Name ________________________ Date ____________ Class ____________

LESSON 3-1

Review for Mastery

Using Graphs and Tables to Solve Linear Systems

A **linear system** of equations is a set of two or more linear equations. To **solve a linear system**, find all the ordered pairs (x, y) that make both equations true. Use a table and a graph to solve a system of equations.

$\begin{cases} y + x = 2 \\ y - 2x = 5 \end{cases}$ Solve each equation for y.→ $\begin{cases} y = -x + 2 \\ y = 2x + 5 \end{cases}$

Make a table of values for each equation.

$y = -x + 2$	
x	y
-2	4
-1	3
0	2
1	1

↔

$y = 2x + 5$	
x	y
-2	1
-1	3
0	5
1	7

When $x = -1$, $y = 3$ for both equations.

On a graph, the point where the lines intersect is the solution.

Use the table to draw the graph of each equation.

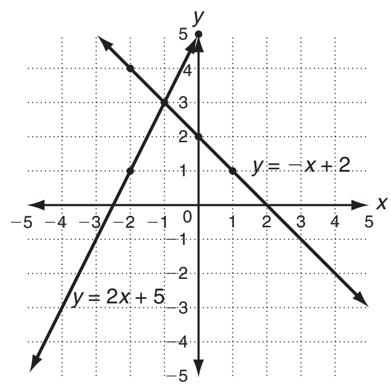
The lines appear to intersect at $(-1, 3)$.

Substitute $(-1, 3)$ into the original equations to check.

$y + x = 2$ $\quad$ $y - 2x = 5$

$3 + (-1) \stackrel{?}{=} 2$ $\quad$ $3 - 2(-1) \stackrel{?}{=} 5$

$2 = 2$✓ $\quad$ $5 = 5$✓



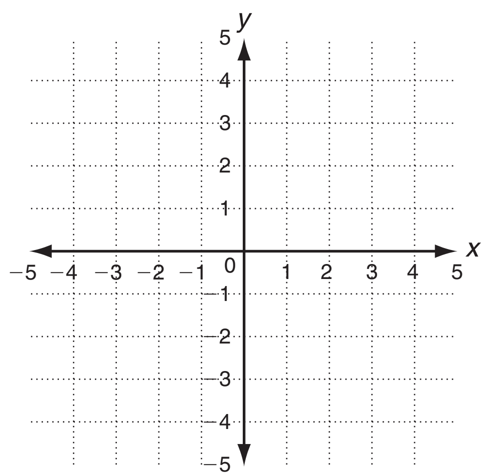
Solve the system using a table and a graph. Give the ordered pair that solves both equations.

1. $\begin{cases} x + y = 1 \\ 2x - y = 5 \end{cases}$

Solution: ________________________

$y = -x + 1$	
x	y
0	
1	
2	
3	

$y = 2x - 5$	
x	y
0	
1	
2	
3	



Name ______________________________________ Date ___________ Class ____________

LESSON **3-1**

Review for Mastery

Using Graphs and Tables to Solve Linear Systems (continued)

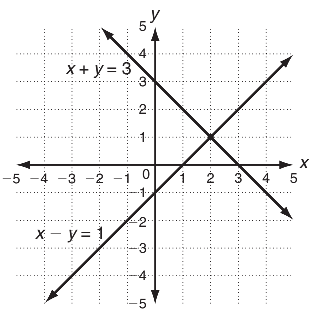
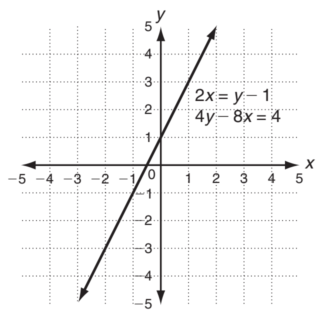
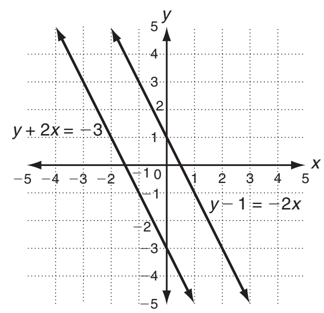
To classify a linear system:

Step 1 Write each equation in the form $y = mx + b$.

Step 2 Compare the slopes and y-intercepts.

Step 3 Classify by the number of solutions of the system.

Remember: m = slope and b = y-intercept.

Exactly One Solution Independent	**Infinitely Many Solutions Dependent**	**No Solution Inconsistent**
The lines have **different slopes** and intersect at one point.	The lines have the **same slope and *y*-intercept.** Their graph is the same line.	The lines have the **same slope and different *y*-intercepts.** The lines are parallel.
$\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ Solve each equation for y. $\begin{cases} y = -x + 3;\ m = -1 \\ y = x - 1;\ m = 1 \end{cases}$ The slopes are different. The system has one solution and is independent.	$\begin{cases} 2x = y - 1 \\ 4y - 8x = 4 \end{cases}$ Solve each equation for y. $\begin{cases} y = 2x + 1;\ m = 2,\ b = 1 \\ y = 2x + 1;\ m = 2,\ b = 1 \end{cases}$ The slopes and the y-intercepts are the same. The system has infinitely many solutions and is dependent.	$\begin{cases} y + 2x = -3 \\ y - 1 = -2x \end{cases}$ Solve each equation for y. $\begin{cases} y = -2x - 3;\ m = -2,\ b = -3 \\ y = -2x + 1;\ m = -2,\ b = 1 \end{cases}$ The slopes are the same but the y-intercepts are different. The system has no solution and is inconsistent.
		

Classify each system and determine the number of solutions.

2. $\begin{cases} y + x = 2 \\ y + 1 = -x \end{cases}$

y = ________ , m = ____, b = ____

y = ________, m = ____, b = ____

Number of solutions: _______________

____

3. $\begin{cases} y + 1 = 3x \\ 2y - 6x = -2 \end{cases}$

y = ________, m = ____, b = ____

y = ________, m = ____, b = ____

Number of solutions: _________________

____

Name ______________________________ Date __________ Class __________

LESSON 3-2

Review for Mastery

Using Algebraic Methods to Solve Linear Systems

To use the **substitution method** to solve a system of linear equations:

1. Solve one equation for one variable.
2. Substitute this expression into the other equation.
3. Solve for the other variable.
4. Substitute the value of the known variable in the equation in Step 1.
5. Solve for the other variable.
6. Check the values in both equations.

$$\begin{cases} y = x + 2 \\ 2x + y = 17 \end{cases}$$

Use this equation. It is solved for y.

