was 23.84 mph, with a standard deviation of 3.56 mph. To see how many cars are speeding, John subtracts 20 mph from all speeds.
a) What is the mean speed now? What is the new standard deviation?
b) His friend in Berlin wants to study the speeds, so John converts all the original miles-per-hour readings to kilometers per hour by multiplying all speeds by 1.609 (km per mile). What is the mean now? What is the new standard deviation?

21. Cattle, part III. Suppose the auctioneer in Exercise 19 sold a herd of cattle whose minimum weight was 980 pounds, median was 1140 pounds, standard deviation 84 pounds, and IQR 102 pounds. They sold for 40 cents a pound, and the auctioneer took a \$20 commission on each animal. Then, for example, a steer weighing 1100 pounds would net the owner $0.40(1100) - 20 = \$420$. Find the minimum, median, standard deviation, and IQR of the net sale prices.

22. Caught speeding. Suppose police set up radar surveillance on the Stanford street described in Exercise 18. They handed out a large number of tickets to speeders going a mean of 28 mph, with a standard deviation of 2.4 mph, a maximum of 33 mph, and an IQR of 3.2 mph. Local law prescribes fines of \$100, plus \$10 per mile per hour over the 20 mph speed limit. For example, a driver convicted of going 25 mph would be fined $100 + 10(5) = \$150$. Find the mean, maximum, standard deviation, and IQR of all the potential fines.

23. Professors. A friend tells you about a recent study dealing with the number of years of teaching experience among current college professors. He remembers the mean but can't recall whether the standard deviation was 6 months, 6 years, or 16 years. Tell him which one it must have been, and why.

24. Rock concerts. A popular band on tour played a series of concerts in large venues. They always drew a large crowd, averaging 21,359 fans. While the band did not announce (and probably never calculated) the standard deviation, which of these values do you think is most likely to be correct: 20, 200, 2000, or 20,000 fans? Explain your choice.

25. Guzzlers? Environmental Protection Agency (EPA) fuel economy estimates for automobile models tested recently predicted a mean of 24.8 mpg and a standard deviation of 6.2 mpg for highway driving. Assume that a Normal model can be applied.

a) Draw the model for auto fuel economy. Clearly label it, showing what the 68–95–99.7 Rule predicts.
b) In what interval would you expect the central 68% of autos to be found?
c) About what percent of autos should get more than 31 mpg?
d) About what percent of cars should get between 31 and 37.2 mpg?
e) Describe the gas mileage of the worst 2.5% of all cars.

26. IQ. Some IQ tests are standardized to a Normal model, with a mean of 100 and a standard deviation of 16.

a) Draw the model for these IQ scores. Clearly label it, showing what the 68–95–99.7 Rule predicts.
b) In what interval would you expect the central 95% of IQ scores to be found?
c) About what percent of people should have IQ scores above 116?
d) About what percent of people should have IQ scores between 68 and 84?
e) About what percent of people should have IQ scores above 132?

27. Small steer. In Exercise 17 we suggested the model $N(1152, 84)$ for weights in pounds of yearling Angus steers. What weight would you consider to be unusually low for such an animal? Explain.

28. High IQ. Exercise 26 proposes modeling IQ scores with $N(100, 16)$. What IQ would you consider to be unusually high? Explain.

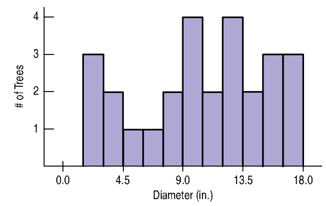
29. Trees. A forester measured 27 of the trees in a large woods that is up for sale. He found a mean diameter of 10.4 inches and a standard deviation of 4.7 inches. Suppose that these trees provide an accurate description of the whole forest and that a Normal model applies.

a) Draw the Normal model for tree diameters.
b) What size would you expect the central 95% of all trees to be?
c) About what percent of the trees should be less than an inch in diameter?
d) About what percent of the trees should be between 5.7 and 10.4 inches in diameter?
e) About what percent of the trees should be over 15 inches in diameter?

