

AP Statistics – Probability Models Review

Review Chapters 16 – 17

1. Each Spring, students in the fictitious realm of Podunk take the Comic Design* AP Exam. Scores on this exam range from a 1 (lowest) to a 5 (highest), with most colleges and universities granting college credit for an introductory Comic Design course for a score of at least a 3 or 4. The following probability model summarizes the results for the 2012 Comic Design AP Exam:

**before you ask: NO – there is no “Comic Design” AP class.*

score (x)	1	2	3	4	5
P(X = x)	0.214	0.197	0.241	0.224	0.124

Using the probability model, answer the following questions:

- What is the probability that a randomly selected student earned a score of at least a 3?
 - If eight students are selected randomly, what is the probability that all of them scored at least a 3?
 - If five students are randomly selected, what is the probability that at least one of them earned a score of at least a 3?
 - Find the expected value and standard deviation of the global score on the Comic Design AP Exam.
2. In the last problem, we established that the probability that a randomly selected student from this group earned a score of a 3 or above was 0.589. Use that probability to answer the following questions.
- In a random sample of 8 students, what is the probability that exactly 4 of them score at least a 3?
 - In a random sample of 30 students, what is the probability that fewer than half of them score a 3 or above?
 - In a random sample of 30 students, what is the probability that at least 20 of them score a 3 or above?
 - In a random sample of 30 students, what is the probability that at least 10 of them score a 3 or above?
 - On average, how many students would you expect to have to randomly sample from this group until you found one that scored a 3 or above on this exam?
3. In the last problem, we established that the probability that a randomly selected student from this group earned a score of a 3 or above was 0.589. Suppose that a very large number of students took the Comic Design AP Exam, such that the probability of score a 3 or above does not change if we select more than one student.
- If we take a random sample of 150 students to see how many of them scored a 3 or above, would this meet the requirements of a Binomial model? (HINT: WHAT ARE THE REQUIREMENTS FOR BERNOULLI TRIALS?)
 - In a random sample of 150 students who took the AP Comic Design Exam, what are the mean and standard deviation for the expected number of students who score a 3 or above?
 - Suppose we are interested in finding the probability that fewer than 100 of the 150 randomly selected students would score a 3 or above. First show that it is appropriate to use a Normal model approximation to find this probability. Then use the mean and standard deviation calculated in part (b) to find this probability by using the Normal model.
4. The amount of money that a local donut stand earns from a typical afternoon customer has a mean of \$6.07 and a standard deviation of \$1.13.
- If the owner of the stand expects 80 customers during a particular afternoon hour, what are the mean and standard deviation for the total amount of money earned during that hour?
 - The owner wants to buy a new espresso machine that costs \$500 to better serve their customers, and is hoping to make enough money for it in the next hour. The amount of money earned from 80 customers during a typical afternoon hour is normally distributed. Based on the mean and standard deviation calculated in part (a), what is the probability that the donut stand will make at least \$500 during a particular afternoon hour?

5. You are preparing to take two math tests in the next week. You estimate that the probability that you pass the first test is 0.55. If you pass the first test, the probability you also pass the second is 0.7. If you fail the first test, the probability that you pass the second is 0.4. **Complete the probability model in the table below for the number of tests that you pass.** (HINT: YOU MAY WISH TO USE A TREE DIAGRAM TO AID YOUR THINKING)

x = # of tests passed	0	1	2
P(x)			

6. Assume the heights of high school basketball players are normally distributed. For boys the mean height is 74 inches with a standard deviation of 4.5 inches, while girl players have a mean height of 70 inches and standard deviation 3 inches. At a mixed 2-on-2 tournament teams are formed by randomly pairing boys with girls as teammates. (A boy-girl team competes against another boy-girl team.)
- On average, how much taller do you expect the boy to be on a boy-girl team?
 - What condition must be true in order to find the standard deviation of the difference in teammates' heights?
 - Since we are using random pairing to match the boys and girls, we will assume that the condition from part (b) is true. What will be the standard deviation of the difference in teammates' heights?
 - What is the probability that on a randomly selected team, the girl is at least 2 inches taller than her male teammate? (in other words, what is the probability that the difference "G – B" is at least 2 inches?)
7. X and Y are two independent random variables with the following attributes:

$$E(X) = 19 \quad SD(X) = 9 \quad E(Y) = 24 \quad SD(Y) = 5$$

Find the mean and standard deviation of each of these random variables:

- $3X$
 - $Y_1 + Y_2 + Y_3$
 - $X + 4Y$
8. On a physical fitness test, middle school boys are awarded one point for each push-up they can do and a point for each sit-up. National results showed that boys average 18 pushups with a standard deviation of 4 push-ups and 34 sit-ups with standard deviation of 11 sit-ups. The mean of their combined (total) scores was therefore 52 points. What is the standard deviation of their combined scores?
- A) 5.3 B) 11.7 C) 15 D) 137 E) It cannot be determined

9. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs.

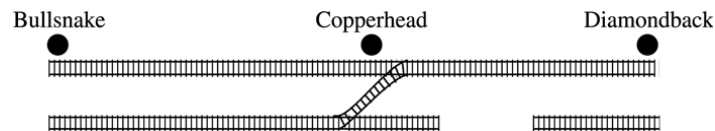
The weights of the empty cardboard containers, C , have a mean of 20 grams and a standard deviation of 1.7 grams.

The weights of the individual Grade A eggs, E , have a mean of 68.3 grams and a standard deviation of 2.23 grams.

It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

Let the random variable X represent the weight (in grams) of a full carton of Grade A eggs (empty cardboard container **plus** 12 randomly selected eggs).

- a) What is the mean of X ?
- b) What is the standard deviation of X ?
10. Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one usable track from Copperhead to Diamondback, as shown in the figure below. Having only one usable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X , it takes the train leaving Bullsnake to get to Copperhead is approximately normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y , it takes the train leaving Diamondback to get to Copperhead is approximately normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

- a) What is the distribution of $Y - X$?
Hint: this means you must describe shape, center (mean), and spread (standard deviation) of " $Y - X$ "
- b) Over the long run, what proportion of the days will the train from Bullsnake have to wait at Copperhead for the train from Diamondback to arrive?
Hint: this means that $Y > X$... or $Y - X > 0$. Use the normal model and the mean & standard deviation from (a) to calculate this.
- c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsnake so that the probability that it has to wait is only 0.01?
Hint: This one is meant to be a challenge.