Use the substitution method when the coefficient of one of the variables is 1 or -1.

$$2x + y = 17$$

$$2x + (x + 2) = 17 \quad \textit{Substitute } x + 2 \textit{ for } y.$$

$$3x + 2 = 17 \quad \textit{Simplify and solve for } x.$$

$$3x = 15$$

$$x = 5$$

Substitute $x = 5$ into $y = x + 2$ and solve for y: $y = x + 2$

$$y = 5 + 2$$

$$y = 7$$

The solution of the system is the ordered pair $(5, 7)$.

Check using both equations: $y = x + 2$; $7 \stackrel{?}{=} (5) + 2$; $7 = 7$✓

$2x + y = 17$; $2(5) + 7 \stackrel{?}{=} 17$; $17 = 17$✓

Use substitution to solve each system of equations.

1. $\begin{cases} y = 2x - 5 \\ 3x + y = 10 \end{cases}$

   Use $y = 2x - 5$.

   $3x +$ ________ $= 10$

   ____

   ____

   ____

   Ordered pair solution: ______________

2. $\begin{cases} 3x + 2y = 1 \\ x - y = 2 \end{cases}$

   Solve for x: $x - y = 2$.

   $x =$ ______________

   $3($________$) + 2y = 1$

   ____

   ____

   Ordered pair solution: ______________

Name ______________________________ Date __________ Class __________

LESSON 3-2

Review for Mastery

Using Algebraic Methods to Solve Linear Systems (continued)

To use the **elimination method** to solve a system of linear equations:

1. Add or subtract the equations to eliminate one variable.
2. Solve the resulting equation for the other variable.
3. Substitute the value for the known variable into one of the original equations.
4. Solve for the other variable.
5. Check the values in both equations.

Use the elimination method when the coefficients of one of the variables are the same or opposite.

$$\begin{cases} 3x + 2y = 7 \\ 5x - 2y = 1 \end{cases}$$

The y terms have opposite coefficients, so add.

$$\begin{array}{r} 3x + 2y = 7 \\ +\ 5x - 2y = 1 \\ \hline 8x \qquad = 8 \end{array}$$

Add the equations.

Solve for x.

$$x = 1$$

Substitute $x = 1$ into $3x + 2y = 7$ and solve for y:

$$\begin{aligned} 3x + 2y &= 7 \\ 3(1) + 2y &= 7 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

The solution to the system is the ordered pair $(1, 2)$.

Check using both equations:

$$\begin{array}{rr} 3x + 2y = 7 & 5x - 2y = 1 \\ 3(1) + 2(2) \stackrel{?}{=} 7 & 5(1) - 2(2) \stackrel{?}{=} 1 \\ 7 = 7 \checkmark & 1 = 1 \checkmark \end{array}$$

Use elimination to solve each system of equations.

3. $\begin{cases} 2x + y = 1 \\ -2x - 3y = 5 \end{cases}$

$$\begin{array}{r} 2x + y = 1 \\ +(-2x - 3y = 5) \\ \hline \end{array}$$

$-2y =$ ____

$y =$ ____

____

Ordered pair solution: ______________

4. $\begin{cases} 3x + 4y = 13 \\ 5x - 4y = -21 \end{cases}$

$$\begin{array}{r} 3x + 4y = 13 \\ +\ 5x - 4y = -21 \\ \hline \end{array}$$

____

$x =$ ____

____

Ordered pair solution: ______________

Name ______________________________________ Date ___________ Class ____________

LESSON **3-3**

Review for Mastery

Solving Systems of Linear Inequalities

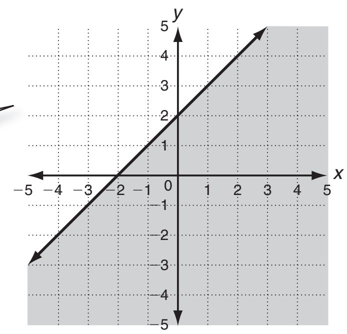
To use graphs to find the solution to a system of inequalities:

1. Draw the graph of the boundary for the first inequality. Remember to use a solid line for $\leq$ or $\geq$ and a dashed line for $<$ or $>$.
2. Shade the region above or below the boundary line that is a solution of the inequality.
3. Draw the graph of the boundary for the second inequality.
4. Shade the region above or below the boundary line that is a solution of the inequality using a different pattern.
5. The region where the shadings overlap is the solution region.

Graph $\begin{cases} y \leq x + 2 \\ x > 1 \end{cases}$ Graph $y \leq x + 2$.

Graph $y = x + 2$.

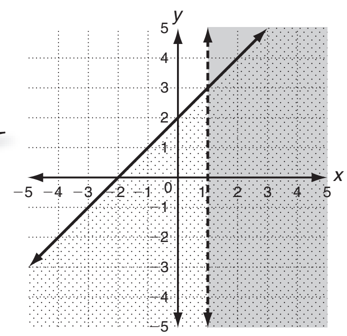
Use a solid line for the boundary.
Shade the region below the line.



On the same plane, graph $x > 1$.

Graph $x = 1$.

Use a dashed line for the boundary.
Shade the region to the right of the line.



Check: Test a point in the solution region in both inequalities.
Try $(2, 2)$.

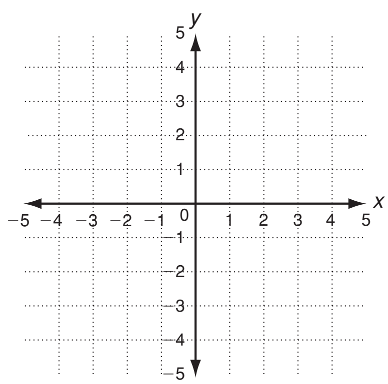
$y \leq x + 2$ $\qquad$ $x > 1$

$2 \overset{?}{\leq} 2 + 2$ $\qquad$ $2 > 1$

$2 \leq 4$

Graph the system of inequalities.

1. $\begin{cases} y > -x + 1 \\ y \leq 2 \end{cases}$



a. Shade ______________ the line for $y > -x + 1$.

b. Shade ______________ the line for $y \leq 2$.

c. Check: ___________________________________

d. Check: ___________________________________

Name ______________________________ Date ___________ Class ___________

LESSON **3-3**

Review for Mastery

Solving Systems of Linear Inequalities (continued)

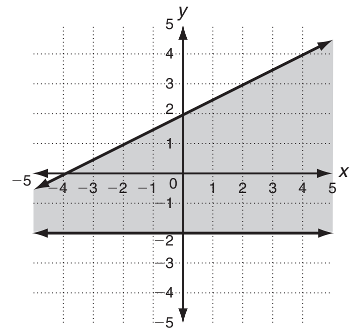
The solution of a system of inequalities may create a geometric figure.

Graph $\begin{cases} y \le \frac{1}{2}x + 2 \\ y \ge -2 \\ x \le 3 \\ x \ge -2 \end{cases}$

The graph of $y = -2$ is a horizontal line.
The graphs of $x = 3$ and $x = -2$ are vertical lines.