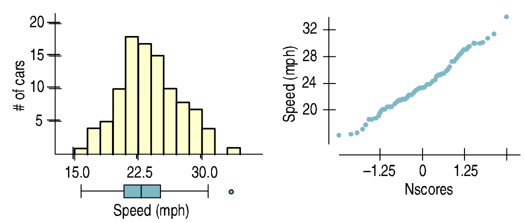
30. Rivets. A company that manufactures rivets believes the shear strength (in pounds) is modeled by $N(800, 50)$.

a) Draw and label the Normal model.
b) Would it be safe to use these rivets in a situation requiring a shear strength of 750 pounds? Explain.
c) About what percent of these rivets would you expect to fall below 900 pounds?
d) Rivets are used in a variety of applications with varying shear strength requirements. What is the maximum shear strength for which you would feel comfortable approving this company's rivets? Explain your reasoning.

31. Trees, part II. Later on, the forester in Exercise 29 shows you a histogram of the tree diameters he used in analyzing the woods that was for sale. Do you think he was justified in using a Normal model? Explain, citing some specific concerns.



T **32. Car speeds, the picture.** For the car speed data of Exercise 18, here is the histogram, boxplot, and Normal probability plot of the 100 readings. Do you think it is appropriate to apply a Normal model here? Explain.



T **33. Winter Olympics 2006 downhill.** Fifty-three men qualified for the men's alpine downhill race in Torino. The gold medal winner finished in 1 minute, 48.8 seconds. All competitors' times (in seconds) are found in the following list:

108.80	109.52	109.82	109.88	109.93	110.00
110.04	110.12	110.29	110.33	110.35	110.44
110.45	110.64	110.68	110.70	110.72	110.84
110.88	110.88	110.90	110.91	110.98	111.37
111.48	111.51	111.55	111.70	111.72	111.93
112.17	112.55	112.87	112.90	113.34	114.07
114.65	114.70	115.01	115.03	115.73	116.10
116.58	116.81	117.45	117.54	117.56	117.69
118.77	119.24	119.41	119.79	120.93	

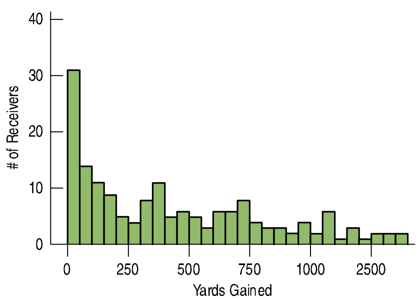
a) The mean time was 113.02 seconds, with a standard deviation of 3.24 seconds. If the Normal model is appropriate, what percent of times will be less than 109.78 seconds?
b) What is the actual percent of times less than 109.78 seconds?
c) Why do you think the two percentages don't agree?
d) Create a histogram of these times. What do you see?

34. Check the model. The mean of the 100 car speeds in Exercise 20 was 23.84 mph, with a standard deviation of 3.56 mph.
a) Using a Normal model, what values should border the middle 95% of all car speeds?
b) Here are some summary statistics.

Percentile		Speed
100%	**Max**	34.060
97.5%		30.976
90.0%		28.978
75.0%	**Q3**	25.785
50.0%	**Median**	23.525
25.0%	**Q1**	21.547
10.0%		19.163
2.5%		16.638
0.0%	**Min**	16.270

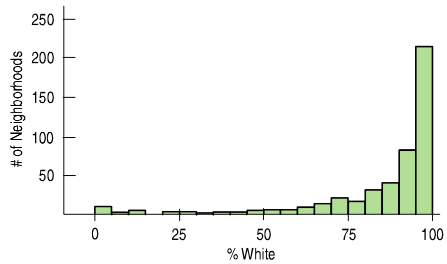
From your answer in part a, how well does the model do in predicting those percentiles? Are you surprised? Explain.