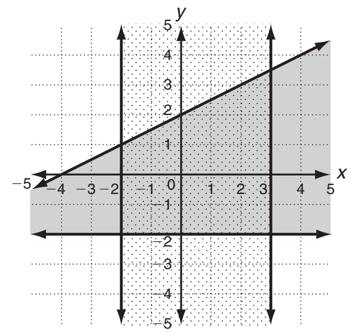
Graph $y \le \frac{1}{2}x + 2$ and $y \ge -2$.

Use solid boundary lines.
Shade the region below $y \le \frac{1}{2}x + 2$ and above $y \ge -2$.



On the same plane, graph $x \le 3$ and $x \ge -2$.

Use solid boundary lines.
Shade the region to the left of $x \le 3$ and to the right of $x \ge -2$.



The figure created by the overlapping pattern is a quadrilateral with one pair of parallel sides. The figure is a trapezoid.

Graph the system of inequalities. Classify the figure created by the solution region.

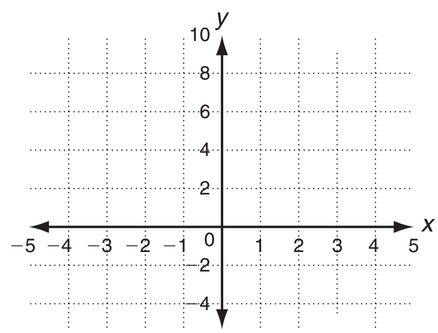
2. $\begin{cases} y \le 2x + 1 \\ y \ge -x + 1 \\ x \le 3 \end{cases}$

a. Shade ______________ the line for $y \le 2x + 1$.

b. Shade ______________ the line for $y \ge -x + 1$.

c. Shade to the ______________ of the line for $x \le 3$.

d. The figure is a ______________________.



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LESSON **3-4**

Review for Mastery

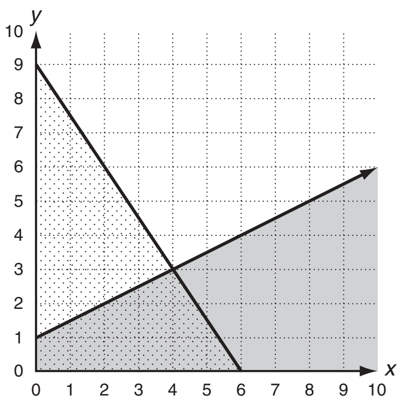
Linear Programming

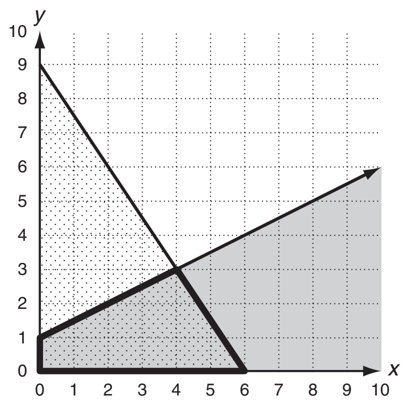
Linear programming is used to maximize or minimize a function based on conditions that have to be met. These conditions are called **constraints**. The constraints are a system of inequalities. The graph of their solution is the **feasible region**.

To graph the feasible region, graph the system of inequalities.

$$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 0.5x + 1 \\ y \le -1.5x + 9 \end{cases}$$

When $x \ge 0$ and $y \ge 0$, the graph lies in the first quadrant, so the x- and y-values must be positive.





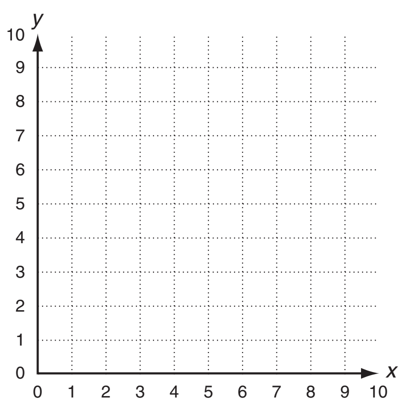
Check a point in the feasible region. Try $(2, 1)$.

$x \ge 0$	$y \ge 0$	$y \le 0.5x + 1$	$y \le -1.5x + 9$
$2 \ge 0$	$1 \ge 0$	$1 \overset{?}{\le} 0.5(2) + 1$	$1 \overset{?}{\le} -1.5(2) + 9$
		$1 \le 2$	$1 \le 6$

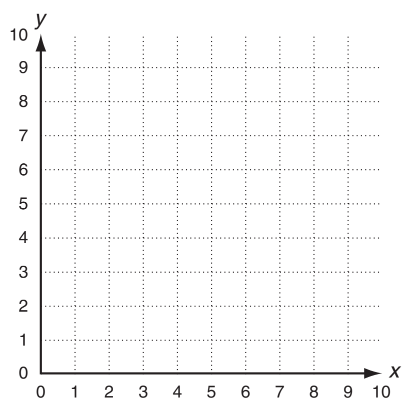
Since all of the inequalities are true, the constraints are satisfied.

Graph each feasible region.

1. $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 1.5x + 1 \\ y \le -x + 6 \end{cases}$



2. $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \ge 2x + 1 \\ y \le -2x + 9 \end{cases}$



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LESSON **3-4**

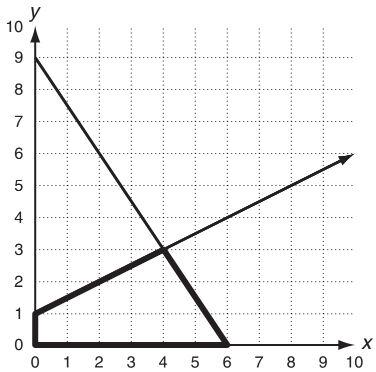
Review for Mastery

Linear Programming (continued)

The **objective function** is the best combination of values to maximize or minimize a function subject to the constraints graphed in the feasible region. The maximum or minimum occurs at one or more of the vertices of the feasible region. Evaluate the objective function for each vertex to find the maximum or minimum.

Maximize $P = 5x + 7y$ for the constraints $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 0.5x + 1 \\ y \leq -1.5x + 9 \end{cases}$

Step 1 Graph the feasible region.



Step 2 Identify the vertices.
$(0, 0), (0, 1), (4, 3), (6, 0)$

Step 3 Evaluate the objective function at each vertex. Find the maximum value.

$P = 5x + 7y$

$P(0, 0) = 5(0) + 7(0) = 0$

$P(0, 1) = 5(0) + 7(1) = 7$

$P(4, 3) = 5(4) + 7(3) = 41$ ⟵

$P(6, 0) = 5(6) + 7(0) = 30$

The objective function is maximized at $(4, 3)$.