35. Receivers. NFL data from the 2006 football season reported the number of yards gained by each of the league's 167 wide receivers:



The mean is 435 yards, with a standard deviation of 384 yards.
a) According to the Normal model, what percent of receivers would you expect to gain fewer yards than 2 standard deviations below the mean number of yards?
b) For these data, what does that mean?
c) Explain the problem in using a Normal model here.

36. Customer database. A large philanthropic organization keeps records on the people who have contributed to their cause. In addition to keeping records of past giving, the organization buys demographic data on neighborhoods from the U.S. Census Bureau. Eighteen of these variables concern the ethnicity of the neighborhood of the donor. Here are a histogram and summary statistics for the percentage of whites in the neighborhoods of 500 donors:



Count	500
Mean	83.59
Median	93
StdDev	22.26
IQR	17
Q1	80
Q3	97

a) Which is a better summary of the percentage of white residents in the neighborhoods, the mean or the median? Explain.
b) Which is a better summary of the spread, the IQR or the standard deviation? Explain.
c) From a Normal model, about what percentage of neighborhoods should have a percent white within one standard deviation of the mean?
d) What percentage of neighborhoods actually have a percent white within one standard deviation of the mean?
e) Explain the discrepancy between parts c and d.

37. Normal cattle. Using $N(1152, 84)$, the Normal model for weights of Angus steers in Exercise 17, what percent of steers weigh
a) over 1250 pounds?
b) under 1200 pounds?
c) between 1000 and 1100 pounds?

38. IQs revisited. Based on the Normal model $N(100, 16)$ describing IQ scores, what percent of people's IQs would you expect to be
a) over 80?
b) under 90?
c) between 112 and 132?

39. More cattle. Based on the model $N(1152, 84)$ describing Angus steer weights, what are the cutoff values for
a) the highest 10% of the weights?
b) the lowest 20% of the weights?
c) the middle 40% of the weights?

40. More IQs. In the Normal model $N(100, 16)$, what cutoff value bounds
a) the highest 5% of all IQs?
b) the lowest 30% of the IQs?
c) the middle 80% of the IQs?

41. Cattle, finis. Consider the Angus weights model $N(1152, 84)$ one last time.
a) What weight represents the 40th percentile?
b) What weight represents the 99th percentile?
c) What's the IQR of the weights of these Angus steers?

42. IQ, finis. Consider the IQ model $N(100, 16)$ one last time.
a) What IQ represents the 15th percentile?
b) What IQ represents the 98th percentile?
c) What's the IQR of the IQs?

43. Cholesterol. Assume the cholesterol levels of adult American women can be described by a Normal model with a mean of 188 mg/dL and a standard deviation of 24.
a) Draw and label the Normal model.
b) What percent of adult women do you expect to have cholesterol levels over 200 mg/dL?
c) What percent of adult women do you expect to have cholesterol levels between 150 and 170 mg/dL?
d) Estimate the IQR of the cholesterol levels.
e) Above what value are the highest 15% of women's cholesterol levels?

44. Tires. A tire manufacturer believes that the treadlife of its snow tires can be described by a Normal model with a mean of 32,000 miles and standard deviation of 2500 miles.
a) If you buy a set of these tires, would it be reasonable for you to hope they'll last 40,000 miles? Explain.
b) Approximately what fraction of these tires can be expected to last less than 30,000 miles?
c) Approximately what fraction of these tires can be expected to last between 30,000 and 35,000 miles?
d) Estimate the IQR of the treadlives.
e) In planning a marketing strategy, a local tire dealer wants to offer a refund to any customer whose tires fail to last a certain number of miles. However, the dealer does not want to take too big a risk. If the dealer is willing to give refunds to no more than 1 of every 25 customers, for what mileage can he guarantee these tires to last?