Solve using your graphs from Exercises 1–2 on the previous page.

3. Maximize $P = 2x + 5y$ for:

$\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq 1.5x + 1 \\ y \leq -x + 6 \end{cases}$

Vertices: ____________________

$P($____, ____$)$ = ____________________

$P($____, ____$)$ = ____________________

$P($____, ____$)$ = ____________________

$P($____, ____$)$ = ____________________

Maximum value at ____________________

4. Minimize $P = 3x + 6y$ for:

$\begin{cases} x \geq 0 \\ y \geq 0 \\ y \geq 2x + 1 \\ y \leq -2x + 9 \end{cases}$

Vertices: ____________________

____

____

____

Minimum value at ____________________

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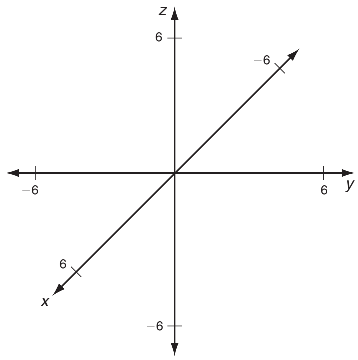
LESSON 3-5

Review for Mastery

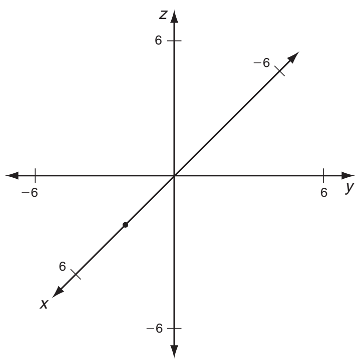
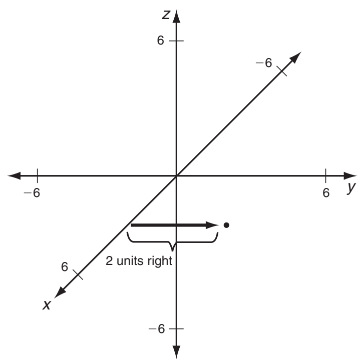
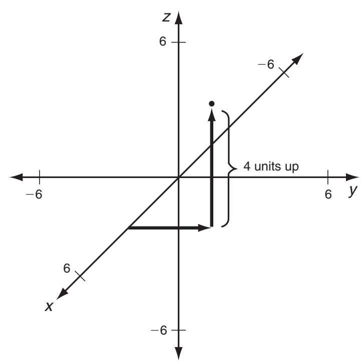
Linear Equations in Three Dimensions

In a three-dimensional coordinate system, the *x*-axis projects out from the paper and the *y*- and *z*-axes lie in the plane of the paper.



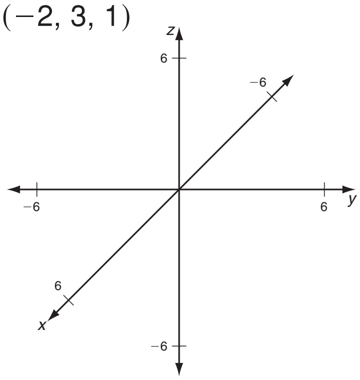
An **ordered triple** (x, y, z) is used to locate points in coordinate space. Points in three-dimensional space are graphed similarly to points graphed in two-dimensional space. First count *x* units along the projected *x*-axis, then move *y* units to the right or left, and finally move *z* units up or down.

To graph $(3, 2, 4)$, start at the origin.

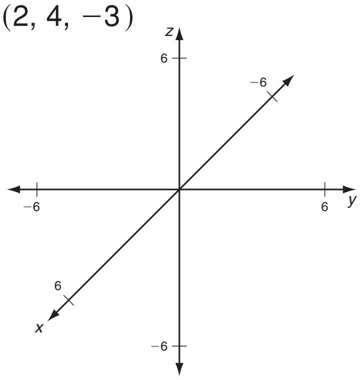
Move 3 units forward along the *x*-axis. This is the point $(3, 0, 0)$.	Move 2 units to the right. This is the point $(3, 2, 0)$.	Move 4 units up. This is the point $(3, 2, 4)$.
		

Graph each point in three-dimensional space.

1. $(-2, 3, 1)$



2. $(2, 4, -3)$



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LESSON 3-5

Review for Mastery

Linear Equations in Three Dimensions (continued)

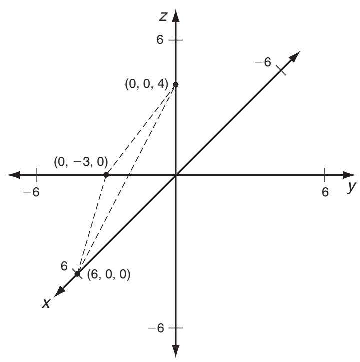
In three-dimensional space, the graph of a linear equation is a plane. You can graph the plane by finding its *x*-, *y*-, and *z*-intercepts.

Graph $2x - 4y + 3z = 12$.

Step 1 Find the intercepts.

Find the *x*-intercept.	Find the *y*-intercept.	Find the *z*-intercept.
Set $y = z = 0$.	Set $x = z = 0$.	Set $x = y = 0$.
$2x - 4(0) + 3(0) = 12$	$2(0) - 4y + 3(0) = 12$	$2(0) - 4(0) + 3z = 12$
$2x = 12$	$-4y = 12$	$3z = 12$
$x = 6$	$y = -3$	$z = 4$
The *x*-intercept is at $(6, 0, 0)$.	The *y*-intercept is at $(0, -3, 0)$.	The *z*-intercept is at $(0, 0, 4)$.

Step 2 Plot each point.
Use a dashed line to connect the points. The triangle represents the plane.



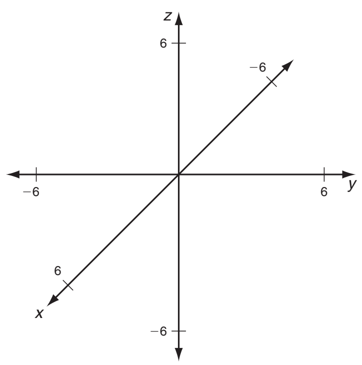
Graph each linear equation in three-dimensional space.

3. $3x + 4y + 6z = 12$

x-intercept is at $(4, \_\_\_\_, \_\_\_\_)$

y-intercept is at $(\_\_\_\_, 3, \_\_\_\_)$

z-intercept is at $(\_\_\_\_, \_\_\_\_, 2)$

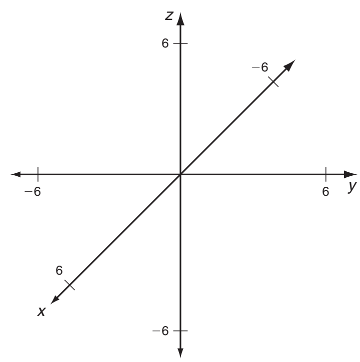


4. $2x - 2y + 5z = 10$

x-intercept is at ____________

y-intercept is at ____________

z-intercept is at ____________



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LESSON 3-6

Review for Mastery

Solving Linear Systems in Three Variables

You know how to solve a system of two linear equations in two variables using the **elimination method**. The same method can be used to solve a system of three linear equations in three variables.

$$\begin{cases} x - y + 2z = 8 \\ 2x + y - z = -2 \\ x + 2y + z = 2 \end{cases}$$

The first and second equations have opposite coefficients of y. So adding these two equations will eliminate y.

$$\begin{array}{r} x - y + 2z = 8 \\ +2x + y - z = -2 \\ \hline 3x + z = 6 \end{array}$$

Multiply the first equation by 2 and add to the third equation to eliminate y.

$$\begin{array}{r} 2x - 2y + 4z = 16 \\ + x + 2y + z = 2 \\ \hline 3x + 5z = 18 \end{array}$$

Now you have two equations in two variables. Solve using the elimination method for a system of two equations.

$$\begin{cases} 3x + z = 6 \\ 3x + 5z = 18 \end{cases}$$

Solving this system gives $x = 1$ and $z = 3$. Substituting these values in any of the original equations gives $y = -1$.

So the solution is the ordered triple $(1, -1, 3)$

Show the steps you would use to eliminate the variable *z*.

1. $\begin{cases} 2x - y + z = -3 \\ x + 2y - z = 2 \\ x + 3y - 2z = 3 \end{cases}$

a.
$$\begin{array}{r} 2x - y + z = -3 \\ + \underline{\qquad\qquad} \\ \hline \underline{\qquad\qquad} \end{array}$$

b.
$$\begin{array}{r} 2(\underline{\qquad\qquad}) = \underline{\qquad\qquad} \\ + x + 3y - 2z = 3 \\ \hline \underline{\qquad\qquad} \end{array}$$

c. Give the resulting system of two equations. ______________________________

Name ______________________ Date __________ Class __________

LESSON 3-6

Review for Mastery

Solving Linear Systems in Three Variables (continued)

Linear systems in three variables are classified by their solutions.

Exactly One Solution Independent	Infinitely Many Solutions Dependent	No Solution Inconsistent
Three planes intersect at one point.	Three planes intersect at a line.	All three planes never intersect.

Classify: $\begin{cases} x + z = 1 \\ x + y + z = 2 \\ x - y + z = 1 \end{cases}$

Add the second and third equations to eliminate *y*.

$$\begin{array}{r} x + y + z = 2 \\ + \; x - y + z = 1 \\ \hline 2x + 2z = 3 \end{array}$$

Solve: $\begin{cases} x + z = 1 \\ 2x + 2z = 3 \end{cases}$

Multiply the first equation by −2. Then add.

$$\begin{array}{r} -2x - 2z = -2 \\ +2x + 2z = 3 \\ \hline 0 = 1 \end{array}$$

Since 0 does not equal 1, the system has no solution and is inconsistent.

Classify: $\begin{cases} x + 2y + 4z = 3 \\ 4x - 2y - 6z = 2 \\ 2x - y - 3z = 1 \end{cases}$

Add the first and second equations.

$$\begin{array}{r} x + 2y + 4z = 3 \\ + \; 4x - 2y - 6z = 2 \\ \hline 5x \qquad -2z = 5 \end{array}$$

Multiply the third equation by 2. Add to the first equation.

$$\begin{array}{r} 4x - 2y - 6z = 2 \\ + \; x + 2y + 4z = 3 \\ \hline 5x \qquad -2z = 5 \end{array}$$

Now you have a system with two identical equations. $\begin{cases} 5x - 2z = 5 \\ 5x - 2z = 5 \end{cases}$

Subtracting the equations gives $0 = 0$.

The system has infinitely many solutions and is dependent.

Classify each system and determine the number of solutions.

2. $\begin{cases} x + z = 0 \\ x + y + 2z = 3 \\ y + z = 2 \end{cases}$

3. $\begin{cases} y - z = 0 \\ x - 3z = -1 \\ -x + 3y = 1 \end{cases}$

______________________ ______________________

LESSON 3-1

Review for Mastery

Using Graphs and Tables to Solve Linear Systems

A **linear system** of equations is a set of two or more linear equations. To **solve a linear system**, find all the ordered pairs (x, y) that make both equations true. Use a table and a graph to solve a system of equations.

$\begin{cases} y + x = 2 \\ y - 2x = 5 \end{cases}$ Solve each equation for y.→ $\begin{cases} y = -x + 2 \\ y = 2x + 5 \end{cases}$

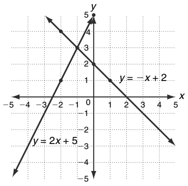
Make a table of values for each equation.

$y = -x + 2$	
x	y
-2	4
-1	3
0	2
1	1

↔

$y = 2x + 5$	
x	y
-2	1
-1	3
0	5
1	7

When $x = -1$, $y = 3$ for both equations.



On a graph, the point where the lines intersect is the solution.

Use the table to draw the graph of each equation.

The lines appear to intersect at $(-1, 3)$.

Substitute $(-1, 3)$ into the original equations to check.

$y + x = 2$ $y - 2x = 5$

$3 + (-1) \stackrel{?}{=} 2$ $3 - 2(-1) \stackrel{?}{=} 5$

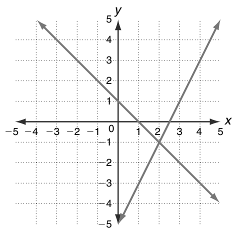
$2 = 2$✓ $5 = 5$✓

Solve the system using a table and a graph. Give the ordered pair that solves both equations.

1. $\begin{cases} x + y = 1 \\ 2x - y = 5 \end{cases}$ Solution: (2, −1)

$y = -x + 1$	
x	y
0	1
1	0
2	−1
3	−2

$y = 2x - 5$	
x	y
0	−5
1	−3
2	−1
3	1



LESSON 3-1

Review for Mastery

Using Graphs and Tables to Solve Linear Systems (continued)

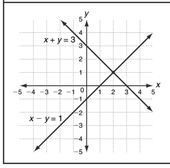
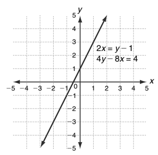
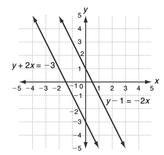
To classify a linear system:

Step 1 Write each equation in the form $y = mx + b$.

Step 2 Compare the slopes and y-intercepts.

Step 3 Classify by the number of solutions of the system.

Remember: m = slope and b = y-intercept.