45. Kindergarten. Companies that design furniture for elementary school classrooms produce a variety of sizes for kids of different ages. Suppose the heights of kindergarten children can be described by a Normal model with a mean of 38.2 inches and standard deviation of 1.8 inches.
a) What fraction of kindergarten kids should the company expect to be less than 3 feet tall?
b) In what height interval should the company expect to find the middle 80% of kindergarteners?
c) At least how tall are the biggest 10% of kindergarteners?

46. Body temperatures. Most people think that the "normal" adult body temperature is 98.6°F. That figure, based on a 19th-century study, has recently been challenged. In a 1992 article in the *Journal of the American Medical Association*, researchers reported that a more accurate figure may be 98.2°F. Furthermore, the standard deviation appeared to be around 0.7°F. Assume that a Normal model is appropriate.
a) In what interval would you expect most people's body temperatures to be? Explain.
b) What fraction of people would be expected to have body temperatures above 98.6°F?
c) Below what body temperature are the coolest 20% of all people?

47. Eggs. Hens usually begin laying eggs when they are about 6 months old. Young hens tend to lay smaller eggs, often weighing less than the desired minimum weight of 54 grams.
a) The average weight of the eggs produced by the young hens is 50.9 grams, and only 28% of their eggs exceed the desired minimum weight. If a Normal model is appropriate, what would the standard deviation of the egg weights be?
b) By the time these hens have reached the age of 1 year, the eggs they produce average 67.1 grams, and 98% of them are above the minimum weight. What is the standard deviation for the appropriate Normal model for these older hens?
c) Are egg sizes more consistent for the younger hens or the older ones? Explain.

48. Tomatoes. Agricultural scientists are working on developing an improved variety of Roma tomatoes. Marketing research indicates that customers are likely to bypass Romas that weigh less than 70 grams. The current variety of Roma plants produces fruit that averages 74 grams, but 11% of the tomatoes are too small. It is reasonable to assume that a Normal model applies.
a) What is the standard deviation of the weights of Romas now being grown?
b) Scientists hope to reduce the frequency of undersized tomatoes to no more than 4%. One way to accomplish this is to raise the average size of the fruit. If the standard deviation remains the same, what target mean should they have as a goal?
c) The researchers produce a new variety with a mean weight of 75 grams, which meets the 4% goal. What is the standard deviation of the weights of these new Romas?
d) Based on their standard deviations, compare the tomatoes produced by the two varieties.

JUST CHECKING
Answers

1. **a)** On the first test, the mean is 88 and the SD is 4, so $z = (90 - 88)/4 = 0.5$. On the second test, the mean is 75 and the SD is 5, so $z = (80 - 75)/5 = 1.0$. The first test has the lower z-score, so it is the one that will be dropped.
   b) No. The second test is 1 standard deviation above the mean, farther away than the first test, so it's the better score relative to the class.
2. **a)** The mean would increase to 500.
   b) The standard deviation is still 100 points.
   c) The two boxplots would look nearly identical (the shape of the distribution would remain the same), but the later one would be shifted 50 points higher.
3. The standard deviation is now 2.54 millimeters, which is the same as 0.1 inches. Nothing has changed. The standard deviation has "increased" only because we're reporting it in millimeters now, not inches.
4. The mean is 184 centimeters, with a standard deviation of 8 centimeters. 2 meters is 200 centimeters, which is 2 standard deviations above the mean. We expect 5% of the men to be more than 2 standard deviations below or above the mean, so half of those, 2.5%, are likely to be above 2 meters.
5. **a)** We know that 68% of the time we'll be within 1 standard deviation (2 min) of 20. So 32% of the time we'll arrive in less than 18 or more than 22 minutes. Half of those times (16%) will be greater than 22 minutes, so 84% will be less than 22 minutes.
   b) 24 minutes is 2 standard deviations above the mean. Because of the 95% rule, we know 2.5% of the times will be more than 24 minutes.
   c) Traffic incidents may occasionally increase the time it takes to get to school, so the driving times may be skewed to the right, and there may be outliers.
   d) If so, the Normal model would not be appropriate and the percentages we predict would not be accurate.