Exactly One Solution **Independent**	**Infinitely Many Solutions** **Dependent**	**No Solution** **Inconsistent**
The lines have **different slopes** and intersect at one point.	The lines have the **same slope and y-intercept.** Their graph is the same line.	The lines have the **same slope and different y-intercepts.** The lines are parallel.
$\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$ Solve each equation for y. $\begin{cases} y = -x + 3;\ m = -1 \\ y = x - 1;\ m = 1 \end{cases}$ The slopes are different. The system has one solution and is independent.	$\begin{cases} 2x = y - 1 \\ 4y - 8x = 4 \end{cases}$ Solve each equation for y. $\begin{cases} y = 2x + 1;\ m = 2,\ b = 1 \\ y = 2x + 1;\ m = 2,\ b = 1 \end{cases}$ The slopes and the y-intercepts are the same. The system has infinitely many solutions and is dependent.	$\begin{cases} y + 2x = -3 \\ y - 1 = -2x \end{cases}$ Solve each equation for y. $\begin{cases} y = -2x - 3;\ m = -2,\ b = -3 \\ y = -2x + 1;\ m = -2,\ b = 1 \end{cases}$ The slopes are the same but the y-intercepts are different. The system has no solution and is inconsistent.
		

Classify each system and determine the number of solutions.

2. $\begin{cases} y + x = 2 \\ y + 1 = -x \end{cases}$

$y =$ −x + 2, $m =$ −1, $b =$ 2

$y =$ −x −1, $m =$ −1, $b =$ −1

Number of solutions: none

inconsistent

3. $\begin{cases} y + 1 = 3x \\ 2y - 6x = -2 \end{cases}$

$y =$ 3x − 1, $m =$ 3, $b =$ −1

$y =$ 3x − 1, $m =$ 3, $b =$ −1

Number of solutions: infinitely many

dependent

LESSON 3-2

Review for Mastery

Using Algebraic Methods to Solve Linear Systems

To use the **substitution method** to solve a system of linear equations:

1. Solve one equation for one variable.
2. Substitute this expression into the other equation.
3. Solve for the other variable.
4. Substitute the value of the known variable in the equation in Step 1.
5. Solve for the other variable.
6. Check the values in both equations.

$\begin{cases} y = x + 2 \\ 2x + y = 17 \end{cases}$

Use this equation. It is solved for y.

Use the substitution method when the coefficient of one of the variables is 1 or −1.

$2x + y = 17$

$2x + (x + 2) = 17$ *Substitute $x + 2$ for y.*

$3x + 2 = 17$ *Simplify and solve for x.*

$3x = 15$

$x = 5$

Substitute $x = 5$ into $y = x + 2$ and solve for y: $y = x + 2$

$y = 5 + 2$

$y = 7$

The solution of the system is the ordered pair $(5, 7)$.

Check using both equations: $y = x + 2$; $7 \stackrel{?}{=} (5) + 2$; $7 = 7$✓

$2x + y = 17$; $2(5) + 7 \stackrel{?}{=} 17$; $17 = 17$✓

Use substitution to solve each system of equations.

1. $\begin{cases} y = 2x - 5 \\ 3x + y = 10 \end{cases}$

Use $y = 2x - 5$.

$3x +$ 2x − 5 $= 10$

5x − 5 = 10

x = 3

y = 2(3) − 5 = 1

Ordered pair solution: (3, 1)

2. $\begin{cases} 3x + 2y = 1 \\ x - y = 2 \end{cases}$

Solve for x: $x - y = 2$.

$x =$ y + 2

$3($ y + 2 $) + 2y = 1$

y = −1

x = −1 + 2 = 1

Ordered pair solution: (1, −1)

LESSON 3-2

Review for Mastery

Using Algebraic Methods to Solve Linear Systems (continued)

To use the **elimination method** to solve a system of linear equations:

1. Add or subtract the equations to eliminate one variable.
2. Solve the resulting equation for the other variable.
3. Substitute the value for the known variable into one of the original equations.
4. Solve for the other variable.
5. Check the values in both equations.

$\begin{cases} 3x + 2y = 7 \\ 5x - 2y = 1 \end{cases}$

The y terms have opposite coefficients, so add.

Use the elimination method when the coefficients of one of the variables are the same or opposite.

$3x + 2y = 7$

$+\ 5x - 2y = 1$ *Add the equations.*

$8x = 8$ *Solve for x.*

$x = 1$

Substitute $x = 1$ into $3x + 2y = 7$ and solve for y: $3x + 2y = 7$

$3(1) + 2y = 7$

$2y = 4$

$y = 2$

The solution to the system is the ordered pair $(1, 2)$.

Check using both equations:

$3x + 2y = 7$ $5x - 2y = 1$

$3(1) + 2(2) \stackrel{?}{=} 7$ $5(1) - 2(2) \stackrel{?}{=} 1$

$7 = 7$✓ $1 = 1$✓

Use elimination to solve each system of equations.

3. $\begin{cases} 2x + y = 1 \\ -2x - 3y = 5 \end{cases}$

$2x + y = 1$

$+(-2x - 3y = 5)$

$-2y =$ 6

$y =$ −3

x = 2

Ordered pair solution: (2, −3)

4. $\begin{cases} 3x + 4y = 13 \\ 5x - 4y = -21 \end{cases}$

$3x + 4y = 13$

$+\ 5x - 4y = -21$

8x = −8

$x =$ −1

y = 4

Ordered pair solution: (−1, 4)

LESSON **3-3**

Review for Mastery

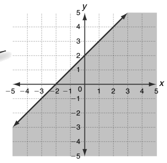
Solving Systems of Linear Inequalities

To use graphs to find the solution to a system of inequalities:

1. Draw the graph of the boundary for the first inequality. Remember to use a solid line for $\le$ or $\ge$ and a dashed line for $<$ or $>$.
2. Shade the region above or below the boundary line that is a solution of the inequality.
3. Draw the graph of the boundary for the second inequality.
4. Shade the region above or below the boundary line that is a solution of the inequality using a different pattern.
5. The region where the shadings overlap is the solution region.

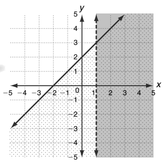
Graph $\begin{cases} y \le x + 2 \\ x > 1 \end{cases}$ Graph $y \le x + 2$.

Graph $y = x + 2$.
Use a solid line for the boundary.
Shade the region below the line.



On the same plane, graph $x > 1$.

Graph $x = 1$.
Use a dashed line for the boundary.
Shade the region to the right of the line.



Check: Test a point in the solution region in both inequalities.
Try $(2, 2)$.

$y \le x + 2$	$x > 1$
$2 \overset{?}{\le} 2 + 2$	$2 > 1$
$2 \le 4$	

Graph the system of inequalities.

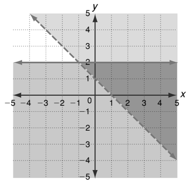
1. $\begin{cases} y > -x + 1 \\ y \le 2 \end{cases}$

a. Shade Above the line for $y > -x + 1$.

b. Shade below the line for $y \le 2$.

c. Check: possible answer: (1, 3)

d. Check: possible answer: (4, 0)



LESSON **3-3**

Review for Mastery

Solving Systems of Linear Inequalities (continued)

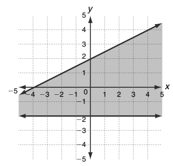
The solution of a system of inequalities may create a geometric figure.

Graph $\begin{cases} y \le \frac{1}{2}x + 2 \\ y \ge -2 \\ x \le 3 \\ x \ge -2 \end{cases}$

The graph of $y = -2$ is a horizontal line.
The graphs of $x = 3$ and $x = -2$ are vertical lines.

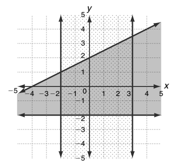
Graph $y \le \frac{1}{2}x + 2$ and $y \ge -2$.

Use solid boundary lines.
Shade the region below $y \le \frac{1}{2}x + 2$ and above $y \ge -2$.



On the same plane, graph $x \le 3$ and $x \ge -2$.

Use solid boundary lines.
Shade the region to the left of $x \le 3$ and to the right of $x \ge -2$.



The figure created by the overlapping pattern is a quadrilateral with one pair of parallel sides. The figure is a trapezoid.

Graph the system of inequalities. Classify the figure created by the solution region.

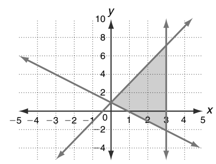
2. $\begin{cases} y \le 2x + 1 \\ y \ge -x + 1 \\ x \le 3 \end{cases}$

a. Shade Below the line for $y \le 2x + 1$.

b. Shade above the line for $y \ge -x + 1$.

c. Shade to the left of the line for $x \le 3$.

d. The figure is a triangle.



LESSON **3-4**

Review for Mastery

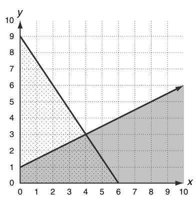
Linear Programming

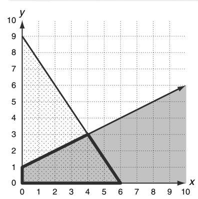
Linear programming is used to maximize or minimize a function based on conditions that have to be met. These conditions are called **constraints**. The constraints are a system of inequalities. The graph of their solution is the **feasible region**.

To graph the feasible region, graph the system of inequalities.

$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 0.5x + 1 \\ y \le -1.5x + 9 \end{cases}$

When $x \ge 0$ and $y \ge 0$, the graph lies in the first quadrant, so the x- and y-values must be positive.





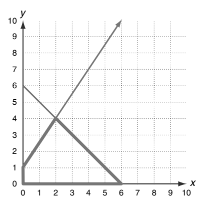
Check a point in the feasible region. Try $(2, 1)$.

$x \ge 0$	$y \ge 0$	$y \le 0.5x + 1$	$y \le -1.5x + 9$
$2 \ge 0$	$1 \ge 0$	$1 \overset{?}{\le} 0.5(2) + 1$	$1 \overset{?}{\le} -1.5(2) + 9$
		$1 \le 2$	$1 \le 6$

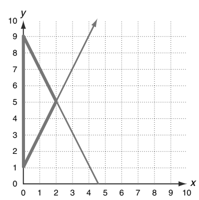
Since all of the inequalities are true, the constraints are satisfied.

Graph each feasible region.

1. $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 1.5x + 1 \\ y \le -x + 6 \end{cases}$



2. $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \ge 2x + 1 \\ y \le -2x + 9 \end{cases}$



LESSON **3-4**

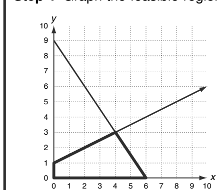
Review for Mastery

Linear Programming (continued)

The **objective function** is the best combination of values to maximize or minimize a function subject to the constraints graphed in the feasible region. The maximum or minimum occurs at one or more of the vertices of the feasible region. Evaluate the objective function for each vertex to find the maximum or minimum.

Maximize $P = 5x + 7y$ for the constraints $\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 0.5x + 1 \\ y \le -1.5x + 9 \end{cases}$

Step 1 Graph the feasible region.



The objective function is maximized at $(4, 3)$.

Step 2 Identify the vertices.
$(0, 0), (0, 1), (4, 3), (6, 0)$

Step 3 Evaluate the objective function at each vertex. Find the maximum value.

$P = 5x + 7y$

$P(0, 0) = 5(0) + 7(0) = 0$

$P(0, 1) = 5(0) + 7(1) = 7$

$P(4, 3) = 5(4) + 7(3) = 41 \leftarrow$

$P(6, 0) = 5(6) + 7(0) = 30$

Solve using your graphs from Exercises 1–2 on the previous page.

3. Maximize $P = 2x + 5y$ for:
$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \le 1.5x + 1 \\ y \le -x + 6 \end{cases}$

Vertices: (0, 0), (0, 1), (2, 4), (6, 0)

$P($ 0 , 0 $) =$ 0

$P($ 0 , 1 $) =$ 5

$P($ 2 , 4 $) =$ 24

$P($ 6 , 0 $) =$ 12

Maximum value at max at (2, 4)

4. Minimize $P = 3x + 6y$ for:
$\begin{cases} x \ge 0 \\ y \ge 0 \\ y \ge 2x + 1 \\ y \le -2x + 9 \end{cases}$

Vertices: (0, 1), (0, 9), (2, 5)

$P(0, 1) = 6$

$P(0, 9) = 54$

$P(2, 5) = 36$

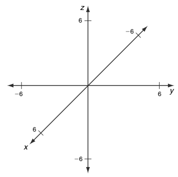
Minimum value at min at (0, 1)

LESSON 3-5

Review for Mastery

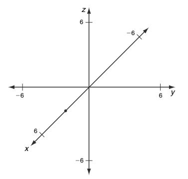
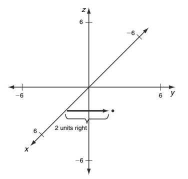
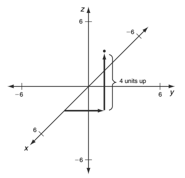
Linear Equations in Three Dimensions

In a three-dimensional coordinate system, the *x*-axis projects out from the paper and the *y*- and *z*-axes lie in the plane of the paper.



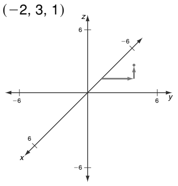
An **ordered triple** (x, y, z) is used to locate points in coordinate space. Points in three-dimensional space are graphed similarly to points graphed in two-dimensional space. First count *x* units along the projected *x*-axis, then move *y* units to the right or left, and finally move *z* units up or down.

To graph $(3, 2, 4)$, start at the origin.

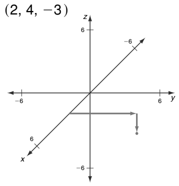
Move 3 units forward along the *x*-axis. This is the point $(3, 0, 0)$.	Move 2 units to the right. This is the point $(3, 2, 0)$.	Move 4 units up. This is the point $(3, 2, 4)$.
		

Graph each point in three-dimensional space.

1\. $(-2, 3, 1)$



2\. $(2, 4, -3)$



LESSON 3-5

Review for Mastery

Linear Equations in Three Dimensions (continued)

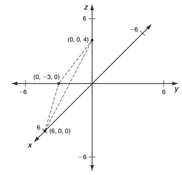
In three-dimensional space, the graph of a linear equation is a plane. You can graph the plane by finding its *x*-, *y*-, and *z*-intercepts.

Graph $2x - 4y + 3z = 12$.

Step 1 Find the intercepts.

Find the *x*-intercept.	Find the *y*-intercept.	Find the *z*-intercept.
Set $y = z = 0$.	Set $x = z = 0$.	Set $x = y = 0$.
$2x - 4(0) + 3(0) = 12$	$2(0) - 4y + 3(0) = 12$	$2(0) - 4(0) + 3z = 12$
$2x = 12$	$-4y = 12$	$3z = 12$
$x = 6$	$y = -3$	$z = 4$
The *x*-intercept is at $(6, 0, 0)$.	The *y*-intercept is at $(0, -3, 0)$.	The *z*-intercept is at $(0, 0, 4)$.

Step 2 Plot each point.
Use a dashed line to connect the points. The triangle represents the plane.



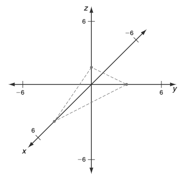
Graph each linear equation in three-dimensional space.

3\. $3x + 4y + 6z = 12$

x-intercept is at $(4, \underline{0}, \underline{0})$

y-intercept is at $(\underline{0}, 3, \underline{0})$

z-intercept is at $(\underline{0}, \underline{0}, 2)$

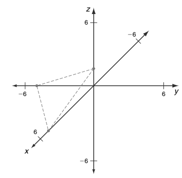


4\. $2x - 2y + 5z = 10$

x-intercept is at $\underline{(5, 0, 0)}$

y-intercept is at $\underline{(0, -5, 0)}$

z-intercept is at $\underline{(0, 0, 2)}$



LESSON 3-6

Review for Mastery

Solving Linear Systems in Three Variables

You know how to solve a system of two linear equations in two variables using the **elimination method**. The same method can be used to solve a system of three linear equations in three variables.

$$\begin{cases} x - y + 2z = 8 \\ 2x + y - z = -2 \\ x + 2y + z = 2 \end{cases}$$

The first and second equations have opposite coefficients of *y*. So adding these two equations will eliminate *y*.

$$\begin{array}{r} x - y + 2z = 8 \\ +2x + y - z = -2 \\ \hline 3x + z = 6 \end{array}$$

Multiply the first equation by 2 and add to the third equation to eliminate *y*.

$$\begin{array}{r} 2x - 2y + 4z = 16 \\ +\ x + 2y + z = 2 \\ \hline 3x + 5z = 18 \end{array}$$

Now you have two equations in two variables. Solve using the elimination method for a system of two equations.

$$\begin{cases} 3x + z = 6 \\ 3x + 5z = 18 \end{cases}$$

Solving this system gives $x = 1$ and $z = 3$. Substituting these values in any of the original equations gives $y = -1$.

So the solution is the ordered triple $(1, -1, 3)$

Show the steps you would use to eliminate the variable *z*.

1\. $\begin{cases} 2x - y + z = -3 \\ x + 2y - z = 2 \\ x + 3y - 2z = 3 \end{cases}$

a\.
$$\begin{array}{r} 2x - y + z = -3 \\ +\ x + 2y - z = 2 \\ \hline 3x + y = -1 \end{array}$$

b\.
$$2\,(2x - y + z = -3) = \begin{array}{r} 4x - 2y + 2z = -6 \\ +\ x + 3y - 2z = 3 \\ \hline 5x + y = -3 \end{array}$$

c\. Give the resulting system of two equations. $\begin{cases} 3x + y = -1 \\ 5x + y = -3 \end{cases}$

LESSON 3-6

Review for Mastery

Solving Linear Systems in Three Variables (continued)

Linear systems in three variables are classified by their solutions.

Exactly One Solution Independent	Infinitely Many Solutions Dependent	No Solution Inconsistent
Three planes intersect at one point.	Three planes intersect at a line.	All three planes never intersect.

Classify: $\begin{cases} x + z = 1 \\ x + y + z = 2 \\ x - y + z = 1 \end{cases}$

Add the second and third equations to eliminate *y*.

$$\begin{array}{r} x + y + z = 2 \\ +\ x - y + z = 1 \\ \hline 2x + 2z = 3 \end{array}$$

Solve: $\begin{cases} x + z = 1 \\ 2x + 2z = 3 \end{cases}$

Multiply the first equation by -2. Then add.

$$\begin{array}{r} -2x - 2z = -2 \\ +2x + 2z = 3 \\ \hline 0 = 1 \end{array}$$

X

Since 0 does not equal 1, the system has no solution and is inconsistent.

Classify: $\begin{cases} x + 2y + 4z = 3 \\ 4x - 2y - 6z = 2 \\ 2x - y - 3z = 1 \end{cases}$

Add the first and second equations.

$$\begin{array}{r} x + 2y + 4z = 3 \\ +\ 4x - 2y - 6z = 2 \\ \hline 5x \quad -2z = 5 \end{array}$$

Multiply the third equation by 2. Add to the first equation.

$$\begin{array}{r} 4x - 2y - 6z = 2 \\ +\ x + 2y + 4z = 3 \\ \hline 5x \quad -2z = 5 \end{array}$$

Now you have a system with two identical equations. $\begin{cases} 5x - 2z = 5 \\ 5x - 2z = 5 \end{cases}$

Subtracting the equations gives $0 = 0$.

The system has infinitely many solutions and is dependent.

Classify each system and determine the number of solutions.

2\. $\begin{cases} x + z = 0 \\ x + y + 2z = 3 \\ y + z = 2 \end{cases}$

No solution; inconsistent

3\. $\begin{cases} y - z = 0 \\ x - 3z = -1 \\ -x + 3y = 1 \end{cases}$

Infinitely many solutions; dependent